

ESTIMATING CONFIDENCE INTERVALS FOR ZERO-INFLATED RAYLEIGH DISTRIBUTIONS

MR. SASIPONG KIJSASON

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN
APPLIED STATISTICS
DEPARTMENT OF APPLIED STATISTICS
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KING MONGKUT'S UNIVERSITY OF TECHNOLOGY NORTH BANGKOK
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ABSTRACT

The zero-inflated two-parameter Rayleigh (ZITR) distribution is a mixture distribution that combines right-skewed continuous data with excess zero observations, where the occurrence of zeros follows a binomial distribution. This thesis aims to investigate parameter estimation for the ZITR distribution, with particular interest in the means, variances, and coefficients of variation (CV). Furthermore, the study extends the investigation to the difference and ratio of the mean, variance, and CV between two independent populations. For interval estimation of the parameters, several confidence interval methods were considered, including Bayesian, bootstrap, generalized confidence interval (GCI), and approximate normal (AN) methods. The performance of these methods was evaluated in terms of coverage probability (CP) and expected length (EL) through Monte Carlo simulation. A satisfactory method was expected to achieve a CP close to or greater than the nominal confidence level of 0.95 while providing the shortest expected length. In addition, real-world data were used to demonstrate the practical applicability of the proposed confidence interval methods. The applications included road traffic fatality data and road traffic fatality rates, illustrating the usefulness of the ZITR distribution for modeling data characterized by right skewness and excess zero observations.

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บทคัดย่อ

การแจกแจงเรย์ลีแบบสองพารามิเตอร์ที่มีค่าศูนย์ เป็นการแจกแจงแบบผสมที่รวมข้อมูลต่อเนื่องซึ่งมีความเบ้ขวาเข้ากับข้อมูลที่มีจำนวนค่าศูนย์มากผิดปกติ โดยการเกิดค่าศูนย์มีการแจกแจงแบบทวินาม วิทยานิพนธ์นี้มีวัตถุประสงค์เพื่อศึกษาการประมาณค่าพารามิเตอร์ของการแจกแจง โดยมุ่งเน้นเป็นพิเศษที่ค่าเฉลี่ย ความแปรปรวน และสัมประสิทธิ์การแปรผัน นอกจากนี้ ยังขยายการศึกษาไปสู่การประมาณค่าผลต่างและอัตราส่วนของค่าเฉลี่ย ความแปรปรวน และสัมประสิทธิ์การแปรผันระหว่างประชากรอิสระสองประชากร สำหรับการประมาณค่าแบบช่วงของพารามิเตอร์ ได้พิจารณาวิธีการสร้างช่วงความเชื่อมั่นหลายวิธี ได้แก่ วิธีแบบเบย์ วิธีบูตสแตรป วิธีช่วงความเชื่อมั่นทั่วไป และวิธีการประมาณแบบปกติ ประสิทธิภาพของวิธีการดังกล่าวได้รับการประเมินโดยพิจารณาจากความน่าจะเป็นครอบคลุม และความยาวคาดหมาย ด้วยการจำลองแบบมอนติคาร์โล โดยวิธีที่มีประสิทธิภาพเป็นที่น่าพอใจควรให้ค่า ความน่าจะเป็นครอบคลุม ใกล้เคียงหรือไม่น้อยกว่าระดับความเชื่อมั่นที่กำหนดไว้ที่ 0.95 และมีค่าความยาวคาดหมายสั้นที่สุด

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LIST OF ABBREVIATIONS

ZITR	Zero-inflated two-parameter Rayleigh distribution
TR	Two-parameter Rayleigh distribution
CP	Coverage probability
EL	Expected length
MLE	Maximum likelihood estimation
PB	Percentile bootstrap
BS	Bootstrap with standard error
GCI	Generalized confidence interval
VST	Variance stabilized transformation
STD	Standard confidence interval based on MLE
MOVER	Method of variance recovery estimates
AN	Approximate normal
MCMC	Bayesian Markov chain Monte carlo
HPD	Bayesian Highest posterior density
AIC	Akaike's information criterion
BIC	Bayesian information criterion

LIST OF SYMBOLS

ψ	Mean
ξ	Variance
κ	Coefficient of variation
$\theta_{(\cdot)}$	Difference parameter of interest
$\tau_{(\cdot)}$	Ratio parameter of interest
δ	Probability of zero
λ	Scale parameter
μ	Location parameter

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Chapter 1
AN OVERVIEW

1.1 Introduction

Parameter estimation for statistical distributions plays a crucial role in both theoretical development and practical applications, where a high level of accuracy is required. In general, estimation involves using data collected from a sample to infer the value of population parameters. Estimation can be classified into two main types: point estimation and interval estimation. Point estimation provides a single numerical value as an estimate of the parameter, which may lead to uncertainty or potential estimation error. In contrast, interval estimation expresses the parameter in terms of a range of plausible values. This range is constructed based on a point estimator together with the sampling distribution of that estimator. As a result, interval estimation enables researchers to assess the degree of confidence that the estimated interval contains the true parameter of interest. Confidence intervals, therefore, offer more informative insight into population parameters than point estimates alone. Commonly studied parameters include the mean, variance, and coefficient of variation, among others. These parameters are widely applied across various fields such as insurance, accident analysis, and lifetime data analysis, where understanding variability and central tendency is essential for effective decision-making and modeling.

The Rayleigh distribution (Rayleigh, 1880) is a right-skewed distribution defined on the non-negative real line and has been widely applied in fields such as reliability engineering, quality control, medical research, and lifetime data analysis. In its original form, it was introduced as a single-parameter model characterized by a scale parameter. Later developments led to the two-parameter Rayleigh distribution, in which an additional location parameter was incorporated. The inclusion of this parameter improves the flexibility of the model, enabling it to better represent the characteristics of real-world data. This extension was first reported by Johnson et al. (1994).

Numerous studies have demonstrated the applicability of the Rayleigh distribution and its extensions to real-world data. For instance, Arıkan (2015) applied the Rayleigh distribution to assess the wind energy potential of Elmadag, Turkey, using hourly wind speed data collected from 2012 to 2013 and evaluated the economic feasibility of wind power investment. Fundi et al. (2016) analyzed the survival times of chemotherapy patients using the two-parameter Rayleigh distribution. Seo et al. (2019) applied the two-parameter Rayleigh distribution to carbon fiber strength data, whereas Liu et al. (2020) employed it to model lifetime data from experiments involving small electrical appliances. Moreover, Almongy et al. (2021) developed a Rayleigh-based statistical model for analyzing COVID-19 mortality rates in Italy, Mexico, and the Netherlands by introducing an extended lifetime distribution derived from the Rayleigh distribution. These studies highlight the versatility of the Rayleigh distribution and its extensions in modeling positive-valued data arising from diverse application domains. Similarly, Kilai (2022) employed a Rayleigh-based flexible lifetime distribution to model COVID-19 mortality rates in Italy and Canada. These studies highlight the versatility of the Rayleigh distribution and its extensions in modeling positive-valued data arising from diverse application domains. More recently, Fuxiang et al. (2023) proposed a novel zero-inflated Rayleigh distribution and investigated its statistical properties, extending the Rayleigh framework to accommodate data with an excess of zero

observations. These studies highlight the versatility of the Rayleigh distribution and its extensions in modeling positive-valued data and data containing excess zeros arising from diverse application domains.

Parameter estimation for the two-parameter Rayleigh distribution presents certain challenges, as the maximum likelihood estimators do not have closed-form solutions and must be obtained through numerical procedures. Consequently, a range of estimation methods has been proposed in the literature. For example, Khan et al. (2010) investigated inferential procedures for this model. Dey et al. (2014) provided a detailed comparison of several estimation techniques, including maximum likelihood estimation, the method of moments, L-moments, percentile-based estimators, and least squares estimation, as well as Bayesian approaches based on a gamma prior for the scale parameter and a uniform prior for the location parameter. In a subsequent study, Krishnamoorthy et al. (2019) developed confidence intervals for important quantities such as the mean, quantiles, survival probabilities, and prediction intervals using a pivotal approach. More recently, Seal B. et al. (2023) examined Bayesian estimation under a distance-based loss function within the Rayleigh framework.

In this study, attention is focused on the mean, variance, and coefficients of variation as parameters of primary interest. The mean is widely used as a measure of central tendency and is commonly employed for analyzing and comparing multiple quantitative datasets. A substantial body of literature has been devoted to the development of confidence intervals for the mean.

Variance, on the other hand, serves as a measure of dispersion around the central value and provides important information regarding the level of variability or risk in the data. It is also useful for comparing the degree of risk across different datasets.

Accordingly, this study investigates methods for constructing confidence intervals for the mean, as well as for the difference and ratio of means, under the zero-inflated two-parameter Rayleigh distribution (ZITR). In addition, for the variance, confidence intervals are developed for the variance itself, along with the difference and ratio of variances under the same distributional framework. The details of the thesis are organized as follow

In Chapter 2, methods for constructing confidence intervals for the mean, the difference of means, and the ratio of means of the ZITR distributions are presented. For single-mean parameter estimation, the proposed methods include the percentile bootstrap (PB), generalized confidence interval (GCI), standard confidence interval based on the maximum likelihood estimator (MLE), approximate normal (AN), Bayesian Markov chain Monte Carlo (MCMC), and Bayesian highest posterior density (HPD) interval methods. For the difference and ratio of means, the considered methods include PB, bootstrap with standard error (BS), AN, GCI, method of variance estimates recovery (MOVER), MCMC, and HPD interval methods.

In Chapter 3, confidence intervals for the variance, difference of variances, and ratio of variances of the ZITR distribution are constructed. For single-variance parameter estimation, the proposed methods include MCMC, HPD, STD, AN, PB, BS, and GCI. For the difference and ratio of variances, the proposed methods include MCMC, HPD, GCI, MOVER, AN, PB, and BS.

In Chapter 4 presents the concepts of confidence intervals of coefficient of variation, difference of coefficient of variations, and ratio of coefficients of variation for ZITR distribution. For single coefficient proposed Bayesian MCMC, Bayesian HPD

and AN comparing, PB, BS, and GCI. For the difference and ratio coefficient of variation, the proposed methods include MCMC, HPD, GCI, MOVER, AN, PB, and BS.

1.2 Definition

The definitions introduced in this section are employed consistently throughout this thesis.

1.2.1 Confidence intervals are defined by Casella and Berger (2002)

Consider a random sample $X=(X_1,X_2,\ldots,X_n)$ of size n drawn from a distribution with probability density function $f(x;\theta)$, where θ is an unknown parameters. Let $x=(x_1,x_2,\ldots,x_n)$ denote the observed value of X. A pair of statistics $L(X)$ and $U(X)$ defines a confidence interval for θ if the probability that the interval contains the true parameter value satisfies

$$P\big(\theta\in[L(X),U(X)]\big)=1-\alpha,\quad 0<\alpha<1.$$

Accordingly, the interval $[L(X),U(X)]$ is referred to as a $(1-\alpha)100\%$ confidence interval for θ.

1.2.2 Coverage probability

Consider $[L(X),U(X)]$ be interval estimate of the parameter θ. The coverage probability estimates obtained by

$$CP=\frac{\#\big(L(x)\le\theta\le U(x)\big)}{M},$$

where $\#\big(L(x)\le\theta\le U(x)\big)$ denotes the number of simulation replications for which the true parameter θ is contained within the interval $[L(X),U(X)]$, and M denotes the total number of simulation replications.

1.2.3 Expected length

Let $[L(X),U(X)]$ denote an interval estimator for the parameter θ. The length of the interval is defined as the difference between its upper and lower bounds, given by $l(X)=U(X)-L(X)$. The expected length of the confidence interval is defined as the expectation of $l(X)$, which can be expressed as $E[l(X)]=E\big[U(X)-L(X)\big]$. Accordingly, the estimated expected length is obtained as follows:

$$EL=\sum_{i=1}^{M}\frac{U_i(X)-L_i(X)}{M}$$

where $L_i(X)$ and $U_i(X)$ denote the lower and upper bounds of the confidence interval obtained from the i-th simulation replication, respectively.

1.3 Review of distribution

In this thesis, the zero-inflated two-parameter Rayleigh distribution is considered. Let $X=(X_1,X_2,\ldots,X_n)$ be a random sample of size n drawn from zero-inflated two-parameter Rayleigh distribution with μ is the location parameter, λ is the scale parameter and the probability of zero defined δ, denoted as $ZITR(\mu,\lambda,\delta)$. The probability density function (PDF) of X given by

$$f(x;\mu,\lambda,\delta)=\begin{cases}\delta & ;x=0\\ (1-\delta)2\lambda(x-\mu)e^{-\lambda(x-\mu)^2} & ;x>\mu.\end{cases} \tag{1-1}$$

Parameter estimation for the zero-inflated two-parameter Rayleigh distribution can be carried out using the maximum likelihood estimation (MLE) approach. The maximum likelihood estimator of the zero-inflation parameter is $\hat{\delta}=\frac{n_0}{n}$. However, this procedure is not straightforward, as the resulting estimators do not admit closed-form expressions and must be determined numerically. In particular, the estimator of the location parameter is obtained as the root of the following equation:

$$\frac{2n_1^2(\bar{x}-\mu)}{\sum_{i=1}^{n_1}(x_i-\mu)^2}-\sum_{i=1}^{n_1}(x_i-\mu)^{-1}=0.$$

Similarly, the maximum likelihood estimators of λ are defined by

$$\hat{\lambda}=\frac{n_1}{\sum_{i=1}^{n_1}(x_i-\hat{\mu})^2}.$$

The population mean (ψ), variance (ξ) and the coefficient of variation (κ) for X are

$$E(X)=\psi=(1-\delta)\left(\mu+\frac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right), \tag{1-2}$$

and

$$V(X)=\xi=\frac{4(1-\delta)-(1-\delta)^2\pi}{4\lambda}+\frac{\mu}{\sqrt{\lambda}}\Gamma\left(\frac{1}{2}\right)\delta(1-\delta)+\mu^2\delta(1-\delta), \tag{1-3}$$

and

$$CV(X)=\kappa=\frac{\sqrt{\xi}}{\psi}=\frac{\sqrt{\frac{4(1-\delta)-(1-\delta)^2\pi}{4\lambda}+\frac{\mu}{\sqrt{\lambda}}\Gamma\left(\frac{1}{2}\right)\delta(1-\delta)+\mu^2\delta(1-\delta)}}{(1-\delta)\left(\mu+\frac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right)} \quad (1\text{-}4)$$

respectively.

1.4 Data Processing and Management

The R statistical software Venables and Smith (2009) is employed, together with Monte Carlo simulation techniques, to generate simulated data and to compute both the coverage probability and the expected length. The corresponding R code is provided in Appendix A.

1.5 List of Publications and Presentations

1.5.1 Publications

1.5.1.1 Kijsason, S., Niwitpong, S. A., & Niwitpong, S. (2025). Confidence intervals for the parameter mean of zero-inflated two-parameter Rayleigh distribution. *Symmetry*, *17*(7), 1019.

1.5.1.2 Kijsason, S., Niwitpong, S. A., & Niwitpong, S. (2026). Confidence Intervals for the Difference and Ratio Means of Zero-Inflated Two-Parameter Rayleigh Distribution. *Symmetry*, *18*(1), 109.

1.5.1.3 Kijsason, S., Niwitpong, S. A., & Niwitpong, S. (2026). Interval Estimation for the Difference and Ratio of Variances Under the Zero-Inflated Two-Parameter Rayleigh Distribution. *Mathematics*, *14*(9), 1440.

1.5.1.4 Kijsason, S., Niwitpong, S. A., & Niwitpong, S. (2026). Confidence Intervals for Parameter Variance of Zero-Inflated Two-Parameter Rayleigh Distribution. *Symmetry*, *18*(5), 765.

1.5.1.5 Kijsason, S., Niwitpong, S. A., & Niwitpong, S. (2026). Bayesian Estimation of the Difference and Ratio of the Coefficients of Variation Under the Zero-Inflated Two-Parameter Rayleigh Distribution. *Symmetry*, *18*(8), 1257.

1.5.1.6 Kijsason, S., Niwitpong, S. A., & Niwitpong, S. (2026). Bayesian Estimation for the Single Coefficient of Variation of Zero-Inflated Two-Parameter Rayleigh Distribution. *Mathematics*, *14*(17), 3056.

1.5.2 Presentations

1.5.2.1 Kijsason, S., Niwitpong, S. A., & Niwitpong, S. (2025, September). Statistical Computing for Variance Zero-Inflated Two-Parameter Rayleigh Distribution. In *World Conference on Information Systems for Business Management* (pp. 388-400). Cham: Springer Nature Switzerland.

1.5.2.2 Kijsason, S., & Niwitpong, S. (2026, June). Confidence interval for the difference between variances of the zero-inflated two-parameter Rayleigh distribution. In *Journal of Physics: Conference Series* (Vol. 3268, No. 1, p. 012009). IOP Publishing.

Chapter 2
CONFIDENCE INTERVALS FOR MEANS OF ZERO-INFLATED TWO-PARAMETER RAYLEIGH DISTRIBUTIONS

This chapter presents methods for constructing confidence intervals for the single mean, the difference of means, and the ratio of means of the ZITR distribution. For single-mean parameter estimation, the proposed methods include the percentile bootstrap (PB), generalized confidence interval (GCI), standard confidence interval based on the maximum likelihood estimator (STD), approximate normal (AN), Bayesian Markov chain Monte Carlo (MCMC), and Bayesian highest posterior density (HPD) interval methods. For the difference and ratio of means of the ZITR distribution, the proposed methods include PB, bootstrap with standard error (BS), AN, GCI, method of variance estimates recovery (MOVER), MCMC, and HPD interval methods. A simulation study is conducted to evaluate the performance of the proposed confidence intervals in terms of coverage probability (CP) and expected length (EL). In addition, the proposed methods for single-mean confidence interval estimation are illustrated using COVID-19 mortality data, while the analyses of the difference and ratio of means are conducted using road accident data from Southern Thailand during the Songkran festival in 2024 and 2025.

2.1 Introduction

The mean is an important parameter in statistical data analysis because it represents the central tendency and reflects the overall characteristics of a population. Owing to its fundamental role, accurate statistical inference for the mean is essential in many scientific and practical applications. The mean has been widely used in diverse fields, including the measurement of nitrogen-bound bovine serum albumin in mice (Hand et al., 1993; Schaarschmidt, 2013; Sadooghi-Alvandi & Malekzadeh, 2014), the determination of selenium levels in non-fat milk powder (Philip et al., 1999), the assessment of cell counts in HIV patients following antiretroviral therapy (Liang et al., 2008), and the analysis of rainfall patterns in Chiang Mai, Thailand (Maneerat et al., 2019).

In statistical inference, interval estimation plays an important role because it provides a range of plausible values for unknown parameters together with a measure of uncertainty. Consequently, the construction of confidence intervals for the mean has attracted considerable attention in statistical research. Confidence interval methods for the means of various distributions have been extensively investigated (Chen and Mi, 1996; Colosimo & Ho, 1999; Peng, 2004; Krishnamoorthy et al., 2009; Thangjai et al., 2020; Maneerat et al., 2022). Moreover, functions of the mean, such as the difference between two means and the ratio of two means, have also been widely studied (Lee & Lin, 2004; Niwitpong & Niwitpong, 2010; Niwitpong et al., 2012; Thangjai et al., 2017; Maneerat & Niwitpong, 2020). For example, Bebu and Mathew (2008) proposed confidence intervals for the means of the lognormal distribution with applications in the medical field.

Beyond methodological developments for the mean, statistical inference has been widely applied in medicine, industry, and risk management. For example, Xu et al. (2025) proposed a novel class of tail-weighted multivariate degradation models and demonstrated their applications using two real degradation datasets. In another study, Xu et al. (2025) investigated adaptive sampling and ensemble techniques for estimating

the remaining useful life of machinery, contributing to improved operational efficiency and cost-effective maintenance. Li et al. (2025) introduced a Bayesian hierarchical model for public health applications and validated the model using data from mouse brain research and human pancreatic ductal adenocarcinoma studies. Furthermore, Ma et al. (2025) developed a new statistical inference algorithm for estimating parameters in regression models, with applications to chemical sensor data. Collectively, these studies highlight the crucial role of accurate parameter estimation and statistical inference in improving model reliability, increasing predictive performance, and supporting efficient decision-making across diverse real-world applications.

In statistical inference, interval estimation plays an important role because it provides a range of plausible values for unknown parameters together with a measure of uncertainty. Consequently, the construction of confidence intervals for population parameters, particularly the mean, has attracted considerable attention in statistical research. Several studies have supported this perspective. For example, Bebu and Mathew (2008) proposed confidence intervals for the means of the lognormal distribution with applications in the medical field.

The construction of confidence intervals for the mean, the difference between means, or the ratio of means under the Zero-Inflated Two-Parameter Rayleigh (ZITR) distribution has not been the subject of any research to date. Consequently, this chapter delineates techniques for establishing confidence intervals for these parameters. The proposed methods for estimating the single mean parameter of the ZITR distribution encompass the PB, GCI, STD, AN, MCMC, and HPD interval techniques. The proposed methods for assessing the difference between means and the ratio of means of the ZITR distribution include PB, BS, AN, GCI, MOVER, MCMC, and HPD interval methods.

The actual application of the suggested techniques for the single mean parameter was shown using COVID-19 mortality data, while the approaches for the difference between means and the ratio of means were used using road traffic fatality data.

2.2 The confidence intervals for the mean of ZITR distribution

Let $X = (X_1, X_2, \ldots, X_n)$ denote a random sample sized n that are independently drawn from a ZITR, denoted by $ZITR(\mu, \lambda, \delta)$. The probability density function of this distribution is delineated as follows:

$$f\left(x;\mu,\lambda,\delta\right)=\begin{cases}\delta\ ;x=0\\ \left(1-\delta\right)\ 2\lambda(x\text{-}\mu)\mathrm{e}^{-\lambda(x-\mu)^2}\ ;x>\mu.\end{cases} \tag{2-1}$$

where λ, which is greater than zero, represents the scale parameter of the Rayleigh distribution, and μ is the location parameter. The parameter δ determines the likelihood of zeros. The expectation of the distribution is denoted by

$$\psi =(1-\delta)\left(\mu+\frac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right), \tag{2-2}$$

where $\Gamma(\cdot)$ is gamma function. The following section presents the method for constructing confidence intervals for ψ

2.2.1 The Percentile Bootstrap Confidence Interval (PB)

The concept was first introduced by Efron and Tibshirani (1993), where the bootstrap confidence interval relies on percentile-based replication of the target

distribution. The mean can be estimated using Equation (2-2). For each bootstrap sample, the parameters of the ZITR distribution are estimated using the maximum likelihood method, and the corresponding estimate of the mean is then obtained using Equation (2-2). Repeating this procedure for b bootstrap samples produces the bootstrap distribution of the estimated mean. A two-sided percentile bootstrap confidence interval for the mean of the ZITR distribution can then be constructed as shown in Equation (2-3):

$$CI_{\psi(PB)} = \left(L_{\psi(PB)}, U_{\psi(PB)}\right) = \left(\hat{\psi}^{*b}\left(\alpha/2\right), \hat{\psi}^{*b}\left(1-\alpha/2\right)\right). \quad (2\text{-}3)$$

where $\hat{\psi}^{*b}(\alpha)$ represent the $100(\alpha)\%$ percentile of $\hat{\psi}^{*b}$

2.2.2 Generalized confidence interval (GCI)

Weerahandi (1993) introduced the initial concept of the generalized confidence interval (GCI), which is based on the generalized pivotal quantity (GPQ). The pivotal quantity and its generalized form (GPQ) are essentially different concepts that are built upon this foundation. A GPQ incorporates extraneous parameters into its formulation, whereas a pivotal quantity is constructed based on the sampling distribution and the parameters of interest. Nevertheless, the observed value of the GPQ is not influenced by these extraneous parameters. It is crucial to note that the GPQ method is applicable to inference in complex models because it entails quantities with distributions that are devoid of uncertain parameters. Krishnamoorthy et al. (2020) proposed pivotal quantities that were particularly designed for the unknown parameters of the two-parameter Rayleigh distribution. The parameters are represented as $a = \mu$ and $\lambda = \frac{1}{2b^2}$. Let $\hat{a}_0$ and $\hat{b}_0$ denote the observed values of $\hat{a}$ and $\hat{b}$, respectively. The pivotal quantities for the parameters are as follows: $\frac{\hat{a}-a}{b} \sim \hat{a}^*$, $\frac{\hat{b}}{b} \sim \hat{b}^*$, where $\hat{a}$ and $\hat{b}$ are the maximum likelihood estimates from the two-parameter Rayleigh distribution with a = 0 and b = 1, respectively, based on a sample of size n. The distribution of $Q_a = \hat{a} - \hat{b}\frac{\hat{a}^*}{\hat{b}^*}$, and $Q_b = \frac{\hat{b}}{\hat{b}^*}$. Consequently, the pivotal quantities for μ and λ are defined as $Q_a = Q_\mu$, and $Q_\lambda = \frac{1}{2Q_b{}^2}$. Furthermore, Wu and Hsieh (2014) construct the pivotal quantities based on VST to construct intervals, which is

$$Q_\delta = \sin^2\left(\arcsin\sqrt{\hat{\delta}} - \frac{T}{2\sqrt{n}}\right), \quad (2\text{-}4)$$

where $T = 2\sqrt{n}\left(\arcsin\sqrt{\hat{\delta}} - \arcsin\sqrt{\delta}\right) \sim N(0,1)$. Hence, considering the three pivotal Q_a, Q_λ, and Q_δ, which are independent of the parameter values, GPQ for the mean of the zero-inflated Rayleigh distribution is given by

$$Q_{\psi} = (1-Q_{\delta})\left(Q_{\mu} + \frac{1}{\sqrt{Q_{\lambda}}}\Gamma\left(\frac{3}{2}\right)\right). \quad (2\text{-}5)$$

Consequently, Equation (2-6) provides the GCI for the single mean of the zero-inflated Rayleigh distribution:

$$CI_{\psi(GCI)} = \left(L_{\psi(GCI)}, U_{\psi(GCI)}\right) = \left(Q_{\psi}(\alpha/2), Q_{\psi}(1-\alpha/2)\right), \quad (2\text{-}6)$$

where $Q_{\psi}(\alpha/2)$ represent the $100(\alpha/2)\%$ percentile of Q_{ψ}

2.2.3 Standard confidence interval based on Maximum likelihood (STD)

The standard confidence interval is a fundamental approach to estimating the confidence intervals of parameters. Dey et al. (2014) were the first to propose the method for determining the parameters of the two-parameter Rayleigh distribution.

Let $x = (x_1, \ldots x_n)$ denote the observation that was drawn independently with a sample size of n. The approximate confidence intervals of δ, λ, and μ are as follows:

$$CI_{\delta(MLE)} = (L_{\delta}, U_{\delta}) = \left(\hat{\delta} - z_{1-\alpha/2}\sqrt{\frac{\hat{\delta}(1-\hat{\delta})}{n}}, \hat{\delta} + z_{1-\alpha/2}\sqrt{\frac{\hat{\delta}(1-\hat{\delta})}{n}}\right), \quad (2\text{-}7)$$

$$CI_{\lambda(MLE)} = (L_{\lambda}, U_{\lambda}) = \left(\hat{\lambda} - z_{1-\alpha/2}\frac{\hat{\lambda}}{\sqrt{n}}, \hat{\lambda} + z_{1-\alpha/2}\frac{\hat{\lambda}}{\sqrt{n}}\right), \quad (2\text{-}8)$$

and

$$CI_{\mu(MLE)} = (L_{\mu}, U_{\mu}) = \left(\hat{\mu} - z_{1-\alpha/2}\left(\frac{1}{2\hat{\mu} + \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu})^{-2}}\right)^{\frac{1}{2}}, \hat{\mu} + z_{1-\alpha/2}\left(\frac{1}{2\hat{\mu} + \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu})^{-2}}\right)^{\frac{1}{2}}\right), \quad (2\text{-}9)$$

where $\hat{\delta}, \hat{\lambda}$, and $\hat{\mu}$ are the maximum likelihood estimators from random samples, and the standard confidence interval for the single mean, based on the maximum likelihood estimators, is constructed using Equation (2-10):

$$CI_{\psi(STD)} = \left(L_{\psi(STD)}, U_{\psi(STD)}\right) = \left((1-L_{\delta})\left(L_{\mu} + \frac{1}{\sqrt{L_{\lambda}}}\Gamma\left(\frac{3}{2}\right)\right), (1-U_{\delta})\left(U_{\mu} + \frac{1}{\sqrt{U_{\lambda}}}\Gamma\left(\frac{3}{2}\right)\right)\right). \quad (2\text{-}10)$$

2.2.4 Approximation normal (AN)

The Delta method is a technique that is employed within the normal approximation framework to acquire a normal distribution as the limiting distribution of an estimator. The following is a concise summary of this prevalent methodology: Assume that g is a differentiable scalar function with parameters of interest, denoted as $g(u_1, u_2, u_3)$. The objective is to construct a stochastic representation in order to identify the asymptotic distribution of the estimator $G(U_1, U_2, U_3)$. The asymptotic mean and variance of ψ are calculated by applying the delta method to confirm the

asymptotic normality of G as the sample size approaches infinity. Function g is expanded into Taylor series at $\theta_1,\theta_2,\theta_3$, the true values corresponding to U_1,U_2,U_3.

$$g(U_1,U_2,U_3)\approx g(\theta_1,\theta_2,\theta_3)+\frac{\partial g(\theta_1,\theta_2,\theta_3)}{\partial u_1}(U_1-\theta_1)+\frac{\partial g(\theta_1,\theta_2,\theta_3)}{\partial u_2}(U_2-\theta_2)+\frac{\partial g(\theta_1,\theta_2,\theta_3)}{\partial u_3}(U_3-\theta_3)$$

The basic statistics are assumed here as $U_1=\hat{\delta},U_2=\hat{\mu}$ and $U_3=\hat{\lambda}$, where $\hat{\delta},\hat{\mu},\hat{\lambda}$ are the MLE of δ,μ,λ respectively. Then, $\hat{\psi}=g\left(\hat{\delta},\hat{\mu},\hat{\lambda}\right)$, where the function is $\psi=(1-\delta)\left(\mu+\frac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right)$. The partial derivatives are as follows:

$$\frac{\partial}{\partial\delta}g(\delta,\lambda,\mu)=(-1)\left(\mu+\frac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right),$$

$$\frac{\partial}{\partial\mu}g(\delta,\lambda,\mu)=(1-\delta),$$

$$\frac{\partial}{\partial\lambda}g(\delta,\lambda,\mu)=(1-\delta)\left(-\frac{1}{2}\lambda^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right).$$

Consequently, the expression for $\hat{\psi}$ is obtained by expanding the function $g(\delta,\mu,\lambda)$ around the true parameter values using the Taylor series method. This approximation linearizes the function with respect to the parameter estimates $\hat{\delta},\hat{\lambda}$, and $\hat{\mu}$ given by

$$\hat{\psi}\approx g(\delta,\mu,\lambda)+\frac{\partial}{\partial\delta}g(\delta,\lambda,\mu)\left(\hat{\delta}-\delta\right)+\frac{\partial}{\partial\mu}g(\delta,\lambda,\mu)(\hat{\mu}-\mu)+\frac{\partial}{\partial\lambda}g(\delta,\lambda,\mu)\left(\hat{\lambda}-\lambda\right)$$

.

The approximate mean and variance for can be derived by considering the expectation and variance given by

$$E(\hat{\psi})=E\begin{pmatrix}\psi+(-1)\left(\mu+\frac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right)\left(\hat{\delta}-\delta\right)+(1-\delta)(\hat{\mu}-\mu)\\+(1-\delta)\left(-\frac{1}{2}\lambda^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\left(\hat{\lambda}-\lambda\right)\end{pmatrix},$$

$$=\begin{pmatrix}E(\psi)+(-1)\left(\mu+\frac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right)E\left(\hat{\delta}-\delta\right)+(1-\delta)E(\hat{\mu}-\mu)\\+(1-\delta)\left(-\frac{1}{2}\lambda^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)E\left(\hat{\lambda}-\lambda\right)\end{pmatrix},$$

$$=\psi,$$

$$V(\hat{\psi}) \approx V\begin{pmatrix} \psi + (-1)\left(\mu + \frac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right)(\hat{\delta} - \delta) \\ + (1-\delta)(\hat{\mu} - \mu) + (1-\delta)\left(-\frac{1}{2}\lambda^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)(\hat{\lambda} - \lambda) \end{pmatrix}$$

$$= (-1)^2\left(\mu + \frac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right)^2 V(\hat{\delta} - \delta)$$

$$+ (1-\delta)^2 V(\hat{\mu} - \mu) + (1-\delta)^2\left(-\frac{1}{2}\lambda^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)^2 V(\hat{\lambda} - \lambda).$$

Thus, as $n \to \infty$, $\hat{\psi}$ is approximately normally distributed with mean $E(\hat{\psi})$ and variance $V(\hat{\psi})$. The estimations of the variances $V(\hat{\lambda} - \lambda)$ and $V(\hat{\mu} - \mu)$ for the two-parameter Rayleigh distribution were presented in the study by Dey et al. (2014), and the results are given as follows: $V(\hat{\lambda} - \lambda) = \frac{\lambda^2}{n}$ and $V(\hat{\mu} - \mu) \approx \frac{1}{2\mu + \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^{-2}}$.

We use plug-in estimators for $E(\hat{\psi})$ and $V(\hat{\psi})$ as follows:

$$V(\hat{\psi}) = \left(\mu + \frac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right)^2 \frac{\delta(1-\delta)}{n} + (1-\delta)^2 \frac{1}{2\mu + \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^{-2}} + (1-\delta)^2\left(-\frac{1}{2}\lambda^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)^2 \frac{\lambda^2}{n}.$$

The estimate of $V(\hat{\psi})$ is as follows:

$$\hat{V}(\hat{\psi}) = \left(\hat{\mu} + \frac{1}{\sqrt{\hat{\lambda}}}\Gamma\left(\frac{3}{2}\right)\right)^2 \frac{\hat{\delta}(1-\hat{\delta})}{n} + (1-\hat{\delta})^2 \frac{1}{2\mu + \frac{1}{n_{(1)}}\sum_{i=1}^{n_{(1)}}(x_i - \hat{\mu})^{-2}} + (1-\hat{\delta})^2\left(-\frac{1}{2}\hat{\lambda}^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)^2 \frac{\hat{\lambda}^2}{n_{(1)}}. \quad (2\text{-}11)$$

where x_i represents positive observation from the two-parameter Rayleigh distribution, and let $n_{(1)}$ denotes the number of observations with values greater than zero.

Therefore, the confidence interval for the approximate normal is based on the delta method for the single mean constructed by Equation (2-12):

$$CI_{\psi(AN)} = (L_{\psi(AN)}, U_{\psi(AN)}) = \left(\hat{\psi} - z_{1-\alpha/2}\sqrt{\hat{V}(\hat{\psi})}, \hat{\psi} + z_{1-\alpha/2}\sqrt{\hat{V}(\hat{\psi})}\right), \quad (2\text{-}12)$$

where $\hat{\psi} = (1-\hat{\delta})\left(\hat{\mu} + \frac{1}{\sqrt{\hat{\lambda}}}\Gamma\left(\frac{3}{2}\right)\right)$, with $\hat{\delta}, \hat{\lambda}$, and $\hat{\mu}$ being the maximum likelihood estimators. The variance of $\hat{\psi}$ denoted by $\hat{V}(\hat{\psi})$ is estimated using the delta method and computed according to Equation (2-11).

2.2.5 Bayesian credible intervals

The Bayesian method, or Bayesian parameter estimation, is an approach to estimating parameters by combining observed data with a prior distribution. This results in the posterior probability density function based on the given dataset obtained from random sampling. The posterior density function is given by

$$p(\lambda,\mu\,|\,x)\propto \ell(\lambda,\mu\,|\,x)\,p(\lambda)\,p(\mu),$$

where $\ell(\lambda,\mu\,|\,x)$ represents the likelihood function based on the observation. The prior distribution for the scale parameter is represented by $p(\lambda)$, $p(\mu)$ denotes the prior distribution for the location parameter, and $p(\lambda,\mu\,|\,x)$ is the posterior distribution function. Since the posterior distributions of the Rayleigh distribution cannot be computed analytically, we use Gibbs sampling to estimate the posterior distribution and approximate the unknown parameters. In this research, we propose two methods for Bayesian credible intervals: the MCMC and HPD interval.

2.2.5.1 Bayesian MCMC

Assume that an independent observation from a zero-inflated Rayleigh distribution is represented as $x=(x_1,x_2,\ldots,x_n)$. The posterior distribution and the parameters of interest were defined using Markov chain Monte Carlo simulations. The location parameter μ was modeled using a uniform prior, the scale parameter λ was modeled using a gamma distribution, and the prior for δ was modeled using a beta distribution. Informative priors were used to increase the accuracy of parameter estimation by lowering the variability of the posterior mean and to integrate prior knowledge into the study. Reliable parameter inference is also possible with this method, even with insufficient observational data. The Bayesian estimates were calculated using R's R2OpenBUGS package.

Consequently, Equation (2-2) is used to determine the single mean, and Equation (2-13) provides the estimate of the credible intervals for the single mean of the zero-inflated Rayleigh distribution based on Bayesian MCMC for the two-sided case:

$$CI_{\psi(MCMC)}=\left(L_{\psi(MCMC)},U_{\psi(MCMC)}\right)=\left(\psi_{MCMC}\left(\alpha/2\right),\psi_{MCMC}\left(1-\alpha/2\right)\right), \qquad (2\text{-}13)$$

where $\psi_{MCMC}(\alpha/2)$ represent the $100(\alpha/2)\%$ percentile of ψ_{MCMC}

2.2.5.2 Bayesian HPD

Every value inside the given range is more likely than every point outside it, according to the Bayesian HPD interval. Additionally, it is the shortest interval (Box and Tiao, 1973). The HDInterval package in R is used in this study to compute the Bayesian HPD interval. Consequently, Equation (2-2) is used to determine the single mean, and Equation (2-14) provides the two-sided credible intervals for the single mean of the zero-inflated two-parameter Rayleigh distribution based on the Bayesian HPD interval:

$$CI_{\psi(HPD)}=\left(L_{\psi(HPD)},U_{\psi(HPD)}\right). \qquad (2\text{-}14)$$

where $L_{\psi(HPD)}$ and $U_{\psi(HPD)}$ represent the lower and upper bounds of ψ_{HPD} credible interval

2.3 The confidence intervals for difference and ratio means of ZITR distribution

This section analyzes confidence intervals for the difference and ratio of two means, using the principles of PB, BS, AN, GCI, MOVER, MCMC, and HPD interval techniques. Let $X_i = (X_{i1}, \ldots, X_{in_i})$, where $i = 1, 2$, represent two independent random samples extracted from the zero-inflated two-parameter Rayleigh distribution, denoted as $ZITR(\mu_i, \lambda_i, \delta_i)$. In this context, μ_i, λ_i, and δ_i denote the location parameter, the scale parameter, and the probability of zero, respectively. The mean of $x_i = (x_{i1}, \ldots, x_{in_i})$ is expressed as shown in equation (2-15).

$$\psi_i = (1-\delta_i)\left(\mu_i + \frac{1}{\sqrt{\lambda_i}}\Gamma\left(\frac{3}{2}\right)\right). \tag{2-15}$$

The difference between means of $ZITR$ distribution defined as

$$\theta_{mean} = \psi_1 - \psi_2 = (1-\delta_1)\left(\mu_1 + \frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right) - (1-\delta_2)\left(\mu_2 + \frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right). \tag{2-16}$$

The ratio of two means for $ZITR$ distribution defined as

$$\tau_{mean} = \frac{\psi_1}{\psi_2} = \frac{(1-\delta_1)\left(\mu_1 + \frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)}{(1-\delta_2)\left(\mu_2 + \frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}. \tag{2-17}$$

The method for constructing confidence intervals for θ_{mean} and τ_{mean} is delineated in the subsequent section.

2.3.1 Percentile Bootstrap (PB)

The confidence intervals are constructed by resampling from the empirical distribution in the percentile bootstrap approach. In this method, the sampling distribution of the statistics of interest is estimated by drawing repeated samples with replacement from the original data. For each bootstrap replication, the parameters of the ZITR distribution are estimated using the maximum likelihood method for each sample, and the corresponding means are calculated using their analytical expressions. The bootstrap estimates of the difference and ratio of the means are then obtained from these estimated means. Repeating this procedure for b bootstrap replications produces the bootstrap distributions of the difference and ratio of the means. The percentile bootstrap procedure is distribution-free and does not rely on assumptions regarding the underlying distribution. Additionally, it is particularly well-suited for asymmetrical data or distributions, as it can inherently reflect asymmetry in the sampling distribution. Furthermore, the methodology is straightforward to implement and has been extensively employed in the literature.

The corresponding analytical expressions can be used to derive the mean of the zero-inflated two-parameter Rayleigh distribution. The two-sided percentile bootstrap confidence intervals for both the difference and the ratio of the means are provided by

$$CI_{\theta_{mean}(PB)} = \left(L_{\theta_{mean}(PB)}, U_{\theta_{mean}(PB)}\right) = \left(\hat{\theta}^{*b}_{mean}(\alpha/2), \hat{\theta}^{*b}_{mean}(1-\alpha/2)\right), \tag{2-18}$$

and

$$CI_{\tau_{mean}(PB)} = \left(L_{\tau_{mean}(PB)}, U_{\tau_{mean}(PB)}\right) = \left(\hat{\tau}^{*b}_{mean}\left(\alpha/2\right), \hat{\tau}^{*b}_{mean}\left(1-\alpha/2\right)\right), \tag{2-19}$$

where $\hat{\theta}^{*b}_{mean}\left(\frac{\alpha}{2}\right)$ and $\hat{\tau}^{*b}_{mean}\left(\frac{\alpha}{2}\right)$ represent the $100(\alpha/2)\%$ percentile of $\hat{\theta}_{mean}$ and $\hat{\tau}_{mean}$, respectively.

2.3.2 Bootstrap method with standard error (BS)

The bootstrap method with standard error is conceptually straightforward and differs from classical confidence interval approaches in that it does not necessitate an explicit analytical derivation of the variance for the parameter of interest. The method generates dependable standard error measures by continually resampling from the observed data, thereby providing empirical estimates of the estimator's variability. This method is particularly well-suited for situations in which there are limited data sources or small sample sizes (Efron and Tibshirani, 1993).

Let $\hat{\theta}^{*b}$ and $\hat{\tau}^{*b}$ denote the corresponding bootstrap estimates for $b = 1, 2, \ldots, B$, where B is the number of bootstrap replications. For each bootstrap sample, the MLEs of the model parameters are obtained, and the corresponding estimates of the difference and ratio of the means are then calculated. The sample standard deviations of the resulting bootstrap distributions are used to estimate the standard errors of the respective statistics. The two-sided $100(1-\alpha)\%$ confidence intervals for the difference and ratio of the means of the zero-inflated two-parameter Rayleigh distribution, derived using the bootstrap method with standard error estimation, can be expressed as

$$CI_{\theta_{mean}(BS)} = \left(L_{\theta_{mean}(BS)}, U_{\theta_{mean}(BS)}\right) = \left(\bar{\theta}^{*}_{mean} - Z_{\alpha/2}SE\left(\hat{\theta}_{mean}\right), \bar{\theta}^{*}_{mean} + Z_{\alpha/2}SE\left(\hat{\theta}_{mean}\right)\right), \tag{2-20}$$

and

$$CI_{\tau_{mean}(BS)} = \left(L_{\tau_{mean}(BS)}, U_{\tau_{mean}(BS)}\right) = \left(\bar{\tau}^{*}_{mean} - Z_{\alpha/2}SE\left(\hat{\tau}_{mean}\right), \bar{\tau}^{*}_{mean} + Z_{\alpha/2}SE\left(\hat{\tau}_{mean}\right)\right) \tag{2-21}$$

where $SE\left(\hat{\theta}_{mean}\right) = \sum_{i=1}^{b} \frac{\left(\hat{\theta}^{*b}_{mean} - \bar{\theta}^{*}_{mean}\right)}{b-1}$ and $SE\left(\hat{\tau}_{mean}\right) = \sum_{i=1}^{b} \frac{\left(\hat{\tau}^{*b}_{mean} - \bar{\tau}^{*}_{mean}\right)}{b-1}$ be the standard error of θ_{mean} and τ_{mean}.

2.3.3 The generalized confidence interval (GCI)

The GPQ for ψ_i can be defined using for random variable x_i where $i = 1, 2$ as follow

$$Q_{\psi_i} = \left(1 - Q_{\delta_i}\right)\left(Q_{\mu_i} + \frac{1}{\sqrt{Q_{\lambda_i}}}\Gamma\left(\frac{3}{2}\right)\right), \tag{2-22}$$

where Q_{δ_i}, Q_{μ_i} and Q_{λ_i} be pivotal quantities of δ_i, μ_i and λ_i respectively. The estimators of $\hat{\theta}_{mean}$ and $\hat{\tau}_{mean}$ obtained using the generalized confidence interval (GCI) method are denoted by Q_θ and Q_τ respectively, defined as

$$Q_\theta = Q_{\psi_1} - Q_{\psi_1} = \left(1-Q_{\delta_1}\right)\left(Q_{\mu_1} + \frac{1}{\sqrt{Q_{\lambda_1}}}\Gamma\left(\frac{3}{2}\right)\right) - \left(1-Q_{\delta_2}\right)\left(Q_{\mu_2} + \frac{1}{\sqrt{Q_{\lambda_2}}}\Gamma\left(\frac{3}{2}\right)\right), \quad (2\text{-}23)$$

and

$$Q_\tau = \frac{Q_{\psi_1}}{Q_{\psi_2}} = \frac{\left(1-Q_{\delta_1}\right)\left(Q_{\mu_1} + \frac{1}{\sqrt{Q_{\lambda_1}}}\Gamma\left(\frac{3}{2}\right)\right)}{\left(1-Q_{\delta_2}\right)\left(Q_{\mu_2} + \frac{1}{\sqrt{Q_{\lambda_2}}}\Gamma\left(\frac{3}{2}\right)\right)}. \quad (2\text{-}24)$$

These estimators are derived based on the generalized pivotal quantity framework, which enables reliable interval estimation even when the exact sampling distributions of the parameters are analytically intractable. Consequently, the confidence intervals for Q_θ and Q_τ are constructed using the GCI approach defined as

$$CI_{\theta(GCI)} = \left(L_{\theta(GCI)}, U_{\theta(GCI)}\right) = \left(Q_\theta\left(\alpha/2\right), Q_\theta\left(1-\alpha/2\right)\right) \quad (2\text{-}25)$$

and

$$CI_{\tau(GCI)} = \left(L_{\tau(GCI)}, U_{\tau(GCI)}\right) = \left(Q_\tau\left(\alpha/2\right), Q_\tau\left(1-\alpha/2\right)\right) \quad (2\text{-}26)$$

2.3.4 Method of Variance Estimates Recovery (MOVER)

Donner and Zou (2012) developed confidence intervals for the difference and ratio of two distribution parameters. This approach is especially effective for establishing confidence intervals for the difference and ratio of means when the variance of these parameters is difficult to ascertain analytically. The confidence intervals for the difference and ratio of means are defined as

$$CI_{\theta_{mean}(MOVER)} = \left(L_{\theta_{mean}(MOVER)}, U_{\theta_{mean}(MOVER)}\right), \quad (2\text{-}27)$$

and

$$CI_{\tau_{mean}(MOVER)} = \left(L_{\tau_{mean}(MOVER)}, U_{\tau_{mean}(MOVER)}\right), \quad (2\text{-}28)$$

respectively, where the lower limit and the upper limit for θ are given by

$$L_{\theta_{mean}(MOVER)} = \left(\hat{\psi}_1 - \hat{\psi}_2\right) - \sqrt{\left(\hat{\psi}_1 - l_1\right)^2 - \left(u_2 - \hat{\psi}_2\right)^2},$$

and

$$U_{\theta_{mean}(MOVER)} = \left(\hat{\psi}_1 - \hat{\psi}_2\right) + \sqrt{\left(u_1 - \hat{\psi}_1\right)^2 - \left(\hat{\psi}_2 - l_2\right)^2},$$

where the lower limit and upper limit for ψ_x / ψ_y are given by

$$L_{\tau_{mean}(MOVER)} = \frac{\left(\hat{\psi}_1\hat{\psi}_2\right) - \sqrt{\left(\hat{\psi}_1\hat{\psi}_2\right)^2 - l_1u_2\left(2\hat{\psi}_1 - l_1\right)\left(2\hat{\psi}_2 - u_2\right)}}{u_2\left(2\hat{\psi}_2 - u_2\right)}$$

and

$$U_{\tau_{mean}(MOVER)} = \frac{\left(\hat{\psi}_1\hat{\psi}_2\right) + \sqrt{\left(\hat{\psi}_1\hat{\psi}_2\right)^2 - u_1l_2\left(2\hat{\psi}_1 - u_1\right)\left(2\hat{\psi}_2 - l_2\right)}}{l_2\left(2\hat{\psi}_2 - l_2\right)},$$

where for $\left(l_1, u_1\right)$ and $\left(l_2, u_2\right)$, we construct the confidence interval using the GCI

method.

2.3.5 Approximate normal (AN)

The Delta method is a recognized technique within the normal approximation framework, often used to estimate the limiting distribution of an estimator using a normal distribution. Examine a differentiable scalar function $g(u_1,u_2,u_3,u_4,u_5,u_6)$, articulated in relation to the parameters of interest. Let $G(U_1,U_2,U_3,U_4,U_5,U_6)$ denote its corresponding estimator. The objective is to get the asymptotic distribution of G by a stochastic expansion. As the sample size becomes large, the Delta method ensures that G follows an asymptotically normal distribution, which enables the determination of the asymptotic mean and variance of θ and τ. The function g is represented by a Taylor series expansion centered on the true parameter values $(\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6)$, corresponding to $(U_1,U_2,U_3,U_4,U_5,U_6)$.

$$g(U_1,U_2,U_3,U_4,U_5,U_6)\approx g\left((\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6)\right)+\sum_{i=1}^{6}\frac{\partial g\left((\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6)\right)}{\partial u_i}(U_i-\omega_i).$$

The fundamental statistics are designated as $U_1=\hat{\delta}_1, U_2=\hat{\mu}_1, U_3=\hat{\lambda}_1, U_4=\hat{\delta}_2, U_5=\hat{\mu}_2$ and $U_6=\hat{\lambda}_2$, where $\hat{\delta}_1,\hat{\mu}_1,\hat{\lambda}_1,\hat{\delta}_2,\hat{\mu}_2,\hat{\lambda}_2$ represent the maximum likelihood estimates (MLEs) of $\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2$, respectively.

Then,

$$\hat{\theta}_{mean}=g\left(\hat{\delta}_1,\hat{\mu}_1,\hat{\lambda}_1,\hat{\delta}_2,\hat{\mu}_2,\hat{\lambda}_2\right)=\left(1-\hat{\delta}_1\right)\left(\hat{\mu}_1+\frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)-\left(1-\hat{\delta}_2\right)\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)$$

and $\hat{\tau}_{mean}=g\left(\hat{\delta}_1,\hat{\mu}_1,\hat{\lambda}_1,\hat{\delta}_2,\hat{\mu}_2,\hat{\lambda}_2\right)=\dfrac{\left(1-\hat{\delta}_1\right)\left(\hat{\mu}_1+\frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)}{\left(1-\hat{\delta}_2\right)\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)}.$

The partial derivatives of $\hat{\theta}$ are as follows:

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\delta_1}=-1\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right),$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\mu_1}=(1-\delta_1),$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\lambda_1}=(1-\delta_1)\left(-\frac{1}{2}\right)\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right),$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\delta_2}=\left(\mu+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right),$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\mu_2}=-(1-\delta_2),$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \lambda_2}=(1-\delta_2)\left(\frac{1}{2}\right)\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right),$$

Consequently, the estimator $\hat{\theta}$ can be obtained by applying a Taylor series expansion of the function $g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)$ around the corresponding true parameter values. This expansion provides a linear approximation of the function in terms of the estimated parameters $\hat{\delta}_1,\hat{\lambda}_1,\hat{\mu},\hat{\delta}_2,\hat{\lambda}_2$ and $\hat{\mu}_2$ given by

$$\begin{aligned}\hat{\theta}_{mean} \approx\ & g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)\left(\hat{\delta}_1-\delta_1\right)}{\partial \delta_1}+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)(\hat{\mu}_1-\mu_1)}{\partial \mu_1}\\ &+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)\left(\hat{\lambda}_1-\hat{\lambda}_1\right)}{\partial \lambda_1}+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)\left(\hat{\delta}_2-\delta_2\right)}{\partial \delta_2}\\ &+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)(\hat{\mu}_2-\mu_2)}{\partial \mu_2}+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)\left(\hat{\lambda}_2-\hat{\lambda}_2\right)}{\partial \lambda_2}.\end{aligned}$$

The approximate expectation and variance of $\hat{\theta}_{mean}$ can be obtained by evaluating its expectation and variance. In particular, the expectation of $\hat{\theta}_{mean}$ is expressed

$$\begin{aligned}\hat{\theta}_{mean} \approx\ & g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)+(-1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)\left(\hat{\delta}_1-\delta_1\right)+(1-\delta_1)(\hat{\mu}_1-\mu_1)\\ &+(1-\delta_1)\left(-\frac{1}{2}\right)\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\left(\hat{\lambda}_1-\lambda_1\right)+\left(\mu+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\left(\hat{\delta}_2-\delta_2\right)+(-1)(1-\delta_2)(\hat{\mu}_2-\mu_2)\\ &+(1-\delta_2)\left(\frac{1}{2}\right)\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\left(\hat{\lambda}_2-\lambda_2\right)\end{aligned}$$

$$\begin{aligned}E\left(\hat{\theta}_{mean}\right) \approx\ & \theta_{mean}+(-1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)E\left(\hat{\delta}_1-\delta_1\right)+(1-\delta_1)E\left(\hat{\mu}_1-\mu_1\right)\\ &+(1-\delta_1)\left(-\frac{1}{2}\right)\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)E\left(\hat{\lambda}_1-\lambda_1\right)+\left(\mu+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)E\left(\hat{\delta}_2-\delta_2\right)+(-1)(1-\delta_2)E\left(\hat{\mu}_2-\mu_2\right)\\ &+(1-\delta_2)\left(\frac{1}{2}\right)\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)E\left(\hat{\lambda}_2-\lambda_2\right)\end{aligned}$$

so $E\left(\hat{\theta}_{mean}\right)=\theta_{mean}$, and

$$\begin{aligned}V\left(\hat{\theta}_{mean}\right)=\ & \left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)^2 V\left(\hat{\delta}_1-\delta_1\right)+(1-\delta_1)^2 V\left(\hat{\mu}_1-\mu_1\right)\\ &+\left((1-\delta_1)\left(\frac{1}{2}\right)\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)^2 V\left(\hat{\lambda}_1-\lambda_1\right)+\left(\mu+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)^2 V\left(\hat{\delta}_2-\delta_2\right)\\ &+(1-\delta_2)^2 V\left(\hat{\mu}_2-\mu_2\right)+\left((1-\delta_2)\left(\frac{1}{2}\right)\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)^2 V\left(\hat{\lambda}_2-\lambda_2\right)\end{aligned}$$

$$V\left(\hat{\theta}_{mean}\right)=\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)^2\frac{\delta_1\left(1-\delta_1\right)}{n_1}+\left(1-\delta_1\right)^2\frac{1}{2\mu_1+\frac{1}{n_{1(1)}}\sum_{j=1}^{n_{1(1)}}\left(x_{1j(1)}-\mu_1\right)^{-2}}$$

$$+\left(\left(1-\delta_1\right)\left(\frac{1}{2}\right)\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)^2\frac{{\lambda_1}^2}{n_{1(1)}}+\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)^2\frac{\delta_2\left(1-\delta_2\right)}{n_2}$$

$$+\left(1-\delta_2\right)^2\frac{1}{2\mu_2+\frac{1}{n_{2(1)}}\sum_{j=1}^{n_{2(1)}}\left(x_{2j(1)}-\mu_2\right)^{-2}}+\left(\left(1-\delta_2\right)\left(\frac{1}{2}\right)\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)^2\frac{{\lambda_2}^2}{n_{2(1)}}.$$

The estimate of $V\left(\hat{\theta}_{mean}\right)$ is as follows:

$$\hat{V}\left(\hat{\theta}_{mean}\right)=\left(\hat{\mu}_1+\frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)^2\frac{\hat{\delta}_1\left(1-\hat{\delta}_1\right)}{n_1}+\left(1-\hat{\delta}_1\right)^2\frac{1}{2\hat{\mu}_1+\frac{1}{n_{1(1)}}\sum_{j=1}^{n_{1(1)}}\left(x_{1j(1)}-\hat{\mu}_1\right)^{-2}}$$

$$+\left(\left(1-\hat{\delta}_1\right)\left(\frac{1}{2}\right)\hat{\lambda}_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)^2\frac{\hat{\lambda}_1^{\,2}}{n_{1(1)}}+\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)^2\frac{\hat{\delta}_2\left(1-\hat{\delta}_2\right)}{n_2} \qquad (2\text{-}29)$$

$$+\left(1-\hat{\delta}_2\right)^2\frac{1}{2\hat{\mu}_2+\frac{1}{n_{2(1)}}\sum_{j=1}^{n_{2(1)}}\left(x_{2j(1)}-\hat{\mu}_2\right)^{-2}}+\left(\left(1-\hat{\delta}_2\right)\left(\frac{1}{2}\right)\hat{\lambda}_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)^2\frac{\hat{\lambda}_2^{\,2}}{n_{2(1)}}.$$

The partial derivatives of $\hat{\tau}_{mean}$ are as follows:

$$\frac{\partial g\left(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2\right)}{\partial\hat{\delta}_1}=\frac{(-1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)}{\left(1-\delta_2\right)\left(\hat{\mu}_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)},$$

$$\frac{\partial g\left(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2\right)}{\partial\mu_1}=\frac{\left(1-\delta_1\right)}{\left(1-\delta_2\right)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)},$$

$$\frac{\partial g\left(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2\right)}{\partial\lambda_1}=\frac{\left(1-\delta_1\right)\left(\left(-\frac{1}{2}\right)\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)}{\left(1-\delta_2\right)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)},$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \delta_2}=\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2},$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \mu_2}=\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)(-1)(1-\delta_2)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2},$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \lambda_2}=\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)(-1)(1-\delta_2)\left(-\frac{1}{2}\right)\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}.$$

The approximate expectation and variance of $\hat{\tau}$ can be obtained by evaluating its expectation and variance. In particular, the expectation of $\hat{\tau}$ is expressed

$$\hat{\tau}_{mean} \approx g\left(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2\right)+\frac{(-1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)}{(1-\delta_2)\left(\hat{\mu}_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}\left(\hat{\delta}_1-\delta_1\right)$$

$$+\frac{(1-\delta_1)}{(1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}\left(\hat{\mu}_1-\mu_1\right)$$

$$+\frac{(1-\delta_1)\left(\left(-\frac{1}{2}\right)\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)}{(1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}\left(\hat{\lambda}_1-\lambda_1\right)$$

$$+\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}\left(\hat{\delta}_2-\delta_2\right)$$

$$+\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)(-1)(1-\delta_2)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}\left(\hat{\mu}_2-\mu_2\right)$$

$$+\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)(-1)(1-\delta_2)\left(-\frac{1}{2}\right)\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}\left(\hat{\lambda}_2-\lambda_2\right)$$

$$E\left(\hat{\tau}_{mean}\right)\approx\tau_{mean}+\frac{(-1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)}{(1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}E\left(\hat{\delta}_1-\delta_1\right)+\frac{(1-\delta_1)}{(1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}E\left(\hat{\mu}_1-\mu_1\right)$$

$$+\frac{(1-\delta_1)\left(\left(-\frac{1}{2}\right)\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)}{(1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}E\left(\hat{\lambda}_1-\lambda_1\right)$$

$$+\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}E\left(\hat{\delta}_2-\delta_2\right)$$

$$+\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)(-1)(1-\delta_2)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}E\left(\hat{\mu}_2-\mu_2\right)$$

$$+\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)(-1)(1-\delta_2)\left(-\frac{1}{2}\right)\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}E\left(\hat{\lambda}_2-\lambda_2\right)$$

so $E\left(\hat{\tau}_{mean}\right)=\tau_{mean}$,

and

$$V\left(\hat{\tau}_{mean}\right)=\left(\frac{(-1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)}{(1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}\right)^2 V\left(\hat{\delta}_1-\delta_1\right)+$$

$$\left(\frac{(1-\delta_1)}{(1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}\right)^2 V\left(\hat{\mu}_1-\mu_1\right)$$

$$+\left(\frac{(1-\delta_1)\left(\left(-\frac{1}{2}\right)\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)}{(1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}\right)^2 V\left(\hat{\lambda}_1-\lambda_1\right)+$$

$$\left(\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}\right)^2 V\left(\hat{\delta}_2-\delta_2\right)$$

$$+\left(\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)(-1)(1-\delta_2)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}\right)^2 V\left(\hat{\mu}_2-\mu_2\right)$$

$$+\left(\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)(-1)(1-\delta_2)\left(-\frac{1}{2}\right)\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}\right)^2 V\left(\hat{\lambda}_2-\lambda_2\right)$$

$$V\left(\hat{\tau}_{mean}\right)=\left(\left(\frac{(-1)\left(\mu_1+\frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)}{(1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}\right)^2\right)\frac{\delta_1(1-\delta_1)}{n_1}$$

$$+\left(\frac{(1-\delta_1)}{(1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}\right)^2\frac{1}{2\mu_1+\frac{1}{n_{1(1)}}\sum_{i=1}^{n_{1(1)}}\left(x_{1j(1)}-\mu_1\right)^{-2}}$$

$$+\left(\frac{(1-\delta_1)\left(\left(-\frac{1}{2}\right)\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)}{(1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}\right)^2\frac{{\lambda_1}^2}{n_{1(1)}}$$

$$+\left(\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}\right)^2\frac{\delta_2(1-\delta_2)}{n_2}$$

$$+\left(\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)(-1)(1-\delta_2)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}\right)^2\frac{1}{2\mu_2+\frac{1}{n_{2(1)}}\sum_{j=1}^{n_{2(1)}}\left(x_{2j(1)}-\mu_2\right)^{-2}}$$

$$+\left(\frac{(1-\delta_1)\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)(-1)(1-\delta_2)\left(-\frac{1}{2}\right)\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)}{\left((1-\delta_2)\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}\right)^2\frac{{\lambda_2}^2}{n_{2(1)}}.$$

The estimate of $V\left(\hat{\tau}_{mean}\right)$ is as follows:

$$\hat{V}\left(\hat{\tau}_{mean}\right)=\left(\frac{(-1)\left(\hat{\mu}_1+\frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)}{\left(1-\hat{\delta}_2\right)\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)}\right)^2\frac{\hat{\delta}_1\left(1-\hat{\delta}_1\right)}{n_1}$$

$$+\left(\frac{\left(1-\hat{\delta}_1\right)}{\left(1-\hat{\delta}_2\right)\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)}\right)^2\frac{1}{2\hat{\mu}_1+\frac{1}{n_{1(1)}}\sum_{j=1}^{n_{1(1)}}\left(x_{1j(1)}-\hat{\mu}_1\right)^{-2}}$$

$$+\left(\frac{\left(1-\hat{\delta}_1\right)\left(\left(-\frac{1}{2}\right)\hat{\lambda}_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)}{\left(1-\hat{\delta}_2\right)\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)}\right)^2\frac{\hat{\lambda}_1^{\,2}}{n_{1(1)}}$$

$$+\left(\frac{\left(1-\hat{\delta}_1\right)\left(\hat{\mu}_1+\frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)}{\left(\left(1-\hat{\delta}_2\right)\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}\right)^2\frac{\hat{\delta}_2\left(1-\hat{\delta}_2\right)}{n_2}$$

$$+\left(\frac{\left(1-\hat{\delta}_1\right)\left(\hat{\mu}_1+\frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)(-1)\left(1-\hat{\delta}_2\right)}{\left(\left(1-\hat{\delta}_2\right)\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}\right)^2\frac{1}{2\hat{\mu}_2+\frac{1}{n_{2(1)}}\sum_{j=1}^{n_{2(1)}}\left(x_{2j(1)}-\hat{\mu}_2\right)^{-2}}$$

$$+\left(\frac{\left(1-\hat{\delta}_1\right)\left(\hat{\mu}_1+\frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)(-1)\left(1-\hat{\delta}_2\right)\left(-\frac{1}{2}\right)\hat{\lambda}_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)}{\left(\left(1-\hat{\delta}_2\right)\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)^2}\right)^2\frac{\hat{\lambda}_2^{\,2}}{n_{2(1)}}, \qquad (2\text{-}30)$$

respectively. The asymptotic standard normal distribution is $Z=\frac{\hat{\theta}_{mean}-\theta_{mean}}{\sqrt{\hat{V}\left(\hat{\theta}_{mean}\right)}}$ and $Z=\frac{\hat{\tau}_{mean}-\tau_{mean}}{\sqrt{\hat{V}\left(\hat{\tau}_{mean}\right)}}$. Therefore, the confidence interval for approximately normal based on the

delta method for the difference and ratio means constructed by Equations (2-31) and (2-32):

$$CI_{\theta_{mean}(AN)} = \left(L_{\theta_{mean}(AN)}, U_{\theta_{mean}(AN)}\right) = \left(\hat{\theta}_{mean} - z_{1-\alpha/2}\sqrt{\hat{V}\left(\hat{\theta}_{mean}\right)}, \hat{\theta}_{mean} + z_{1-\alpha/2}\sqrt{\hat{V}\left(\hat{\theta}_{mean}\right)}\right) \quad (2\text{-}31)$$

and

$$CI_{\tau_{mean}(AN)} = \left(L_{\tau_{mean}(AN)}, U_{\tau_{mean}(AN)}\right) = \left(\hat{\tau}_{mean} - z_{1-\alpha/2}\sqrt{\hat{V}\left(\hat{\tau}_{mean}\right)}, \hat{\tau}_{mean} + z_{1-\alpha/2}\sqrt{\hat{V}\left(\hat{\tau}_{mean}\right)}\right), \quad (2\text{-}32)$$

respectively, where $\hat{\theta}_{mean} = \left(1-\hat{\delta}_1\right)\left(\hat{\mu}_1 + \frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right) - \left(1-\hat{\delta}_2\right)\left(\hat{\mu}_2 + \frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)$ and

$$\hat{\tau}_{mean} = \frac{\left(1-\hat{\delta}_1\right)\left(\hat{\mu}_1 + \frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)}{\left(1-\hat{\delta}_2\right)\left(\hat{\mu}_2 + \frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)}.$$

2.3.6 Bayesian MCMC

Let $X_{ij} = (X_{i1}, \ldots, X_{in_i})$ for $i = 1, 2$ and $j = 1, 2, \ldots, n_i$ represent a random sample separately extracted from a zero-inflated two-parameter Rayleigh distribution. Markov chain Monte Carlo (MCMC) methods were used to derive the posterior distribution and estimate the parameters of interest. A gamma prior was allocated to the scale parameter λ, a uniform prior was selected for the location parameter (Dey et al, 2014), and a beta prior was designated for the parameter δ. Informative priors were used to embody prior information, reduce posterior variability, and enhance the accuracy of the resultant parameter estimations. This method also improves the reliability of inference, especially when the sample size is constrained. The Bayesian analyses were conducted using the R2OpenBUGS tool inside the R programming environment.

This research examines the difference and ratio of the means of the zero-inflated two-parameter Rayleigh distribution. The Bayesian MCMC method is used to generate the associated two-sided credible intervals. Estimating the posterior distributions of the parameters of interest ensures that inference is not only influenced by the likelihood or the observed data, leading to more accurate parameter estimations (Robert, 1999; Gelman, 1995). Thus, the two-sided credible intervals for the difference and ratio of means are derived from the relevant quantiles of the posterior samples, as shown below:

$$CI_{\theta_{mean}(MCMC)} = \left(L_{\theta_{mean}(MCMC)}, U_{\theta_{mean}(MCMC)}\right) = \left(\theta_{mean(MCMC)}\left(\alpha/2\right), \theta_{mean(MCMC)}\left(1-\alpha/2\right)\right), \quad (2\text{-}33)$$

and

$$CI_{\tau_{mean}(MCMC)} = \left(L_{\tau_{mean}(MCMC)}, U_{\tau_{mean}(MCMC)}\right) = \left(\tau_{mean(MCMC)}\left(\alpha/2\right), \tau_{mean(MCMC)}\left(1-\alpha/2\right)\right). \quad (2\text{-}34)$$

2.3.7 Bayesian HPD

The highest posterior density (HPD) interval is defined as the narrowest interval that satisfies the property that the posterior density at any position within the interval is greater than that at any point outside the interval. The HDInterval utility in the R program was employed to calculate the HPD interval in this study. Consequently, the two-sided credible intervals for the ratio of means and the difference of means can

be determined as follows:

$$CI_{\theta_{mean}(HPD)} = \left(L_{\theta_{mean}(HPD)}, U_{\theta_{mean}(HPD)}\right), \quad (2\text{-}35)$$

and

$$CI_{\tau_{mean}(HPD)} = \left(L_{\tau_{mean}(HPD)}, U_{\tau_{mean}(HPD)}\right). \quad (2\text{-}36)$$

2.4 Simulation study and results

This study evaluates the performance of the proposed methods for constructing confidence intervals for the single mean, as well as the difference and ratio of means between two ZITR distributions. For estimating the confidence interval of the mean, the methods PB, GCI, STD, AN, Bayesian MCMC, and HPD are employed. For estimating the confidence intervals of the difference and ratio of means, the methods PB, BS, GCI, MOVER, AN, MCMC, and HPD are used. The evaluation process was conducted by simulating data using the R software with the Monte Carlo simulation technique. The performance of the methods was assessed based on Coverage Probability (CP) and Expected Length (EL). Methods with CP values close to or greater than the nominal confidence level of 0.95 were considered appropriate. Among these methods, the one with the shortest EL was regarded as the most efficient. For the single mean, the data were generated for $X \sim ZITR(\mu, \lambda, \delta)$ with sample sized n = 20, 50, 70, 100, and 200, and probability of zero δ = 0.2, 0.4, and 0.6, where we set λ = 0.2 and 0.5 and μ = 0.2 and 0.5. We used M = 1000 replications for the Monte Carlo simulation. For the bootstrap method, each situation involved B = 1000 replications, along with q = 2500 pivotal quantities, and we generated T = 20,000 realizations of Bayesian using Gibbs sampling with a burn-in of C = 5000. For difference and ratio means, the data generate for two independent ZITR distribution: $X_i \sim ZITR(\mu_i, \lambda_i, \delta_i), i = 1, 2.$ Sample sizes were considered $(n_1, n_2) = (10,10), (10,40), (10,70), (10,100), (40,40), (40,70), (40,100), (70,70), (70,100), (100,100), (100,200)$ and $(200,200)$. The probability of zeros were set at $(\delta_1, \delta_2) = (0.1,0.1), (0.1,0.3)$, and $(0.3,0.3)$. The scale parameter $(\lambda_1, \lambda_2) = (0.1,0.1), (0.1,0.5), (0.5,0.5), (0.5,1)$ and $(1,1)$. The location parameter is fixed $(\mu_1, \mu_2) = (0.1,0.1)$. The number of generated M = 1000 replications for the Monte Carlo simulation, 1000 resampling for bootstrap method, 2500 pivotal quantities were utilized for the GCI, and we generated T = 20,000 realizations of Bayesian using Gibbs sampling with a burn-in of C = 5000.

The simulation study evaluating the performance of confidence interval estimation methods for the single mean, as presented in Tables 2-1 to 2-2 and Figure 2-1, was conducted under various combinations of the scale and location parameters. The results showed that the standard confidence interval based on the maximum likelihood estimator generally provided coverage probabilities (CPs) close to or below the nominal confidence level of 0.95, particularly for small sample sizes, and tended to produce relatively longer expected lengths (ELs) than most of the competing methods. The approximate normal method based on the delta method generally improved the coverage probability compared with the standard method, although its expected lengths were still relatively long in several simulation scenarios. In general, the expected

lengths of the confidence intervals decreased as the sample size increased across all methods. Among the competing methods, the Bayesian approaches, particularly the Bayesian Highest Posterior Density (HPD) method, demonstrated good overall performance when both CP and EL were considered. The HPD method generally provided coverage probabilities close to the nominal confidence level while yielding relatively short confidence intervals. In many simulation scenarios, the HPD interval produced the shortest expected length among the methods with satisfactory coverage probabilities. The Bayesian MCMC method also performed well. Coverage probabilities were generally close to 0.95 and expected lengths were relatively small. The results show that the Bayesian HPD method is a good and efficient method to construct confidence intervals for the single mean under the ZITR distribution.

The simulation study evaluating the performance of confidence interval estimation methods for the difference and ratio of two means is presented in Tables 2-3 to 2-12 and Figures 2-2 to 2-3, where the figures display only selected representative scenarios for clarity. Overall, the results revealed similar patterns for both measures across different parameter settings and sample sizes. For small sample sizes, particularly when $(n_1, n_2) = (10,10)$, the GCI, MOVER, AN, MCMC, and HPD methods produced coverage probabilities (CPs) close to or greater than the nominal confidence level of 95%. Among all methods, the HPD credible interval consistently yielded the shortest expected length (EL). However, under certain parameter configurations, such as $(\mu_1, \mu_2) = (0.1, 0.1)$, $(\lambda_1, \lambda_2) = (0.1, 0.1)$, and $(\delta_1, \delta_2) = (0.1, 0.3)$, some methods showed CP values noticeably different from the nominal level. This was mainly due to model error caused by extremely small sample sizes, which intensified the zero-inflation effect and increased estimator variability. Nevertheless, the GCI method continued to perform well under these conditions. Similar results were observed for other scenarios with $n_1 = 10$, namely $(10, 40)$, $(10, 70)$, and $(10,100)$, where GCI, MOVER, AN, MCMC, and HPD remained the most reliable methods, while HPD consistently provided the shortest EL. When the sample size increased to $(40, 40)$, the PB method showed some improvement, although its CP values were still below the nominal confidence level. The best-performing methods at this stage remained GCI, MOVER, AN, MCMC, and HPD, with HPD again producing the shortest EL. Comparable findings were obtained for $(40, 70)$ and $(40,100)$. For larger sample sizes, including $(70, 70)$, $(70,100)$, $(100,100)$, $(100, 200)$, and $(200, 200)$, the CP values of PB, GCI, MOVER, AN, MCMC, and HPD were generally close to or exceeded the nominal 95% confidence level. In terms of interval precision, the HPD credible interval continued to outperform the competing methods by providing the shortest EL across nearly all scenarios.

In summary, the simulation study showed that the Bayesian HPD credible interval is suitable for constructing confidence intervals for both the difference and the ratio of means for the zero-inflated two-parameter Rayleigh distribution, as the coverage probabilities remained consistently close to the nominal level in nearly all scenarios.

TABLE 2-1 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Mean of the ZITR Distribution with $\mu = 0.2$ and $\lambda = 0.2, 0.5$.

μ	λ	δ	n	Coverage probability					
				PB	GCI	STD	AN	MCMC	HPD
0.2	0.2	0.2	20	0.947	0.948	0.912	0.950	0.952	0.950
				(1.0613)	(1.1424)	(3.0895)	(1.2023)	(0.9762)	**(0.9686)**
			50	0.962	0.949	1.000	0.971	0.949	0.947
				(0.6865)	(0.7072)	(3.9650)	(0.7712)	(0.6571)	(0.6544)
			70	0.950	0.951	1.000	0.971	0.952	0.950
				(0.5819)	(0.5947)	(4.0601)	(0.6544)	(0.5619)	**(0.5599)**
			100	0.967	0.951	1.000	0.963	0.954	0.953
				(0.4900)	(0.4965)	(4.2144)	(0.5476)	(0.4766)	**(0.4750)**
			200	0.960	0.965	1.000	0.973	0.967	0.966
				(0.3464)	(0.3493)	(4.3454)	(0.3867)	(0.3426)	**(0.3416)**
		0.4	20	0.948	0.944	0.615	0.953	0.960	0.948
				(1.1087)	(1.1957)	(1.7170)	(1.2250)	**(0.9917)**	(0.9806)
			50	0.957	0.953	0.990	0.957	0.960	0.956
				(0.7255)	(0.7438)	(2.7476)	(0.7907)	(0.6817)	**(0.6780)**
			70	0.936	0.936	0.994	0.954	0.946	0.941
				(0.6166)	(0.6272)	**(2.9714)**	(0.6680)	(0.5889)	(0.5861)
			100	0.946	0.935	1.000	0.955	0.939	0.938
				(0.5137)	(0.5206)	(3.0712)	**(0.5594)**	(0.4964)	(0.4943)
			200	0.943	0.949	1.000	0.956	0.950	0.950
				(0.3667)	(0.3678)	(3.2043)	(0.3955)	(0.3587)	**(0.3576)**
		0.6	20	0.956	0.961	0.247	0.920	0.968	0.961
				(1.0357)	(1.1699)	(0.0278)	(1.1187)	(0.9261)	**(0.9069)**
			50	0.954	0.951	0.793	0.952	0.962	0.956
				(0.6793)	(0.7006)	(1.4736)	(0.7292)	(0.6350)	**(0.6282)**
			70	0.942	0.943	0.927	0.953	0.951	0.943
				(0.5763)	(0.5887)	(1.7723)	(0.6168)	**(0.5472)**	(0.5429)
			100	0.954	0.941	0.986	0.960	0.944	0.940
				(0.4834)	(0.4909)	(1.8280)	(0.5163)	(0.4649)	(0.4620)
			200	0.949	0.951	1.000	0.963	0.955	0.950
				(0.3451)	(0.3482)	(1.9969)	(0.3665)	(0.3385)	**(0.3369)**
	0.5	0.2	20	0.955	0.936	0.996	0.949	0.954	0.950
				(0.6829)	(0.7340)	(4.3833)	(0.7788)	(0.6483)	**(0.6437)**
			50	0.961	0.957	1.000	0.962	0.961	0.958
				(0.4458)	(0.4580)	(4.1724)	(0.5018)	(0.4316)	**(0.4300)**
			70	0.947	0.957	1.000	0.955	0.961	0.962
				(0.3771)	(0.3854)	(4.3153)	(0.4229)	(0.3687)	**(0.3675)**
			100	0.950	0.947	1.000	0.972	0.951	0.949
				(0.3177)	(0.3220)	(4.4340)	(0.3547)	**(0.3123)**	(0.3113)
			200	0.942	0.947	1.000	0.974	0.950	0.954
				(0.2252)	(0.2264)	(4.5241)	(0.2492)	(0.2234)	**(0.2227)**
		0.4	20	0.940	0.960	0.891	0.941	0.976	0.973
				(0.7297)	(0.7795)	(2.2492)	(0.7949)	(0.6673)	**(0.6603)**
			50	0.960	0.948	0.999	0.959	0.957	0.957
				(0.4731)	(0.4842)	(3.0704)	(0.5141)	(0.4508)	**(0.4485)**
			70	0.961	0.940	0.999	0.972	0.948	0.949
				(0.4020)	(0.4079)	(3.0445)	(0.4370)	(0.3874)	(0.3857)
			100	0.960	0.955	1.000	0.963	0.960	0.959
				(0.3374)	(0.3416)	(3.1435)	(0.3637)	(0.3287)	**(0.3274)**
			200	0.958	0.962	1.000	0.961	0.967	0.964
				(0.2392)	(0.2408)	(3.3154)	(0.2571)	(0.2361)	**(0.2354)**
		0.6	20	0.953	0.959	0.553	0.927	0.980	0.979
				(0.6761)	(0.7563)	(0.9973)	(0.7459)	(0.6291)	**(0.6160)**
			50	0.955	0.946	0.954	0.954	0.955	0.959
				(0.4442)	(0.4573)	(1.7898)	(0.4770)	(0.4219)	**(0.4176)**
			70	0.949	0.952	0.990	0.951	0.958	0.952
				(0.3792)	(0.3865)	(1.8977)	(0.4038)	(0.3638)	**(0.3610)**
			100	0.951	0.957	0.999	0.946	0.961	0.959
				(0.3184)	(0.3220)	(1.9650)	(0.3383)	(0.3082)	**(0.3063)**
			200	0.954	0.953	1.000	0.959	0.957	0.955
				(0.2255)	(0.2272)	(2.1061)	(0.2395)	(0.2226)	**(0.2216)**

TABLE 2-2 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Mean of the ZITR Distribution with $\mu = 0.5$ and $\lambda = 0.2, 0.5$.

μ	λ	δ	Coverage probability						
			n	PB	GCI	STD	AN	MCMC	HPD
0.5	0.2	0.2	20	0.950	0.947	0.739	0.946	0.953	0.955
				(1.1351)	(1.2116)	(1.1942)	(1.2745)	(1.0506)	**(1.0443)**
			50	0.927	0.941	0.996	0.970	0.946	0.943
				(0.7321)	(0.7531)	(1.9751)	**(0.8143)**	(0.7069)	(0.7046)
			70	0.943	0.934	0.999	0.969	0.942	0.943
				(0.6171)	(0.6302)	(2.1609)	**(0.6926)**	(0.6033)	(0.6014)
			100	0.940	0.948	1.000	0.974	0.950	0.951
				(0.5209)	(0.5285)	(2.3193)	(0.5777)	(0.5129)	**(0.5114)**
			200	0.952	0.955	1.000	0.976	0.957	0.955
				(0.3705)	(0.3729)	(2.5574)	(0.4086)	(0.3676)	**(0.3666)**
		0.4	20	0.952	0.954	0.343	0.943	0.968	0.965
				(1.2227)	(1.2952)	(0.3320)	(1.3286)	(1.0840)	**(1.0743)**
			50	0.943	0.952	0.878	0.959	0.959	0.958
				(0.7887)	(0.8059)	(1.1292)	(0.8503)	(0.7448)	**(0.7414)**
			70	0.948	0.949	0.970	0.964	0.957	0.955
				(0.6737)	(0.6825)	(1.3055)	(0.7212)	(0.6453)	**(0.6428)**
			100	0.938	0.953	0.998	0.965	0.954	0.954
				(0.5620)	(0.5674)	(1.4985)	(0.6023)	(0.5455)	**(0.5436)**
			200	0.942	0.950	1.000	0.947	0.952	0.953
				(0.3997)	(0.4015)	(1.7494)	(0.4251)	(0.3924)	**(0.3913)**
		0.6	20	0.953	0.966	0.273	0.934	0.984	0.981
				(0.7879)	(0.8559)	(0.0088)	(1.2295)	(0.7201)	**(0.7075)**
			50	0.961	0.947	0.504	0.948	0.961	0.950
				(0.7479)	(0.7667)	(0.4056)	(0.7952)	(0.6979)	**(0.6916)**
			70	0.957	0.949	0.742	0.956	0.963	0.956
				(0.6362)	(0.6478)	(0.6132)	(0.6725)	(0.6057)	**(0.6016)**
			100	0.958	0.947	0.900	0.948	0.951	0.947
				(0.5357)	(0.5422)	(0.7632)	(0.5628)	**(0.5154)**	(0.5126)
			200	0.942	0.946	0.996	0.958	0.949	0.948
				(0.3800)	(0.3826)	(1.0010)	**(0.3983)**	(0.3736)	(0.3720)
	0.5	0.2	20	0.948	0.948	0.960	0.952	0.967	0.966
				(0.7670)	(0.8139)	(1.8159)	(0.8492)	(0.7283)	**(0.7247)**
			50	0.936	0.952	1.000	0.963	0.968	0.961
				(0.4931)	(0.5046)	(2.3294)	(0.5434)	(0.4793)	**(0.4780)**
			70	0.957	0.948	1.000	0.960	0.953	0.954
				(0.4201)	(0.4274)	(2.4583)	(0.4613)	(0.4112)	**(0.4101)**
			100	0.951	0.942	1.000	0.969	0.945	0.948
				(0.3514)	(0.3557)	(2.5764)	(0.3860)	(0.3452)	(0.3442)
			200	0.941	0.942	1.000	0.970	0.943	0.946
				(0.2504)	(0.2513)	(2.7494)	**(0.2731)**	(0.2473)	(0.2467)
		0.4	20	0.942	0.945	0.705	0.939	0.982	0.975
				(0.8398)	(0.8814)	(0.8684)	(0.8953)	(0.7607)	**(0.7546)**
			50	0.959	0.940	0.986	0.949	0.953	0.948
				(0.5415)	(0.5499)	(1.5184)	(0.5768)	**(0.5150)**	(0.5131)
			70	0.957	0.954	0.998	0.965	0.961	0.960
				(0.4604)	(0.4666)	(1.6394)	(0.4898)	(0.4439)	**(0.4424)**
			100	0.959	0.946	0.999	0.957	0.960	0.955
				(0.3870)	(0.3899)	(1.7688)	(0.4093)	(0.3760)	**(0.3749)**
			200	0.952	0.951	1.000	0.960	0.955	0.953
				(0.2744)	(0.2749)	(1.9422)	(0.2899)	(0.2692)	**(0.2684)**
		0.6	20	0.955	0.963	0.278	0.916	0.979	0.977
				(0.7868)	(0.8547)	(0.0174)	(0.8381)	(0.7175)	**(0.7052)**
			50	0.957	0.950	0.793	0.951	0.968	0.964
				(0.5174)	(0.5279)	(0.7450)	(0.5450)	(0.4891)	**(0.4850)**
			70	0.941	0.945	0.914	0.948	0.959	0.950
				(0.4390)	(0.4454)	(0.8821)	(0.4601)	(0.4214)	**(0.4188)**
			100	0.958	0.954	0.985	0.954	0.962	0.961
				(0.3712)	(0.3739)	(1.0177)	(0.3872)	(0.3586)	**(0.3568)**
			200	0.942	0.944	1.000	0.950	0.948	0.945
				(0.2620)	(0.2631)	(1.1865)	**(0.2745)**	(0.2581)	(0.2571)

TABLE 2-3 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference Between Two Means of the ZITR Distribution with $(\mu_1, \mu_2) = (0.1, 0.1)$ and $(\lambda_1, \lambda_2) = (0.1, 0.1)$.

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.1:0.1	0.1:0.1	10:10	0.898	0.784	0.956	0.957	0.986	0.948	0.953
			(2.6068)	(2.5653)	(3.2062)	(3.2172)	(7.3751)	(2.3475)	**(2.3373)**
		10:40	0.887	0.788	0.958	0.956	0.987	0.939	0.943
			(2.5166)	(2.5331)	**(3.0969)**	(3.1071)	(6.982)	(2.3648)	(2.355)
		10:70	0.907	0.774	0.95	0.953	0.982	0.945	0.952
			(2.5077)	(2.5211)	(3.0754)	(3.0882)	(7.7223)	(2.3562)	**(2.3466)**
		10:100	0.884	0.788	0.953	0.953	0.988	0.942	0.946
			(2.4863)	(2.5372)	**(3.0426)**	(3.0535)	(7.5441)	(2.3474)	(2.3375)
		40:40	0.94	0.816	0.953	0.953	0.999	0.948	0.948
			(1.3869)	(1.3854)	**(1.4485)**	(1.4517)	(10.9208)	(1.3034)	(1.2998)
		40:70	0.924	0.817	0.94	0.941	1.000	0.955	0.952
			(1.3451)	(1.3668)	(1.4096)	(1.4111)	(10.7472)	(1.3023)	**(1.2986)**
		40:100	0.905	0.795	0.934	0.937	1.000	0.949	0.952
			(1.3252)	(1.3203)	(1.39)	(1.3938)	(10.8491)	(1.3019)	**(1.2983)**
		70:70	0.949	0.830	0.954	0.954	0.999	0.956	0.955
			(1.0527)	(1.0554)	(1.0864)	(1.0872)	(11.3662)	(1.006)	**(1.0032)**
		70:100	0.934	0.83	0.941	0.944	0.997	0.946	0.946
			(1.0303)	(1.0363)	(1.0639)	(1.0643)	**(12.0405)**	(1.0096)	(1.0068)
		100:100	0.940	0.825	0.947	0.945	0.999	0.94	0.939
			(0.8865)	(0.8689)	(0.9016)	(0.9021)	**(13.0117)**	(0.8527)	(0.8504)
		100:200	0.884	0.79	0.926	0.927	1.000	0.947	0.947
			(0.8559)	(0.8495)	(0.8682)	(0.8684)	**(11.5592)**	(0.8525)	(0.8503)
		200:200	0.937	0.816	0.949	0.951	1.000	0.943	0.943
			(0.6296)	(0.6281)	(0.6321)	**(0.6318)**	(12.9709)	(0.6099)	(0.6084)
	0.1:0.3	10:10	0.922	0.804	0.956	0.959	0.976	0.916	0.921
			(2.763)	(2.7533)	**(3.3452)**	(3.3537)	(6.4866)	(2.5042)	(2.4929)
		10:40	0.803	0.707	0.936	0.937	0.974	0.917	0.925
			(2.5902)	(2.5953)	(3.0856)	(3.0917)	**(6.3127)**	(2.5018)	(2.4902)
		10:70	0.768	0.675	0.935	0.938	0.975	0.923	0.93
			(2.6198)	(2.6683)	(3.0278)	(3.0377)	**(6.6101)**	(2.4991)	(2.488)
		10:100	0.735	0.666	0.919	0.917	0.962	0.911	0.917
			(2.6421)	(2.5941)	(3.0552)	(3.066)	**(6.2433)**	(2.4996)	(2.4885)
		40:40	0.933	0.816	0.951	0.956	0.999	0.933	0.932
			(1.4533)	(1.4506)	**(1.5255)**	(1.5274)	(9.1961)	(1.3819)	(1.3782)
		40:70	0.788	0.689	0.939	0.94	1.000	0.931	0.932
			(1.3784)	(1.3731)	(1.4319)	(1.434)	**(9.2486)**	(1.3876)	(1.3837)
		40:100	0.693	0.621	0.939	0.939	0.996	0.936	0.937
			(1.3775)	(1.3595)	(1.3913)	(1.393)	**(8.5717)**	(1.3896)	(1.3858)
		70:70	0.942	0.829	0.950	0.948	0.997	0.944	0.944
			(1.1125)	(1.1158)	**(1.142)**	(1.1435)	(10.2608)	(1.0705)	(1.0677)
		70:100	0.846	0.738	0.949	0.947	0.997	0.939	0.941
			(1.0651)	(1.0527)	(1.0889)	(1.0893)	**(10.0585)**	(1.0709)	(1.0681)
		100:100	0.942	0.829	0.951	0.951	1.000	0.942	0.939
			(0.9384)	(0.9449)	**(0.9519)**	(0.9525)	(10.6172)	(0.9077)	(0.9054)
		100:200	0.519	0.500	0.934	0.933	1.000	0.937	0.935
			(0.8804)	(0.8817)	(0.878)	(0.878)	**(11.2478)**	(0.9067)	(0.9043)
		200:200	0.953	0.847	0.961	0.959	1.000	0.951	0.951
			(0.6639)	(0.6584)	(0.6688)	(0.669)	(10.993)	(0.6507)	**(0.649)**
	0.3:0.3	10:10	0.898	0.788	0.968	0.966	0.971	0.941	0.947
			(2.8797)	(2.8687)	(3.5343)	**(3.529)**	(5.5203)	(2.5406)	(2.5288)
		10:40	0.828	0.719	0.950	0.949	0.976	0.960	0.967
			(2.729)	(2.7502)	(3.2761)	(3.2684)	(5.7079)	(2.5323)	**(2.5202)**
		10:70	0.786	0.715	0.944	0.946	0.974	0.951	0.956
			(2.7622)	(2.8198)	(3.2682)	(3.2686)	(6.0703)	(2.5303)	**(2.5184)**
		10:100	0.789	0.708	0.943	0.944	0.971	0.973	0.977
			(2.763)	(2.8275)	(3.173)	(3.1766)	(5.4823)	(2.5311)	**(2.5194)**

Table 2-3 (Con)

$\lambda_1:\lambda_2$	$\delta_1:\delta_2$	Coverage probability						
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		40:40	0.946	0.842	0.949	0.947	0.993	0.949	0.949
			(1.5376)	(1.5731)	(1.6033)	(1.6045)	**(7.6538)**	(1.4139)	(1.4099)
		40:70	0.809	0.707	0.939	0.943	0.996	0.945	0.943
			(1.4605)	(1.4402)	(1.5044)	(1.5075)	**(8.6319)**	(1.407)	(1.403)
		40:100	0.695	0.622	0.937	0.934	0.997	0.951	0.952
			(1.4556)	(1.4415)	(1.4741)	(1.4743)	(7.903)	(1.4099)	**(1.4059)**
		70:70	0.948	0.844	0.953	0.952	0.999	0.942	0.944
			(1.1726)	(1.1819)	**(1.1962)**	(1.1966)	(8.9083)	(1.0928)	(1.0898)
		70:100	0.836	0.740	0.938	0.937	0.999	0.941	0.939
			(1.1264)	(1.113)	(1.1491)	(1.15)	**(8.6708)**	(1.094)	(1.0911)
		100:100	0.936	0.831	0.943	0.946	0.997	0.938	0.934
			(0.9823)	(1.0009)	(0.9994)	(1.0002)	**(9.4559)**	(0.9303)	(0.9279)
		100:200	0.549	0.506	0.935	0.935	1.000	0.933	0.933
			(0.9357)	(0.9281)	(0.9311)	(0.9301)	**(9.5176)**	(0.9265)	(0.9241)
		200:200	0.948	0.861	0.951	0.947	1.000	0.944	0.946
			(0.6965)	(0.6977)	**(0.7019)**	(0.7022)	(9.5204)	(0.6672)	(0.6655)

TABLE 2-4 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference Between Two Means of the ZITR Distribution with $(\mu_1,\mu_2)=(0.1,0.1)$ and $(\lambda_1,\lambda_2)=(0.1,0.5)$.

$\lambda_1:\lambda_2$	$\delta_1:\delta_2$	Coverage probability						
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.1:0.5	0.1:0.1	10:10	0.883	0.801	0.955	0.956	0.996	0.941	0.932
			(2.0097)	(2.0398)	**(2.438)**	(2.4452)	(8.2624)	(1.9042)	(1.8868)
		10:40	0.893	0.790	0.966	0.966	0.997	0.938	0.93
			(1.9769)	(1.9819)	**(2.4416)**	(2.4473)	(8.0877)	(1.9016)	(1.8841)
		10:70	0.900	0.805	0.952	0.951	0.999	0.942	0.939
			(2.0093)	(2.0082)	**(2.4309)**	(2.4368)	(9.571)	(1.9176)	(1.9002)
		10:100	0.891	0.787	0.951	0.949	0.996	0.937	0.932
			(1.9898)	(2.0102)	**(2.4161)**	(2.4235)	(8.8907)	(1.9121)	(1.8949)
		40:40	0.937	0.820	0.954	0.950	1.000	0.948	0.947
			(1.0704)	(1.0762)	**(1.1246)**	(1.1266)	(11.7596)	(1.0211)	(1.0167)
		40:70	0.933	0.834	0.951	0.953	1.000	0.95	0.944
			(1.063)	(1.0894)	(1.1111)	(1.112)	(10.9058)	**(1.0283)**	(1.0239)
		40:100	0.931	0.811	0.94	0.941	0.999	0.955	0.951
			(1.0629)	(1.054)	(1.1054)	(1.1076)	(10.8462)	(1.029)	**(1.0246)**
		70:70	0.954	0.825	0.948	0.947	0.999	0.962	0.960
			(0.8184)	(0.8038)	(0.8388)	(0.8396)	(11.6646)	(0.7895)	**(0.7867)**
		70:100	0.932	0.848	0.948	0.948	1.000	0.942	0.942
			(0.8106)	(0.8107)	(0.8348)	(0.8359)	**(11.7842)**	(0.7904)	(0.7876)
		100:100	0.948	0.843	0.939	0.939	1.000	0.945	0.942
			(0.6876)	(0.6853)	(0.7002)	(0.7006)	**(11.9455)**	(0.6647)	(0.6626)
		100:200	0.923	0.804	0.944	0.947	1.000	0.943	0.948
			(0.675)	(0.675)	(0.6913)	(0.6911)	**(12.7108)**	(0.6658)	(0.6636)
		200:200	0.940	0.830	0.952	0.952	1.000	0.941	0.938
			(0.4855)	(0.4882)	**(0.4914)**	(0.4913)	(11.9212)	(0.4766)	(0.4752)
	0.1:0.3	10:10	0.913	0.801	0.968	0.969	0.990	0.905	0.896
			(2.056)	(2.0867)	**(2.5327)**	(2.5299)	(7.1277)	(1.9866)	(1.9728)
		10:40	0.866	0.754	0.950	0.948	0.996	0.928	0.915
			(2.0008)	(2.0053)	**(2.433)**	(2.4392)	(7.2689)	(1.9992)	(1.986)
		10:70	0.826	0.756	0.949	0.947	0.993	0.901	0.887
			(2.0026)	(2.0557)	(2.4388)	(2.4428)	**(7.1381)**	(1.9959)	(1.9823)
		10:100	0.803	0.731	0.939	0.94	0.988	0.888	0.873
			(2.0144)	(2.0499)	(2.4307)	(2.4354)	**(7.3348)**	(1.9774)	(1.9636)
		40:40	0.930	0.820	0.952	0.950	0.999	0.927	0.924
			(1.0992)	(1.1002)	**(1.1495)**	(1.1494)	(9.3931)	(1.0678)	(1.0638)

TABLE 2-4 (Con)

$\lambda_1:\lambda_2$	$\delta_1:\delta_2$	Coverage probability						
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		40:70	0.892	0.775	0.948	0.948	1.000	0.933	0.931
			(1.0694)	(1.0735)	(1.1156)	(1.1169)	**(11.9177)**	(1.0651)	(1.0612)
		40:100	0.830	0.732	0.938	0.934	0.999	0.931	0.923
			(1.0623)	(1.0658)	(1.1048)	(1.1064)	**(9.9227)**	(1.06)	(1.0559)
		70:70	0.948	0.858	0.956	0.954	1.000	0.952	0.952
			(0.8361)	(0.8467)	(0.8578)	(0.8584)	(10.5498)	(0.818)	**(0.8154)**
		70:100	0.894	0.790	0.955	0.954	1.000	0.949	0.945
			(0.8212)	(0.8167)	**(0.8452)**	(0.8454)	(10.3027)	(0.816)	(0.8135)
		100:100	0.947	0.822	0.951	0.952	1.000	0.940	0.938
			(0.7022)	(0.6915)	**(0.7124)**	(0.7124)	(10.692)	(0.6879)	(0.6858)
		100:200	0.794	0.703	0.940	0.939	1.000	0.952	0.953
			(0.6844)	(0.6831)	(0.6913)	(0.692)	(10.6641)	(0.6878)	**(0.6857)**
		200:200	0.958	0.822	0.953	0.957	1.000	0.957	0.952
			(0.4984)	(0.4863)	(0.5032)	(0.503)	(11.4851)	(0.4916)	**(0.4902)**
	0.3:0.3	10:10	0.907	0.809	0.959	0.960	0.990	0.966	0.968
			(2.2497)	(2.2369)	(2.7192)	(2.7144)	(6.7539)	(2.071)	**(2.0513)**
		10:40	0.879	0.766	0.959	0.961	0.995	0.964	0.961
			(2.2067)	(2.2331)	(2.6635)	(2.6621)	(6.4714)	(2.0712)	**(2.0509)**
		10:70	0.863	0.770	0.953	0.956	0.990	0.966	0.968
			(2.2102)	(2.2113)	(2.6407)	(2.6408)	(6.3714)	(2.072)	**(2.0522)**
		10:100	0.86	0.757	0.944	0.946	0.989	0.964	0.961
			(2.213)	(2.2227)	(2.5776)	(2.5776)	(6.8369)	(2.0692)	**(2.049)**
		40:40	0.942	0.832	0.939	0.938	0.999	0.961	0.961
			(1.2012)	(1.2327)	(1.2421)	(1.2424)	(8.193)	(1.1146)	**(1.1096)**
		40:70	0.899	0.784	0.944	0.949	1.000	0.953	0.952
			(1.1722)	(1.176)	(1.2178)	(1.2183)	(8.6506)	(1.1071)	**(1.1022)**
		40:100	0.860	0.744	0.941	0.940	1.000	0.959	0.956
			(1.1668)	(1.191)	(1.2115)	(1.2126)	(8.2399)	(1.1081)	**(1.1033)**
		70:70	0.949	0.839	0.943	0.941	0.999	0.948	0.947
			(0.9167)	(0.9102)	(0.9252)	(0.9261)	(8.8207)	(0.8659)	(0.8628)
		70:100	0.924	0.832	0.961	0.962	0.999	0.956	0.956
			(0.8968)	(0.9092)	(0.9174)	(0.9179)	(9.3795)	(0.859)	**(0.8559)**
		100:100	0.949	0.830	0.941	0.938	1.000	0.952	0.953
			(0.7663)	(0.768)	(0.7761)	(0.7764)	(9.1519)	(0.7284)	**(0.726)**
		100:200	0.792	0.726	0.947	0.948	1.000	0.941	0.940
			(0.7493)	(0.7542)	(0.7594)	(0.7598)	**(9.3836)**	(0.7293)	(0.7269)
		200:200	0.952	0.824	0.952	0.951	1.000	0.955	0.953
			(0.3198)	(0.3165)	(0.5471)	(0.5472)	(9.0699)	(0.3099)	**(0.309)**

TABLE 2-5 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference Between Two Means of the ZITR Distribution with $(\mu_1, \mu_2) = (0.1, 0.1)$ and $(\lambda_1, \lambda_2) = (0.5, 0.5)$.

$\lambda_1:\lambda_2$	$\delta_1:\delta_2$	Coverage probability						
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.5:0.5	0.1:0.1	10:10	0.898	0.790	0.966	0.967	1.000	0.986	0.991
			(1.1877)	(1.1788)	(1.4434)	(1.4512)	(10.7351)	(1.2823)	**(1.2778)**
		10:40	0.897	0.787	0.957	0.958	1.000	0.991	0.99
			(1.1197)	(1.1133)	(1.379)	(1.3857)	(9.3153)	(1.2711)	**(1.2665)**
		10:70	0.904	0.815	0.951	0.953	0.999	0.995	0.997
			(1.1277)	(1.1334)	(1.3898)	(1.396)	(9.5199)	(1.2759)	**(1.2716)**
		10:100	0.885	0.794	0.952	0.952	1.000	0.99	0.992
			(1.1292)	(1.1183)	(1.3645)	(1.3694)	(10.3862)	(1.2699)	**(1.2654)**
		40:40	0.946	0.831	0.95	0.951	1.000	0.968	0.971
			(0.6288)	(0.6338)	(0.6539)	(0.6546)	(12.5557)	(0.6238)	**(0.622)**
		40:70	0.926	0.795	0.933	0.935	1.000	0.973	0.972
			(0.6056)	(0.6057)	(0.6367)	(0.637)	(12.3534)	(0.6237)	**(0.622)**

Table 2-5 (Con)

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		40:100	0.897	0.798	0.950	0.949	1.000	0.964	0.962
			(0.6017)	(0.5999)	(0.6286)	(0.6296)	(12.0819)	(0.6231)	**(0.6214)**
		70:70	0.943	0.828	0.963	0.964	1.000	0.958	0.959
			(0.4779)	(0.4768)	(0.4898)	(0.4908)	(11.9157)	(0.4706)	**(0.4694)**
		70:100	0.931	0.807	0.933	0.932	1.000	0.960	0.957
			(0.4643)	(0.4624)	(0.4795)	(0.4801)	(12.8689)	(0.4715)	**(0.4702)**
		100:100	0.952	0.827	0.954	0.953	1.000	0.957	0.959
			(0.401)	(0.3988)	(0.4073)	(0.4078)	(11.7305)	(0.3954)	**(0.3944)**
		100:200	0.889	0.772	0.931	0.937	1.000	0.949	0.947
			(0.3853)	(0.3828)	(0.3931)	(0.3934)	(11.6899)	(0.3968)	(0.3958)
		200:200	0.946	0.847	0.957	0.961	1.000	0.952	0.952
			(0.2842)	(0.2854)	(0.287)	(0.2871)	(11.1239)	(0.2816)	**(0.2809)**
	0.1:0.3	10:10	0.912	0.815	0.958	0.96	0.999	0.978	0.98
			(1.2637)	(1.2582)	(1.5229)	(1.525)	(8.7031)	(1.3864)	**(1.3804)**
		10:40	0.778	0.692	0.948	0.948	1.000	0.974	0.978
			(1.1657)	(1.1693)	(1.4117)	(1.4146)	(8.7225)	(1.3729)	**(1.3672)**
		10:70	0.761	0.679	0.934	0.937	0.999	0.972	0.973
			(1.1835)	(1.2051)	(1.3643)	(1.3691)	(8.5678)	(1.381)	**(1.375)**
		10:100	0.753	0.657	0.913	0.914	0.999	0.974	0.976
			(1.1973)	(1.19)	(1.353)	(1.3565)	(8.5176)	(1.3826)	**(1.3768)**
		40:40	0.924	0.827	0.948	0.949	1.000	0.945	0.943
			(0.6661)	(0.6608)	(0.6936)	(0.6945)	(11.1649)	(0.6667)	(0.6649)
		40:70	0.812	0.728	0.928	0.927	1.000	0.961	0.961
			(0.6237)	(0.6287)	(0.6481)	(0.6489)	(10.1251)	(0.6686)	**(0.6668)**
		40:100	0.659	0.61	0.932	0.933	1.000	0.961	0.964
			(0.6178)	(0.621)	(0.6243)	(0.6257)	(10.5625)	(0.6681)	**(0.6662)**
		70:70	0.944	0.836	0.942	0.944	1.000	0.952	0.95
			(0.5079)	(0.5058)	(0.5212)	(0.5209)	(10.6052)	(0.5052)	**(0.5039)**
		70:100	0.808	0.725	0.927	0.928	1.000	0.946	0.948
			(0.4838)	(0.4869)	(0.4959)	(0.4962)	**(17.8627)**	(0.507)	(0.5057)
		100:100	0.939	0.84	0.939	0.941	1.000	0.949	0.948
			(0.427)	(0.4272)	(0.4341)	(0.4342)	**(10.8555)**	(0.4243)	(0.4233)
		100:200	0.484	0.469	0.922	0.926	1.000	0.945	0.948
			(0.3974)	(0.3927)	(0.3978)	(0.3976)	**(10.6879)**	(0.424)	(0.4229)
		200:200	0.950	0.823	0.952	0.953	1.000	0.955	0.949
			(0.303)	(0.2983)	**(0.3052)**	(0.3049)	(10.0221)	(0.3014)	(0.3006)
	0.3:0.3	10:10	0.917	0.792	0.959	0.960	0.999	0.999	1.000
			(1.3132)	(1.3114)	(2.7192)	(2.7144)	(7.3613)	(1.4245)	**(1.4182)**
		10:40	0.800	0.708	0.959	0.961	0.996	0.992	0.994
			(1.2514)	(1.2607)	(2.6635)	(2.6621)	(6.8516)	(1.4246)	(1.4181)
		10:70	0.776	0.693	0.934	0.956	0.997	0.996	0.996
			(1.26)	(1.239)	(2.6407)	(2.6408)	(7.3056)	(1.4314)	**(1.4252)**
		10:100	0.781	0.693	0.944	0.946	1.000	0.998	0.998
			(1.2684)	(1.2851)	(2.5776)	(2.5776)	(7.5875)	(1.4219)	**(1.4159)**
		40:40	0.954	0.841	0.939	0.938	1.000	0.966	0.969
			(0.704)	(0.6956)	(1.2421)	(1.2424)	(9.0816)	(0.6796)	**(0.6777)**
		40:70	0.819	0.742	0.944	0.949	1.000	0.968	0.964
			(0.6663)	(0.6768)	(1.2178)	(1.2183)	(9.7567)	(0.6804)	**(0.6785)**
		40:100	0.691	0.63	0.941	0.94	1.000	0.964	0.962
			(0.6567)	(0.6557)	(1.2115)	(1.2126)	(9.2203)	(0.6786)	**(0.6767)**
		70:70	0.934	0.821	0.943	0.941	0.999	0.951	0.951
			(0.5374)	(0.5367)	(0.9252)	(0.9261)	(10.4404)	(0.5172)	**(0.5159)**
		70:100	0.83	0.729	0.961	0.962	1.000	0.960	0.963
			(0.5128)	(0.5123)	(0.9174)	(0.9179)	(9.6943)	(0.5175)	**(0.5161)**
		100:100	0.939	0.834	0.941	0.938	1.000	0.951	0.948
			(0.4497)	(0.4506)	(0.7761)	(0.7764)	(9.2568)	**(0.4336)**	(0.4324)
		100:200	0.519	0.504	0.947	0.948	1.000	0.958	0.956
			(0.4251)	(0.4274)	(0.7594)	(0.7598)	(9.3048)	(0.4344)	**(0.4332)**
		200:200	0.952	0.824	0.952	0.951	1.000	0.955	0.953
			(0.3198)	(0.3165)	(0.5471)	(0.5472)	(9.0699)	(0.3099)	**(0.309)**

TABLE 2-6 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference Between Two Means of the ZITR Distribution with $(\mu_1, \mu_2) = (0.1, 0.1)$ and $(\lambda_1, \lambda_2) = (0.5, 1)$.

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.5:1	0.1:0.1	10:10	0.896	0.817	0.949	0.949	1.000	0.991	0.992
			(1.0285)	(1.0522)	(1.2341)	(1.2397)	(10.0113)	(1.1726)	**(1.1676)**
		10:40	0.881	0.786	0.959	0.955	1.000	0.994	0.996
			(0.988)	(0.9988)	(1.2239)	(1.2267)	(9.557)	(1.1695)	**(1.1647)**
		10:70	0.901	0.821	0.931	0.933	1.000	0.992	0.992
			(1.0013)	(1.0347)	(1.2154)	(1.2205)	(9.97)	(1.1706)	**(1.1659)**
		10:100	0.881	0.795	0.949	0.948	1.000	0.992	0.993
			(0.9998)	(1.0166)	(1.2057)	(1.2112)	(9.5573)	(1.1766)	**(1.1717)**
		40:40	0.954	0.835	0.942	0.942	1.000	0.974	0.974
			(0.5491)	(0.5573)	(0.5674)	(0.5686)	(11.5186)	(0.5556)	**(0.5539)**
		40:70	0.926	0.817	0.943	0.946	1.000	0.958	0.959
			(0.5271)	(0.5236)	(0.5554)	(0.5562)	(11.7558)	(0.5507)	**(0.549)**
		40:100	0.92	0.796	0.946	0.945	1.000	0.963	0.964
			(0.5275)	(0.5181)	(0.5508)	(0.5525)	(12.3368)	(0.5528)	**(0.5511)**
		70:70	0.946	0.844	0.957	0.957	1.000	0.964	0.962
			(0.4139)	(0.4137)	(0.4267)	(0.4272)	(11.5407)	(0.413)	**(0.4119)**
		70:100	0.918	0.791	0.948	0.948	1.000	0.958	0.959
			(0.4047)	(0.3964)	(0.4179)	(0.4184)	(11.5026)	(0.4141)	**(0.413)**
		100:100	0.945	0.827	0.952	0.951	1.000	0.963	0.959
			(0.3489)	(0.3521)	(0.3538)	(0.3542)	(11.5221)	(0.3475)	**(0.3465)**
		100:200	0.905	0.812	0.94	0.938	1.000	0.957	0.956
			(0.3379)	(0.3475)	(0.3451)	(0.3454)	(11.3463)	(0.3463)	**(0.3453)**
		200:200	0.945	0.834	0.967	0.968	1.000	0.953	0.954
			(0.2471)	(0.249)	(0.2492)	(0.2494)	(11.1933)	(0.2466)	**(0.246)**
	0.1:0.3	10:10	0.931	0.82	0.965	0.966	0.999	0.986	0.989
			(1.0833)	(1.0766)	(1.3003)	(1.3013)	(8.8081)	(1.2705)	**(1.2657)**
		10:40	0.830	0.722	0.946	0.941	0.999	0.977	0.975
			(1.0224)	(1.026)	(1.2239)	(1.2268)	(9.4244)	(1.2608)	**(1.256)**
		10:70	0.778	0.698	0.945	0.941	0.998	0.966	0.965
			(1.0146)	(1.0205)	(1.2095)	(1.2191)	(8.6714)	(1.2564)	**(1.2515)**
		10:100	0.769	0.691	0.941	0.941	1.000	0.969	0.966
			(1.0215)	(0.9983)	(1.2026)	(1.2065)	(8.5638)	(1.2561)	**(1.251)**
		40:40	0.938	0.838	0.943	0.942	1.000	0.951	0.951
			(0.572)	(0.5714)	(0.594)	(0.5949)	(10.897)	(0.5891)	**(0.5875)**
		40:70	0.854	0.744	0.932	0.930	1.000	0.964	0.963
			(0.5452)	(0.5437)	(0.5657)	(0.5666)	(10.82)	(0.589)	**(0.5874)**
		40:100	0.732	0.663	0.939	0.940	1.000	0.954	0.952
			(0.5396)	(0.5448)	(0.5534)	(0.5542)	(10.4897)	(0.587)	**(0.5854)**
		70:70	0.937	0.822	0.95	0.949	1.000	0.952	0.951
			(0.4345)	(0.4365)	(0.4455)	(0.4459)	(10.5684)	(0.44)	**(0.4388)**
		70:100	0.839	0.748	0.934	0.936	1.000	0.945	0.949
			(0.4173)	(0.41)	(0.4307)	(0.4307)	(10.4528)	(0.4404)	(0.4392)
		100:100	0.944	0.83	0.944	0.948	1.000	0.949	0.949
			(0.3665)	(0.3652)	(0.3707)	(0.3709)	(10.5046)	(0.3683)	(0.3674)
		100:200	0.602	0.554	0.936	0.933	1.000	0.951	0.951
			(0.3465)	(0.3393)	(0.3487)	(0.3488)	(10.3036)	(0.3671)	**(0.3662)**
		200:200	0.955	0.827	0.953	0.954	1.000	0.958	0.958
			(0.2581)	(0.2566)	(0.2611)	(0.2611)	(9.9241)	(0.2598)	**(0.2591)**
	0.3:0.3	10:10	0.915	0.815	0.970	0.972	1.000	1.000	1.000
			(1.1538)	(1.1389)	(1.411)	(1.4078)	(7.3772)	(1.3262)	**(1.3196)**
		10:40	0.837	0.744	0.941	0.942	1.000	0.996	0.997
			(1.0985)	(1.1306)	(1.313)	(1.3143)	(7.3403)	(1.3196)	**(1.313)**
		10:70	0.809	0.717	0.936	0.941	0.999	0.996	0.997
			(1.0942)	(1.0907)	(1.3069)	(1.3059)	(8.0045)	(1.3117)	**(1.3052)**

TABLE 2-6 (Con)

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability						
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		10:100	0.796	0.712	0.937	0.938	0.999	1.000	1.000
			(1.1194)	(1.0977)	(1.2825)	(1.285)	(7.7049)	(1.3226)	**(1.3162)**
		40:40	0.925	0.819	0.946	0.949	1.000	0.96	0.961
			(0.6136)	(0.6199)	(0.6344)	(0.6355)	(9.2715)	(0.6069)	**(0.605)**
		40:70	0.834	0.738	0.939	0.942	1.000	0.97	0.969
			(0.5879)	(0.5867)	(0.6096)	(0.6099)	(9.131)	(0.6066)	**(0.6047)**
		40:100	0.769	0.682	0.942	0.94	1.000	0.975	0.974
			(0.5817)	(0.584)	(0.5968)	(0.5978)	(9.3129)	(0.6054)	**(0.6036)**
		70:70	0.944	0.829	0.955	0.953	1.000	0.967	0.965
			(0.4683)	(0.464)	(0.4758)	(0.4763)	(9.0969)	(0.4562)	**(0.4549)**
		70:100	0.882	0.759	0.944	0.941	1.000	0.952	0.954
			(0.4519)	(0.4503)	(0.4651)	(0.4654)	(9.0945)	(0.4589)	**(0.4576)**
		100:100	0.942	0.849	0.951	0.953	1.000	0.959	0.956
			(0.3922)	(0.3923)	(0.3976)	(0.3978)	(9.803)	(0.3818)	**(0.3808)**
		100:200	0.671	0.617	0.952	0.953	1.000	0.967	0.966
			(0.376)	(0.3736)	(0.3787)	(0.3785)	(8.9837)	(0.3844)	**(0.3833)**
		200:200	0.947	0.834	0.941	0.944	1.000	0.952	0.952
			(0.2787)	(0.2787)	(0.2798)	(0.2801)	(8.701)	(0.2716)	**(0.2708)**

TABLE 2-7 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference Between Two Means of the ZITR Distribution with $(\mu_1, \mu_2) = (0.1, 0.1)$ and $(\lambda_1, \lambda_2) = (1,1)$.

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
1:1	0.1:0.1	10:10	0.895	0.803	0.958	0.958	0.999	0.999	1.000
			(0.8471)	(0.8497)	**(1.0271)**	(1.0317)	(10.3602)	(1.0545)	(1.0508)
		10:40	0.906	0.795	0.961	0.955	1.000	1.000	1.000
			(0.8037)	(0.7976)	**(0.9798)**	(0.984)	(10.442)	(1.058)	(1.0546)
		10:70	0.887	0.753	0.964	0.965	0.999	1.000	1.000
			(0.8055)	(0.7921)	**(0.9796)**	(0.9832)	(10.9027)	(1.0509)	(1.0472)
		10:100	0.88	0.789	0.948	0.947	1.000	1.000	1.000
			(0.8098)	(0.8044)	(0.973)	(0.9772)	(11.1338)	(1.057)	**(1.0535)**
		40:40	0.939	0.828	0.959	0.959	1.000	0.984	0.985
			(0.4473)	(0.4468)	**(0.4674)**	(0.4684)	(11.7476)	(0.4695)	(0.4682)
		40:70	0.926	0.809	0.949	0.948	1.000	0.975	0.975
			(0.4323)	(0.4259)	(0.4554)	(0.4562)	(11.5642)	(0.4712)	**(0.4699)**
		40:100	0.917	0.805	0.942	0.943	1.000	0.984	0.982
			(0.4274)	(0.429)	(0.4442)	(0.4451)	(11.6768)	(0.4701)	**(0.4688)**
		70:70	0.934	0.801	0.956	0.954	1.000	0.961	0.964
			(0.3413)	(0.3272)	(0.3493)	(0.3499)	(11.1939)	(0.3483)	**(0.3473)**
		70:100	0.931	0.819	0.956	0.956	1.000	0.975	0.974
			(0.3305)	(0.3299)	(0.3418)	(0.3423)	(11.2472)	(0.3485)	**(0.3476)**
		100:100	0.942	0.832	0.947	0.948	1.000	0.957	0.958
			(0.2865)	(0.2818)	(0.2915)	(0.2919)	(11.1646)	(0.2907)	**(0.2899)**
		100:200	0.861	0.773	0.933	0.935	1.000	0.965	0.969
			(0.2742)	(0.2756)	(0.2796)	(0.28)	(11.2008)	(0.2909)	**(0.2901)**
		200:200	0.948	0.826	0.952	0.95	1.000	0.958	0.957
			(0.2035)	(0.2035)	(0.2048)	(0.2049)	(11.0479)	(0.2043)	**(0.2038)**
	0.1:0.3	10:10	0.919	0.789	0.963	0.964	1.000	1.000	1.000
			(0.9594)	(0.9407)	**(1.0855)**	(1.0869)	(8.0349)	(1.1936)	(1.1886)
		10:40	0.795	0.682	0.937	0.939	1.000	0.994	0.994
			(0.8368)	(0.8336)	(0.9902)	(0.9918)	(9.6171)	(1.1555)	**(1.1505)**
		10:70	0.728	0.647	0.925	0.926	1.000	0.993	0.995
			(0.8427)	(0.8405)	(0.972)	(0.9748)	(1.0378)	(1.1579)	**(1.1526)**
		10:100	0.713	0.631	0.924	0.927	1.000	0.992	0.996
			(0.8487)	(0.8434)	(0.9666)	(0.9698)	(9.2505)	(1.1515)	**(1.1462)**

TABLE 2-7 (Con)

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
	0.3:0.3	40:40	0.937	0.812	0.945	0.943	1.000	0.968	0.969
			(0.4782)	(0.4816)	(0.4981)	(0.4984)	(10.6903)	(0.507)	**(0.5055)**
		40:70	0.788	0.708	0.93	0.928	1.000	0.975	0.974
			(0.4446)	(0.4378)	(0.4649)	(0.4653)	(10.5807)	(0.5067)	**(0.5053)**
		40:100	0.637	0.597	0.929	0.927	1.000	0.977	0.975
			(0.4412)	(0.4384)	(0.4493)	(0.4497)	(10.5507)	(0.5093)	**(0.5079)**
		70:70	0.949	0.831	0.953	0.954	1.000	0.971	0.971
			(0.3649)	(0.3624)	(0.3726)	(0.3729)	(10.1877)	(0.3761)	**(0.3751)**
		70:100	0.813	0.735	0.944	0.944	1.000	0.958	0.957
			(0.3443)	(0.3474)	(0.3545)	(0.3549)	(10.312)	(0.3766)	**(0.3756)**
		100:100	0.949	0.831	0.945	0.943	1.000	0.958	0.959
			(0.3064)	(0.308)	(0.3104)	(0.3106)	(10.1941)	(0.3131)	**(0.3123)**
		100:200	0.494	0.481	0.927	0.924	1.000	0.957	0.962
			(0.2842)	(0.2864)	(0.2846)	(0.2848)	(10.2865)	(0.3128)	**(0.312)**
		200:200	0.947	0.809	0.945	0.945	1.000	0.954	0.952
			(0.2174)	(0.2129)	(0.2189)	(0.2191)	(9.8696)	(0.2199)	**(0.2194)**
	0.3:0.3	10:10	0.923	0.814	0.977	0.98	0.999	1.000	1.000
			(0.96)	(0.9701)	(1.1471)	**(1.1444)**	(8.2591)	(1.2028)	(1.1977)
		10:40	0.782	0.705	0.943	0.941	1.000	1.000	1.000
			(0.8831)	(0.8827)	(1.055)	(1.0534)	(7.4316)	(1.1996)	**(1.1942)**
		10:70	0.776	0.681	0.936	0.938	1.000	0.999	1.000
			(0.8868)	(0.8935)	(1.0383)	(1.0386)	(8.4906)	(1.1983)	**(1.1929)**
		10:100	0.767	0.675	0.921	0.924	1.000	1.000	1.000
			(0.8946)	(0.8991)	(1.026)	(1.0262)	(7.4111)	(1.1979)	**(1.1927)**
		40:40	0.943	0.802	0.947	0.948	1.000	0.978	0.978
			(0.5074)	(0.5077)	(0.5244)	(0.525)	(9.1198)	(0.518)	**(0.5165)**
		40:70	0.821	0.733	0.932	0.931	1.000	0.987	0.989
			(0.4777)	(0.481)	(0.4919)	(0.4923)	(9.1155)	(0.5246)	**(0.5231)**
		40:100	0.663	0.625	0.927	0.931	1.000	0.982	0.984
			(0.4742)	(0.4697)	(0.4801)	(0.48)	(9.2141)	(0.5192)	**(0.5178)**
		70:70	0.935	0.84	0.956	0.954	1.000	0.969	0.968
			(0.3882)	(0.3943)	(0.3937)	(0.394)	(9.1637)	(0.3879)	**(0.3869)**
		70:100	0.836	0.767	0.943	0.947	1.000	0.97	0.972
			(0.3693)	(0.3721)	(0.3769)	(0.3774)	(8.8447)	(0.3879)	**(0.3869)**
		100:100	0.938	0.822	0.959	0.956	1.000	0.956	0.954
			(0.3255)	(0.3221)	(0.3281)	(0.3285)	(8.8167)	(0.322)	**(0.3212)**
		100:200	0.509	0.502	0.923	0.923	1.000	0.958	0.956
			(0.3033)	(0.2971)	(0.3043)	(0.305)	(8.8153)	(0.3214)	**(0.3205)**
		200:200	0.952	0.824	0.949	0.95	1.000	0.955	0.953
			(0.3198)	(0.3165)	(0.2313)	**(0.2314)**	(9.0699)	(0.3099)	(0.309)

TABLE 2-8 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the ratio Means of the ZITR Distribution with $(\mu_1, \mu_2) = (0.1, 0.1)$ and $(\lambda_1, \lambda_2) = (0.1, 0.1)$.

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.1:0.1	0.1:0.1	10:10	0.898	0.801	0.956	0.958	0.999	0.948	0.935
			(1.2052)	(1.2072)	**(1.4395)**	(1.4492)	(2.9824)	(0.9513)	(0.9229)
		10:40	0.887	0.790	0.958	0.956	0.986	0.939	0.932
			(0.9467)	(0.946)	**(1.3086)**	(1.3147)	(2.8736)	(0.9556)	(0.9272)
		10:70	0.907	0.779	0.950	0.952	0.986	0.945	0.943
			(0.9434)	(0.946)	**(1.2594)**	(1.2657)	(3.2472)	(0.9631)	(0.9348)
		10:100	0.884	0.791	0.953	0.953	0.996	0.942	0.937
			(0.9218)	(0.9405)	**(1.2868)**	(1.2943)	(7.5441)	(0.9629)	(0.9344)
		40:40	0.94	0.825	0.953	0.954	0.999	0.948	0.949
			(0.5537)	(0.5542)	**(0.5677)**	(0.5695)	(4.2702)	(0.515)	(0.5093)
		40:70	0.924	0.810	0.940	0.941	1.000	0.955	0.952
			(0.494)	(0.5029)	(0.551)	(0.552)	(10.7472)	(0.5134)	**(0.5077)**
		40:100	0.905	0.787	0.934	0.937	1.000	0.949	0.948
			(0.4717)	(0.4695)	(0.5428)	(0.5446)	**(10.8491)**	(0.5116)	(0.5059)
		70:70	0.949	0.835	0.954	0.954	1.000	0.956	0.952
			(0.4129)	(0.4136)	(0.4214)	(0.422)	(4.3749)	(0.3933)	**(0.3902)**
		70:100	0.934	0.824	0.941	0.945	0.999	0.946	0.94
			(0.3795)	(0.3825)	(0.4122)	(0.4123)	**(12.0405)**	(0.3917)	(0.3886)
		100:100	0.94	0.828	0.947	0.945	1.000	0.94	0.939
			(0.3454)	(0.3387)	(0.3486)	(0.3492)	**(5.0112)**	(0.3321)	(0.3301)
		100:200	0.884	0.772	0.926	0.928	1.000	0.947	0.947
			(0.3049)	(0.3026)	(0.3348)	(0.3348)	**(11.5592)**	(0.3302)	(0.3282)
		200:200	0.937	0.817	0.949	0.951	1.000	0.943	0.943
			(0.2421)	(0.2415)	(0.245)	(0.2449)	**(12.9709)**	(0.2348)	(0.2338)
	0.1:0.3	10:10	0.920	0.840	0.959	0.955	0.968	0.914	0.895
			(2.4029)	(2.4899)	**(2.8877)**	(3.019)	(4.0483)	(1.2723)	(1.2207)
		10:40	0.782	0.662	0.930	0.926	0.974	0.915	0.881
			(1.1149)	(1.1277)	(2.2141)	(2.3829)	**(4.1506)**	(1.2838)	(1.2321)
		10:70	0.740	0.621	0.931	0.921	0.979	0.925	0.901
			(1.0946)	(1.1113)	(2.3544)	(2.0362)	**(4.1986)**	(1.2988)	(1.2452)
		10:100	0.704	0.588	0.918	0.906	0.983	0.905	0.874
			(1.0348)	(1.0182)	(2.2334)	(2.2629)	**(6.2433)**	(1.2512)	(1.2006)
		40:40	0.938	0.827	0.952	0.951	1.000	0.938	0.926
			(0.9317)	(0.9348)	**(0.9278)**	(0.9289)	(5.2578)	(0.7772)	(0.7628)
		40:70	0.765	0.638	0.940	0.940	1.000	0.931	0.915
			(0.6268)	(0.6189)	(0.8466)	(0.8481)	**(9.2486)**	(0.7731)	(0.759)
		40:100	0.623	0.535	0.933	0.933	0.999	0.935	0.924
			(0.5655)	(0.5578)	(0.8159)	(0.8169)	**(8.5717)**	(0.77)	(0.7561)

TABLE 2-8 (CON)

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		70:70	0.942	0.849	0.949	0.950	0.999	0.943	0.938
			(0.679)	(0.6794)	(0.6865)	**(0.6879)**	(10.1438)	(0.6104)	(0.6031)
		70:100	0.811	0.693	0.946	0.946	0.997	0.94	0.93
			(0.5139)	(0.508)	(0.6389)	(0.6397)	**(10.0585)**	(0.6047)	(0.5973)
		100:100	0.948	0.834	0.948	0.946	1.000	0.942	0.935
			(0.5543)	(0.5583)	(0.5603)	(0.5611)	**(10.6036)**	(0.5108)	(0.506)
		100:200	0.404	0.385	0.927	0.924	1.000	0.936	0.926
			(0.3712)	(0.3731)	(0.5015)	(0.5016)	**(11.2478)**	(0.5119)	(0.5069)
		200:200	0.953	0.844	0.954	0.954	1.000	0.949	0.947
			(0.3862)	(0.3834)	**(0.3889)**	(0.3892)	(10.993)	(0.37)	(0.3676)
	0.3:0.3	10:10	0.898	0.847	0.97	0.963	0.957	0.941	0.943
			(2.2042)	(2.2803)	(2.6098)	**(2.4737)**	(3.0905)	(1.2364)	(1.1822)
		10:40	0.828	0.687	0.953	0.946	0.961	0.96	0.96
			(1.054)	(1.0609)	(2.1257)	(1.3)	(3.1736)	(1.1936)	**(1.1423)**
		10:70	0.786	0.692	0.946	0.944	0.96	0.951	0.95
			(1.0343)	(1.0554)	(2.028)	(2,1346)	(3.5037)	(1.2122)	**(1.1608)**
		10:100	0.789	0.685	0.945	0.937	0.985	0.973	0.963
			(0.9942)	(1.0215)	(2.111)	(2.0775)	(5.4823)	(1.1998)	**(1.1484)**
		40:40	0.946	0.86	0.949	0.948	0.997	0.949	0.953
			(0.8312)	(0.8492)	(0.8383)	(0.8405)	(3.8788)	(0.7021)	**(0.6895)**
		40:70	0.809	0.675	0.939	0.943	1.000	0.945	0.949
			(0.5872)	(0.5782)	(0.7773)	(0.7796)	**(8.6319)**	(0.6908)	(0.6786)
		40:100	0.695	0.586	0.937	0.934	1.000	0.951	0.952
			(0.5388)	(0.5336)	(0.7607)	(0.7609)	(7.903)	(0.6948)	**(0.6824)**
		70:70	0.948	0.847	0.953	0.951	0.999	0.942	0.942
			(0.6023)	(0.6087)	**(0.6085)**	(0.6089)	(4.4774)	(0.5382)	(0.532)
		70:100	0.836	0.719	0.938	0.936	0.999	0.941	0.942
			(0.4742)	(0.4696)	(0.5877)	(0.5885)	**(8.6708)**	(0.5333)	(0.5269)
		100:100	0.936	0.848	0.943	0.947	0.999	0.938	0.943
			(0.501)	(0.5095)	(0.5058)	(0.5065)	(4.7516)	(0.4601)	(0.4559)
		100:200	0.549	0.468	0.935	0.935	1.000	0.933	0.928
			(0.3558)	(0.3533)	(0.4638)	(0.4635)	**(9.5176)**	(0.4583)	(0.454)
		200:200	0.948	0.862	0.951	0.947	1.000	0.944	0.953
			(0.3488)	(0.3497)	(0.3508)	(0.351)	(9.5204)	(0.3297)	**(0.3277)**

TABLE 2-9 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the ratio Means of the ZITR Distribution with $(\mu_1, \mu_2) = (0.1, 0.1)$ and $(\lambda_1, \lambda_2) = (0.1, 0.5)$.

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.1:0.5	0.1:0.1	10:10	0.899	0.822	0.958	0.958	1.000	0.942	0.917
			(2.5124)	(2.5925)	**(3.061)**	(3.0894)	(13.6701)	(1.8539)	(1.7989)
		10:40	0.893	0.791	0.965	0.967	0.993	0.944	0.92
			(1.9601)	(1.9751)	**(2.6672)**	(2.6805)	(12.8757)	(1.8496)	(1.7941)
		10:70	0.899	0.802	0.946	0.946	0.998	0.954	0.931
			(1.946)	(1.9429)	(2.6814)	(2.6875)	(14.2668)	**(1.846)**	(1.7911)
		10:100	0.890	0.780	0.952	0.952	0.987	0.942	0.915
			(1.9256)	(1.9281)	**(2.688)**	(2.698)	(8.8907)	(1.8628)	(1.8066)
		40:40	0.946	0.840	0.961	0.964	1.000	0.949	0.940
			(1.1529)	(1.161)	**(1.2106)**	(1.2149)	(16.9036)	(1.0563)	(1.0446)
		40:70	0.928	0.823	0.948	0.949	1.000	0.956	0.946
			(1.037)	(1.0553)	(1.1715)	(1.174)	(10.9058)	**(1.0725)**	(1.0605)
		40:100	0.906	0.804	0.936	0.940	0.999	0.953	0.949
			(1.0115)	(1.0055)	(1.1349)	(1.1372)	(10.8462)	**(1.0779)**	(1.0659)
		70:70	0.964	0.832	0.946	0.944	1.000	0.963	0.958
			(0.8752)	(0.8624)	(0.8958)	(0.898)	(16.1578)	(0.8278)	**(0.8215)**
		70:100	0.92	0.834	0.945	0.945	1.000	0.939	0.935
			(0.8047)	(0.803)	(0.8628)	(0.8636)	**(11.7842)**	(0.8269)	(0.8206)
		100:100	0.951	0.853	0.946	0.946	1.000	0.949	0.944
			(0.726)	(0.7264)	(0.7372)	(0.7385)	(16.2402)	(0.6931)	(0.6889)
		100:200	0.884	0.763	0.943	0.939	1.000	0.943	0.940
			(0.6382)	(0.636)	(0.7088)	(0.7087)	**(17.1126)**	(0.6931)	(0.6888)
		200:200	0.935	0.813	0.952	0.957	1.000	0.936	0.934
			(0.5116)	(0.5131)	**(0.5149)**	(0.5146)	(15.84)	(0.5004)	(0.4982)
	0.1:0.3	10:10	0.923	0.866	0.966	0.962	0.994	0.857	0.796
			(5.002)	(5.1631)	**(6.5869)**	(8.5412)	(18.3253)	(2.3842)	(2.2873)
		10:40	0.775	0.66	0.937	0.93	0.994	0.869	0.812
			(2.3925)	(2.3869)	(4.6888)	(4.5245)	**(18.788)**	(2.4419)	(2.3423)
		10:70	0.717	0.606	0.943	0.936	0.994	0.854	0.782
			(2.1867)	(2.2729)	(4.5653)	(3.9663)	**(16.7959)**	(2.3824)	(2.2854)
		10:100	0.685	0.564	0.924	0.914	0.938	0.828	0.775
			(2.1642)	(2.1659)	(4.7188)	(6.0957)	(7.3348)	(2.3689)	(2.2706)
		40:40	0.944	0.844	0.95	0.948	1.000	0.926	0.914
			(1.9878)	(2.0071)	**(1.9817)**	(1.9857)	(19.4643)	(1.6146)	(1.585)
		40:70	0.776	0.631	0.932	0.931	0.999	0.917	0.904
			(1.3175)	(1.3164)	(1.7943)	(1.7957)	**(11.9177)**	(1.6096)	(1.5796)
		40:100	0.585	0.515	0.909	0.914	0.999	0.925	0.908
			(1.1799)	(1.1823)	(1.7121)	(1.7151)	**(9.9227)**	(1.591)	(1.5618)
		70:70	0.955	0.85	0.956	0.957	1.000	0.939	0.925
			(1.4222)	(1.4454)	**(1.4453)**	(1.4458)	(21.1815)	(1.2678)	(1.2522)
		70:100	0.797	0.672	0.932	0.932	1.000	0.934	0.920
			(1.0749)	(1.074)	(1.3618)	(1.3635)	**(10.3027)**	(1.2606)	(1.2452)
		100:100	0.948	0.836	0.953	0.955	1.000	0.945	0.937
			(1.1771)	(1.1572)	**(1.1782)**	(1.1791)	(21.323)	(1.084)	(1.0737)
		100:200	0.390	0.401	0.927	0.926	1.000	0.94	0.934
			(0.7839)	(0.7851)	(1.0566)	(1.0566)	**(10.6641)**	(1.086)	(1.0757)

TABLE 2-9 (CON)

$\lambda_1:\lambda_2$	$\delta_1:\delta_2$	Coverage probability						
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		200:200	0.956	0.839	0.955	0.951	0.999	0.952	0.947
			(0.8179)	(0.8043)	(0.8238)	(0.824)	(21.1228)	**(0.7834)**	(0.7785)
	0.3:0.3	10:10	0.922	0.856	0.968	0.965	0.957	0.953	0.923
			(4.4622)	(4.5134)	(5.3097)	(6.3643)	(6.7539)	**(2.2683)**	(2.1711)
		10:40	0.801	0.712	0.947	0.941	0.993	0.948	0.924
			(2.2505)	(2.2485)	(4.283)	(1.0791)	**(13.5859)**	(2.2813)	(2.1841)
		10:70	0.79	0.676	0.947	0.941	0.988	0.947	0.927
			(2.1603)	(2.1684)	(4.5351)	(1.4561)	**(13.4526)**	(2.2882)	(2.1889)
		10:100	0.758	0.669	0.944	0.938	0.950	0.940	0.915
			(2.1041)	(2.1183)	(4.0039)	(2.3349)	**(6.8369)**	(2.2585)	(2.1613)
		40:40	0.949	0.842	0.948	0.949	1.000	0.957	0.948
			(1.7643)	(1.82)	(1.7857)	(1.7893)	(15.5408)	**(1.4387)**	(1.4127)
		40:70	0.834	0.706	0.945	0.954	0.997	0.947	0.939
			(1.2537)	(1.2581)	(1.6737)	**(1.6747)**	(8.6506)	(1.4325)	(1.4071)
		40:100	0.697	0.589	0.92	0.922	0.998	0.957	0.947
			(1.137)	(1.1571)	(1.5997)	(1.5996)	(8.2399)	**(1.4326)**	(1.4072)
		70:70	0.944	0.852	0.951	0.951	1.000	0.945	0.944
			(1.2906)	(1.2804)	**(1.2894)**	(1.291)	(15.9439)	(1.1402)	(1.1264)
		70:100	0.878	0.759	0.949	0.947	0.999	0.951	0.949
			(1.0159)	(1.0332)	(1.2324)	(1.2335)	(9.3795)	**(1.1337)**	(1.1203)
		100:100	0.949	0.825	0.948	0.948	0.999	0.953	0.945
			(1.0601)	(1.0568)	(1.0751)	(1.0758)	(9.1519)	**(0.9587)**	(0.9496)
		100:200	0.544	0.487	0.936	0.933	1.000	0.938	0.937
			(0.7508)	(0.7511)	(0.9926)	(0.9931)	**(16.8525)**	(0.9643)	(0.9555)
		200:200	0.952	0.828	0.943	0.940	1.000	0.955	0.957
			(0.3431)	(0.3393)	(0.7445)	(0.7453)	(9.6765)	(0.3258)	**(0.3238)**

TABLE 2-10 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the ratio Means of the ZITR Distribution with $(\mu_1, \mu_2) = (0.1, 0.1)$ and $(\lambda_1, \lambda_2) = (0.5, 0.5)$.

$\lambda_1:\lambda_2$	$\delta_1:\delta_2$	Coverage probability							
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.5:0.5	0.1:0.1	10:10	0.898	0.828	0.966	0.967	0.997	0.986	0.986
			(1.1891)	(1.2009)	(1.394)	(1.4063)	(9.7894)	(0.9879)	**(0.957)**
		10:40	0.897	0.798	0.957	0.958	0.999	0.991	0.99
			(0.9039)	(0.9003)	(1.2602)	(1.2655)	(8.2771)	(0.9807)	**(0.9503)**
		10:70	0.904	0.801	0.951	0.952	0.999	0.995	0.993
			(0.8779)	(0.8854)	(1.2226)	(1.2304)	(9.5199)	(0.9757)	**(0.9449)**
		10:100	0.885	0.799	0.952	0.952	0.997	0.99	0.989
			(0.8898)	(0.8749)	(1.2308)	(1.2346)	(9.2473)	(0.9807)	**(0.9505)**
		40:40	0.946	0.827	0.950	0.953	1.000	0.968	0.970
			(0.5316)	(0.5353)	(0.561)	(0.562)	(10.494)	(0.5082)	**(0.5027)**
		40:70	0.926	0.788	0.933	0.935	1.000	0.973	0.971
			(0.4746)	(0.4744)	(0.5334)	(0.5338)	(10.3271)	(0.5104)	**(0.5048)**
		40:100	0.897	0.780	0.950	0.948	1.000	0.964	0.964
			(0.4562)	(0.4541)	(0.5288)	(0.5298)	(10.0574)	(0.5082)	**(0.5026)**
		70:70	0.943	0.825	0.963	0.964	1.000	0.958	0.953
			(0.4014)	(0.4007)	(0.4094)	(0.4104)	(9.9949)	(0.3885)	**(0.3856)**
		70:100	0.931	0.812	0.933	0.933	1.000	0.96	0.962
			(0.3687)	(0.3669)	(0.398)	(0.3986)	(10.6754)	(0.3885)	**(0.3855)**
		100:100	0.952	0.829	0.954	0.952	1.000	0.957	0.960
			(0.3336)	(0.332)	(0.339)	(0.3396)	(9.7897)	(0.3259)	**(0.3239)**
		100:200	0.889	0.755	0.931	0.937	1.000	0.949	0.951
			(0.2947)	(0.2925)	(0.3248)	(0.3253)	(9.7177)	(0.327)	**(0.325)**
		200:200	0.946	0.847	0.957	0.960	1.000	0.952	0.951
			(0.2338)	(0.2351)	(0.2361)	(0.2362)	(9.1347)	(0.2309)	**(0.2299)**

TABLE 2-10 (CON)

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
	0.1:0.3	10:10	0.906	0.869	0.959	0.960	0.999	0.961	0.938
			(2.4326)	(2.5332)	(2.9349)	(2.2742)	(11.7912)	**(1.2824)**	(1.2278)
		10:40	0.750	0.637	0.935	0.928	0.999	0.958	0.932
			(1.0823)	(1.086)	(2.2574)	(1.4859)	(12.5057)	**(1.2739)**	(1.221)
		10:70	0.737	0.617	0.927	0.924	0.991	0.966	0.944
			(1.0023)	(1.0277)	(2.0667)	(2.1796)	(8.5678)	**(1.2496)**	(1.1975)
		10:100	0.720	0.589	0.906	0.902	0.999	0.965	0.942
			(1.0153)	(1.0145)	(2.0539)	(1.5965)	(11.3867)	**(1.2692)**	(1.2165)
		40:40	0.933	0.839	0.945	0.947	1.000	0.940	0.922
			(0.8984)	(0.8983)	(0.9308)	(0.933)	(13.6018)	(0.761)	(0.747)
		40:70	0.781	0.667	0.928	0.929	1.000	0.958	0.945
			(0.6094)	(0.6146)	(0.8287)	(0.8298)	(12.4913)	**(0.7727)**	(0.7585)
		40:100	0.587	0.517	0.927	0.929	1.000	0.954	0.937
			(0.5437)	(0.5459)	(0.7847)	(0.7861)	(12.7442)	**(0.7642)**	(0.7504)
		70:70	0.951	0.846	0.945	0.946	1.000	0.948	0.936
			(0.6572)	(0.654)	(0.6645)	(0.6645)	(12.8456)	(0.5966)	(0.5893)
		70:100	0.772	0.672	0.923	0.926	1.000	0.940	0.936
			(0.4957)	(0.4999)	(0.6244)	(0.625)	**(20.5803)**	(0.5956)	(0.5882)
		100:100	0.933	0.849	0.943	0.941	1.000	0.946	0.936
			(0.5494)	(0.5492)	(0.5522)	(0.5523)	**(13.0645)**	(0.5112)	(0.5062)
		100:200	0.405	0.351	0.920	0.921	1.000	0.948	0.942
			(0.3596)	(0.3553)	(0.4924)	(0.4921)	**(12.8408)**	(0.5073)	(0.5025)
		200:200	0.948	0.826	0.953	0.954	1.000	0.949	0.942
			(0.3768)	(0.3709)	**(0.3823)**	(0.3818)	(11.771)	(0.3638)	(0.3616)
	0.3:0.3	10:10	0.917	0.841	0.968	0.965	0.995	0.999	0.995
			(2.0623)	(2.1062)	(5.3097)	(6.3643)	(8.9779)	(1.2329)	**(1.1771)**
		10:40	0.800	0.683	0.947	0.941	0.999	0.992	0.994
			(1.0287)	(1.0431)	(4.283)	(1.0791)	(8.285)	(1.226)	**(1.1711)**
		10:70	0.776	0.664	0.947	0.941	0.994	0.996	0.991
			(0.9798)	(0.9664)	(4.5351)	(1.4561)	(7.3056)	(1.2399)	**(1.1833)**
		10:100	0.781	0.663	0.944	0.938	0.997	0.998	0.995
			(0.9773)	(0.9897)	(4.0039)	(2.3349)	(9.2067)	(1.2417)	**(1.1853)**
		40:40	0.954	0.854	0.948	0.949	1.000	0.966	0.971
			(0.8161)	(0.8097)	(1.7857)	(1.7893)	(9.9028)	(0.6965)	**(0.684)**
		40:70	0.819	0.721	0.945	0.954	1.000	0.968	0.97
			(0.5767)	(0.588)	(1.6737)	(1.6747)	(10.7333)	(0.6896)	**(0.6775)**
		40:100	0.692	0.593	0.92	0.922	1.000	0.964	0.964
			(0.5245)	(0.5245)	(1.5997)	(1.5996)	(10.3552)	(0.6953)	**(0.6828)**
		70:70	0.934	0.826	0.951	0.951	1.000	0.951	0.954
			(0.5957)	(0.5935)	(1.2894)	(1.291)	(11.1853)	(0.5372)	**(0.5308)**
		70:100	0.830	0.707	0.949	0.947	1.000	0.96	0.957
			(0.4663)	(0.4656)	(1.2324)	(1.2335)	(10.4539)	(0.5355)	**(0.5293)**
		100:100	0.939	0.842	0.948	0.948	1.000	0.951	0.948
			(0.4904)	(0.4907)	(1.0751)	(1.0758)	(9.849)	**(0.4527)**	(0.4486)
		100:200	0.519	0.470	0.936	0.933	1.000	0.958	0.955
			(0.3446)	(0.3455)	(0.9926)	(0.9931)	(9.8866)	(0.4496)	**(0.4454)**
		200:200	0.952	0.828	0.943	0.940	1.000	0.955	0.957
			(0.3431)	(0.3393)	(0.7445)	(0.7453)	(9.6765)	(0.3258)	**(0.3238)**

TABLE 2-11 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the ratio Means of the ZITR Distribution with $(\mu_1, \mu_2) = (0.1, 0.1)$ and $(\lambda_1, \lambda_2) = (0.5, 1)$.

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.5:1	0.1:0.1	10:10	0.902	0.849	0.950	0.951	1.000	0.986	0.973
			(1.6085)	(1.6692)	(1.8641)	(1.8794)	(15.4192)	(1.2385)	**(1.1994)**
		10:40	0.882	0.774	0.957	0.954	0.999	0.991	0.978
			(1.2161)	(1.2259)	(1.6796)	(1.6862)	(14.5058)	(1.2234)	**(1.185)**
		10:70	0.899	0.800	0.927	0.926	1.000	0.987	0.975
			(1.2063)	(1.2425)	(1.6681)	(1.6745)	(9.97)	(1.2251)	**(1.1864)**
		10:100	0.869	0.784	0.945	0.943	0.999	0.992	0.978
			(1.1805)	(1.2033)	(1.6206)	(1.6274)	(13.9863)	(1.2224)	**(1.184)**
		40:40	0.948	0.841	0.944	0.939	1.000	0.971	0.969
			(0.725)	(0.7375)	(0.7492)	(0.7513)	(15.8592)	(0.6806)	**(0.6731)**
		40:70	0.917	0.814	0.939	0.943	1.000	0.96	0.952
			(0.6381)	(0.6325)	(0.7168)	(0.7179)	(16.0949)	(0.6784)	**(0.6707)**
		40:100	0.901	0.774	0.938	0.942	1.000	0.966	0.96
			(0.6176)	(0.6085)	(0.7111)	(0.713)	(16.7339)	(0.6782)	**(0.6706)**
		70:70	0.94	0.847	0.956	0.957	1.000	0.958	0.952
			(0.5417)	(0.5424)	(0.556)	(0.557)	(11.5407)	(0.5199)	**(0.516)**
		70:100	0.915	0.78	0.944	0.944	1.000	0.96	0.951
			(0.4969)	(0.4864)	(0.5394)	(0.5397)	(15.6492)	(0.5217)	**(0.5177)**
		100:100	0.95	0.839	0.946	0.947	1.000	0.961	0.957
			(0.4542)	(0.4608)	(0.459)	(0.4598)	(15.628)	(0.4412)	**(0.4384)**
		100:200	0.891	0.782	0.929	0.932	1.000	0.954	0.948
			(0.4003)	(0.4112)	(0.4401)	(0.4406)	(15.5066)	**(0.4409)**	(0.4382)
		200:200	0.946	0.831	0.965	0.966	1.000	0.955	0.952
			(0.3199)	(0.3213)	(0.3224)	(0.3227)	(15.1855)	(0.3157)	**(0.3143)**
	0.1:0.3	10:10	0.928	0.861	0.968	0.964	1.000	0.917	0.872
			(3.1737)	(3.2835)	**(4.0275)**	(4.2649)	(20.2615)	(1.555)	(1.4894)
		10:40	0.758	0.642	0.921	0.913	0.999	0.915	0.862
			(1.4608)	(1.4689)	(2.9838)	(3.1197)	**(21.7976)**	(1.5491)	(1.4843)
		10:70	0.723	0.591	0.938	0.922	0.980	0.91	0.855
			(1.3487)	(1.3488)	(2.7793)	(1.9569)	**(8.6714)**	(1.5371)	(1.4726)
		10:100	0.682	0.557	0.93	0.924	0.998	0.905	0.858
			(1.3203)	(1.2898)	(2.6547)	(3.0178)	**(19.5596)**	(1.5338)	(1.4701)
		40:40	0.945	0.835	0.947	0.949	1.000	0.931	0.903
			(1.2159)	(1.2074)	(1.2579)	(1.2603)	**(21.9385)**	(1.0053)	(0.9869)
		40:70	0.777	0.662	0.922	0.926	1.000	0.941	0.924
			(0.8288)	(0.8317)	(1.1345)	(1.136)	**(21.4322)**	(1.0251)	(1.0061)
		40:100	0.583	0.488	0.923	0.925	1.000	0.933	0.913
			(0.7344)	(0.7434)	(1.0792)	(1.0805)	**(20.8017)**	(1.0109)	(0.9922)
		70:70	0.939	0.834	0.949	0.952	1.000	0.932	0.911
			(0.8877)	(0.8867)	(0.9089)	**(0.9104)**	(10.5684)	(0.7968)	(0.7871)
		70:100	0.789	0.663	0.938	0.937	1.000	0.932	0.913
			(0.6689)	(0.6598)	(0.8432)	(0.8437)	**(20.3708)**	(0.7967)	(0.7872)
		100:100	0.943	0.839	0.942	0.941	1.000	0.949	0.94
			(0.7475)	(0.7501)	(0.7463)	(0.7468)	**(20.6395)**	(0.6889)	(0.6822)
		100:200	0.362	0.324	0.922	0.924	1.000	0.943	0.934
			(0.4874)	(0.4766)	(0.6602)	(0.6607)	**(10.3036)**	(0.6805)	(0.6739)
		200:200	0.941	0.821	0.946	0.946	1.000	0.946	0.942
			(0.5159)	(0.5122)	(0.5163)	(0.5169)	**(19.4086)**	(0.4978)	(0.4946)
	0.3:0.3	10:10	0.915	0.852	0.975	0.971	0.996	0.996	0.984
			(2.7894)	(2.8138)	(3.6066)	(3.4667)	(14.7114)	(1.5104)	**(1.4414)**
		10:40	0.804	0.706	0.942	0.938	0.996	0.995	0.989
			(1.3841)	(1.4159)	(2.6966)	(2.7398)	(14.132)	(1.5072)	**(1.4393)**
		10:70	0.774	0.659	0.932	0.922	0.990	0.992	0.986
			(1.3176)	(1.3145)	(2.6355)	(1.9825)	(8.0045)	(1.5082)	**(1.44)**

TABLE 2-11 (CON)

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability						
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		10:100	0.751	0.644	0.934	0.925	0.998	0.994	0.982
			(1.3037)	(1.2707)	(2.6254)	(2.4449)	(15.8105)	(1.5153)	**(1.4462)**
		40:40	0.919	0.845	0.948	0.947	1.000	0.953	0.946
			(1.0908)	(1.1033)	(1.1263)	(1.1291)	(16.4584)	**(0.9067)**	(0.8904)
		40:70	0.785	0.664	0.930	0.930	1.000	0.961	0.95
			(0.7711)	(0.7715)	(1.0362)	(1.0373)	(15.9752)	(0.902)	**(0.8858)**
		40:100	0.678	0.588	0.926	0.93	1.000	0.977	0.967
			(0.7086)	(0.71)	(0.992)	(0.9938)	(16.803)	(0.9185)	**(0.9022)**
		70:70	0.942	0.828	0.955	0.955	1.000	0.957	0.953
			(0.809)	(0.7951)	(0.8183)	(0.8194)	(9.0969)	(0.7176)	**(0.7091)**
		70:100	0.841	0.705	0.937	0.94	1.000	0.962	0.953
			(0.6355)	(0.6339)	(0.7835)	(0.7844)	(16.098)	(0.7229)	**(0.7144)**
		100:100	0.941	0.857	0.957	0.956	1.000	0.953	0.95
			(0.6726)	(0.6765)	(0.6714)	(0.6715)	(17.2198)	(0.6128)	**(0.6072)**
		100:200	0.557	0.503	0.946	0.945	1.000	0.958	0.959
			(0.4715)	(0.4701)	(0.6186)	(0.6183)	(15.8078)	(0.6156)	**(0.6099)**
		200:200	0.949	0.844	0.947	0.945	1.000	0.950	0.952
			(0.4668)	(0.4674)	(0.4685)	(0.4689)	(15.1548)	(0.4413)	**(0.4385)**

TABLE 2-12 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the ratio Means of the ZITR Distribution with $(\mu_1, \mu_2) = (0.1, 0.1)$ and $(\lambda_1, \lambda_2) = (1,1)$.

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability						
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
1:1	0.1:0.1	10:10	0.895	0.824	0.958	0.958	0.999	0.999	0.999
			(1.1517)	(1.1724)	(1.3634)	(1.3731)	(12.9024)	(1.0028)	**(0.9708)**
		10:40	0.906	0.793	0.961	0.956	1.000	1.000	0.999
			(0.883)	(0.8741)	(1.1836)	(1.188)	(13.0147)	(1.0081)	**(0.975)**
		10:70	0.887	0.766	0.965	0.965	1.000	1.000	1.000
			(0.8706)	(0.8618)	(1.183)	(1.1876)	(13.7601)	(1.0013)	**(0.9693)**
		10:100	0.88	0.786	0.948	0.947	1.000	1.000	1.000
			(0.8598)	(0.8562)	(1.1741)	(1.1939)	(14.0276)	(1.0004)	**(0.9685)**
		40:40	0.939	0.829	0.959	0.959	1.000	0.984	0.984
			(0.5176)	(0.5144)	(0.5427)	(0.544)	(13.6094)	(0.5087)	**(0.5033)**
		40:70	0.926	0.807	0.949	0.948	1.000	0.975	0.979
			(0.4664)	(0.4584)	(0.5253)	(0.5264)	(13.2948)	(0.5154)	**(0.5095)**
		40:100	0.917	0.790	0.942	0.943	1.000	0.984	0.982
			(0.4471)	(0.4494)	(0.5088)	(0.5095)	(13.4411)	(0.5134)	**(0.5078)**
		70:70	0.934	0.806	0.956	0.954	1.000	0.961	0.962
			(0.3917)	(0.3754)	(0.3996)	(0.4004)	(12.7013)	(0.3861)	**(0.3832)**
		70:100	0.931	0.818	0.956	0.958	1.000	0.975	0.974
			(0.3586)	(0.3575)	(0.3898)	(0.3906)	(12.7383)	(0.3856)	**(0.3827)**
		100:100	0.942	0.825	0.947	0.948	1.000	0.957	0.956
			(0.3271)	(0.3214)	(0.3319)	(0.3325)	(12.6701)	(0.3248)	**(0.3228)**
		100:200	0.861	0.762	0.933	0.935	1.000	0.965	0.97
			(0.2864)	(0.2878)	(0.3173)	(0.3178)	(12.6259)	(0.3231)	**(0.3211)**
		200:200	0.948	0.824	0.952	0.949	1.000	0.958	0.956
			(0.2305)	(0.2307)	(0.2318)	(0.232)	(12.4838)	(0.2293)	**(0.2283)**
	0.1:0.3	10:10	0.919	0.835	0.964	0.958	1.000	1.000	1.000
			(2.0215)	(2.0125)	(2.9968)	(3.093)	(12.9967)	(1.2418)	**(1.1852)**
		10:40	0.774	0.631	0.934	0.926	1.000	0.991	0.977
			(1.0617)	(1.0552)	(2.1676)	(2.0984)	(18.0072)	(1.28)	**(1.2246)**
		10:70	0.692	0.582	0.920	0.912	1.000	0.98	0.963
			(0.985)	(0.9804)	(2.0969)	(1.6644)	(17.5805)	(1.2703)	**(1.2152)**
		10:100	0.673	0.555	0.917	0.906	0.998	0.981	0.964
			(0.9587)	(0.9535)	(2.1455)	(2.0073)	(17.3471)	(1.2546)	**(1.2006)**

TABLE 2-12 (CON)

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
	0.3:0.3	40:40	0.937	0.826	0.945	0.944	1.000	0.957	0.947
			(0.9069)	(0.9183)	(0.9165)	(0.918)	(18.2027)	**(0.769)**	(0.7548)
		40:70	0.764	0.641	0.929	0.926	1.000	0.966	0.947
			(0.5926)	(0.5841)	(0.8076)	(0.8084)	(17.723)	**(0.7603)**	(0.7464)
		40:100	0.564	0.495	0.929	0.929	1.000	0.971	0.948
			(0.5257)	(0.5223)	(0.7658)	(0.766)	(17.5106)	**(0.7584)**	(0.7444)
		70:70	0.951	0.847	0.955	0.957	1.000	0.969	0.963
			(0.6605)	(0.657)	(0.6592)	(0.6598)	(16.7957)	(0.6029)	**(0.5954)**
		70:100	0.784	0.683	0.939	0.940	1.000	0.951	0.94
			(0.4861)	(0.4896)	(0.621)	(0.6215)	(17.0425)	**(0.5937)**	(0.5865)
		100:100	0.946	0.841	0.944	0.944	1.000	0.951	0.949
			(0.5395)	(0.543)	(0.5433)	(0.544)	(16.7848)	**(0.5069)**	(0.5021)
		100:200	0.373	0.364	0.930	0.929	1.000	0.947	0.945
			(0.3534)	(0.3547)	(0.4803)	(0.4805)	**(10.2865)**	(0.5034)	(0.4986)
		200:200	0.943	0.807	0.942	0.949	1.000	0.947	0.944
			(0.3751)	(0.367)	(0.3751)	(0.3755)	**(16.1312)**	(0.3647)	(0.3625)
	0.3:0.3	10:10	0.923	0.866	0.98	0.979	1.000	1.000	1.000
			(2.0376)	(2.1342)	(2.4464)	(2.5956)	(14.0034)	(1.2531)	**(1.1947)**
		10:40	0.782	0.682	0.944	0.942	0.999	1.000	0.999
			(0.9988)	(0.9956)	(1.9536)	(2.1447)	(12.2856)	(1.2556)	**(1.1974)**
		10:70	0.775	0.678	0.941	0.936	0.995	0.999	0.999
			(0.9522)	(0.9504)	(1.8719)	(1.8464)	(14.2481)	(1.2573)	**(1.1993)**
		10:100	0.767	0.655	0.924	0.921	0.998	1.000	1.000
			(0.9263)	(0.9351)	(1.8612)	(1.8936)	(12.253)	(1.254)	**(1.1957)**
		40:40	0.943	0.816	0.947	0.947	1.000	0.978	0.978
			(0.7979)	(0.7975)	(0.818)	(0.8193)	(13.65)	(0.6934)	**(0.681)**
		40:70	0.821	0.699	0.932	0.931	1.000	0.987	0.983
			(0.5697)	(0.5726)	(0.7508)	(0.7517)	(13.7719)	(0.7092)	**(0.6963)**
		40:100	0.664	0.585	0.927	0.931	1.000	0.982	0.982
			(0.5138)	(0.5068)	(0.7262)	(0.7262)	(13.6826)	(0.6906)	**(0.6784)**
		70:70	0.935	0.849	0.956	0.954	1.000	0.969	0.958
			(0.5862)	(0.5976)	(0.5894)	(0.5905)	(13.525)	(0.5375)	**(0.531)**
		70:100	0.836	0.730	0.943	0.948	1.000	0.970	0.969
			(0.4611)	(0.4643)	(0.5678)	(0.5685)	(13.0638)	(0.5404)	**(0.5339)**
		100:100	0.938	0.827	0.959	0.955	1.000	0.956	0.957
			(0.4858)	(0.4822)	(0.4883)	(0.4891)	(12.8257)	(0.4526)	**(0.4485)**
		100:200	0.510	0.468	0.923	0.923	1.000	0.958	0.96
			(0.339)	(0.3321)	(0.4468)	(0.4477)	(12.8912)	(0.4501)	**(0.446)**
		200:200	0.952	0.828	0.949	0.950	1.000	0.955	0.957
			(0.3431)	(0.3393)	(0.3396)	(0.3396)	(9.6765)	(0.3258)	**(0.3238)**

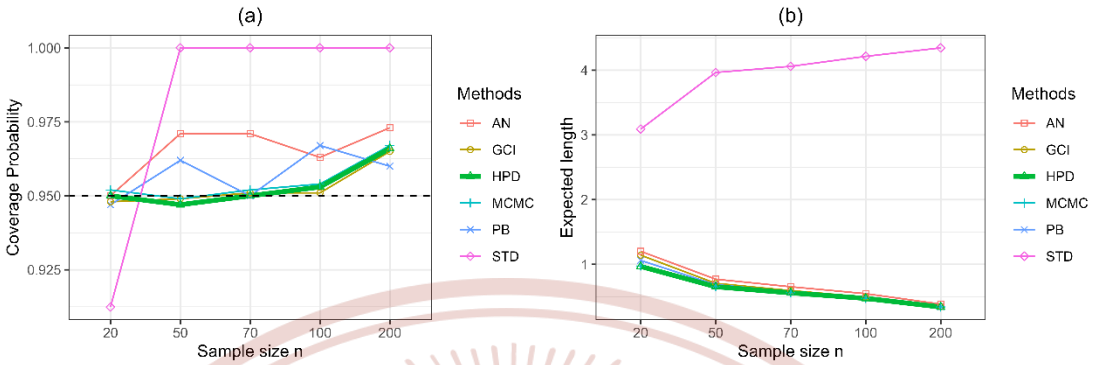


FIGURE 2-1 Graphs comparing the performance of the proposed methods for the single mean parameter in terms of (a) coverage probability and (b) expected length.

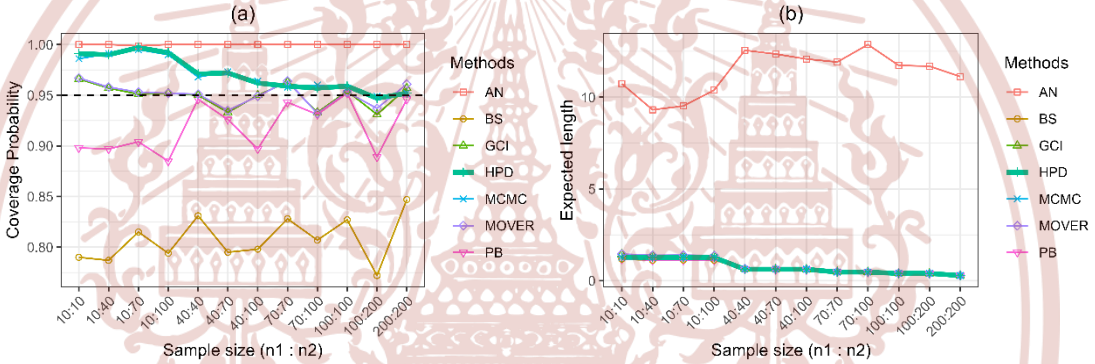


FIGURE 2-2 Graphs comparing the performance of the proposed methods for the difference between means parameters in terms of (a) coverage probability and (b) expected length.

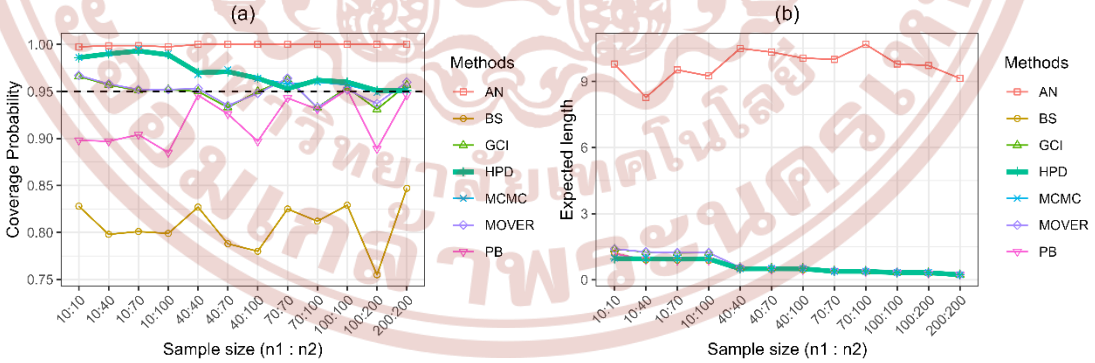


FIGURE 2-3 Graphs comparing the performance of the proposed methods for the ratio means parameters in terms of (a) coverage probability and (b) expected length.

2.5 An empirical application

The performance of the proposed methods was evaluated by utilizing COVID-19 mortality data and road accident fatality data to estimate the single mean, the difference of means, and the ratio of means for the ZITR distribution.

2.5.1 Application of the mean for ZITR

This research employed data on COVID-19 fatalities to illustrate the efficacy of confidence intervals for the mean of the ZITR distribution, calculated using the PB, GCI, STD, AN, MCMC, and HPD methodologies. The data were obtained from the World Health Organization and are publicly available through the World Health Organization COVID-19 database (last accessed on 28 June 2023). The research specifically examined daily COVID-19 mortality data documented in Singapore through October 2022. The dataset consisted of $n = 31$ observations, as presented in Table 2.13. The zero observations in the dataset were modeled using a binomial distribution, while the positive observations were fitted using six probability distributions commonly used in actuarial science, namely the Rayleigh, Normal, Weibull, Lognormal, Exponential, and Gamma distributions. The most appropriate distribution was selected based on the minimum Akaike information criterion (AIC) and Bayesian information criterion (BIC), as reported in Table 2-14 (Akaike, 1974; Schwarz, 1978). The parameter estimates for the dataset are statistically summarized as follows: $n = 31, \hat{\delta} = 0.1936, \hat{\lambda} = 0.1876,$ and $\hat{\mu} = 0.5$. To assess the suitability of the TR distribution for the observed data, Figure 2-4 presents the Q-Q plots and comparisons between the empirical and fitted CDFs for the three provinces. These graphical results provide further support for the adequacy of the two-parameter Rayleigh distribution in describing the datasets. The confidence interval for the single mean of total COVID-19 deaths in Singapore is reported in Table 2.15.

TABLE 2-13 COVID-19 Mortality Data in Singapore, October 2022.

COVID-19 Total Deaths (Singapore, Oct 2022)									
2	1	2	2	0	1	0	4	0	3
2	2	3	0	2	3	2	5	3	5
0	1	2	1	3	2	2	0	4	2
4									

TABLE 2-14 AIC and BIC values of mortality data in Singapore, October 2022.

Distribution	Rayleigh	Normal	Weibull	Lognormal	Exponential	Gamma
AIC	**77.3115**	81.3052	78.8253	77.9169	97.213	77.643
BIC	**79.74922**	83.74298	81.26306	80.35464	99.43182	80.08071

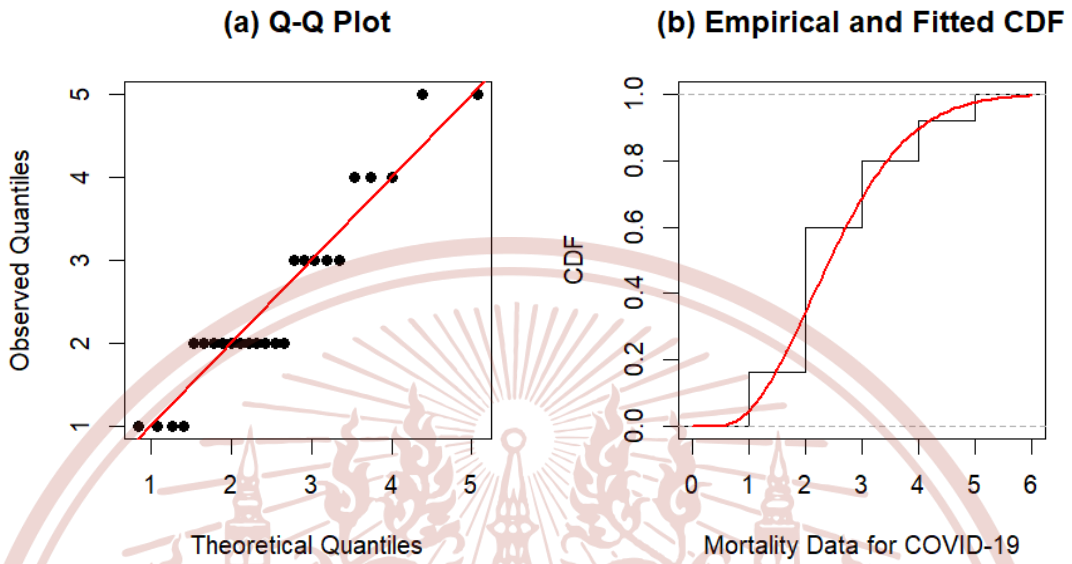


FIGURE 2-4 (a) Q-Q plot of COVID-19 mortality data and (b) CDF of COVID-19 mortality data.

TABLE 2-15 Confidence Intervals for the Single Mean of COVID-19 Mortality in Singapore, October 2022.

Method	Lower	Upper	Length
PB	1.5224	2.5762	1.0538
GCI	1.5858	2.5833	0.9975
STD	1.1280	2.7929	1.6649
AN	1.5056	2.6168	1.1112
MCMC	1.3599	2.2742	0.9142
HPD	1.3602	2.2742	**0.9139**

The findings are presented in Table 2.15. The confidence interval for the total number of COVID-19 fatalities in Singapore is not recommended when using the standard confidence interval based on the maximum likelihood method. The simulation results indicate that, for a small sample size (n), the standard confidence interval often produces a coverage probability that fails to include the true mean. Moreover, as the value of δ increases, the mean estimator becomes less effective in accurately estimating the true mean of the target distribution, resulting in confidence intervals that tend to be wider than those obtained from other methods. Therefore, the percentile bootstrap method, GCI method, AN method, Bayesian MCMC method, and Bayesian HPD method are recommended, as these approaches provide confidence intervals with satisfactory coverage of the mean parameter. Among these methods, the Bayesian HPD approach is found to perform the best, as it provides the shortest expected length of the confidence interval for the mean.

2.5.2 Application of the difference and ratio of means in ZITR distribution

This study utilized road traffic fatality data recorded during the Songkran festival in southern Thailand for 2024 and 2025 to demonstrate the effectiveness of confidence intervals for the mean of the ZITR distribution constructed using the PB, BS, GCI, MOVER, AN, MCMC, and HPD methods. The data were obtained from the Thailand Road Safety Collaboration. The dataset consisted of 14 observations for each year, as presented in Table 2-16. Observations with zero counts were modeled using a binomial distribution, whereas positive observations were analyzed using the distribution selected based on the minimum Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), as reported in Tables 2-17 and 2-18. To further assess the suitability of the selected distribution for the positive observations, Figures 2-5 and 2-6 present the Q-Q plots and comparisons between the empirical and fitted CDFs, providing further support for the adequacy of the selected distribution. In addition, the parameter estimates of the dataset, together with the 95% confidence intervals for the difference and ratio of the two means, are presented in Tables 2-19 and 2-20. The analytical results provide valuable insight into the changes in road traffic fatality risk between the two periods and highlight the public health implications associated with road safety during the Songkran festival.

TABLE 2-16 Road Traffic Fatality Data during the Songkran Festival in Southern Thailand, 2024–2025.

Year	Road Traffic Fatalities						
2024	3	3	6	7	4	2	1
	6	2	3	2	7	0	6
2025	2	3	6	7	3	2	4
	8	0	2	0	4	0	6

TABLE 2-17 AIC and BIC Values for Road Traffic Fatalities during the Seven Dangerous Days of the Songkran Festival in Southern Thailand, 2024.

Distribution	Rayleigh	Normal	Weibull	Lognormal	Exponential	Gamma
AIC	**57.80345**	59.40485	57.96072	58.94112	63.04365	58.22936
BIC	**50.13734**	60.53475	59.09061	60.07102	64.6086	59.35926

TABLE 2-18 AIC and BIC Values for Road Traffic Fatalities during the Seven Dangerous Days of the Songkran Festival in Southern Thailand, 2025.

Distribution	Rayleigh	Normal	Weibull	Lognormal	Exponential	Gamma
AIC	**49.00744**	50.99825	49.63039	49.0718	54.94955	49.23649
BIC	**49.80323**	51.79404	50.42618	49.8676	56.34745	50.03228

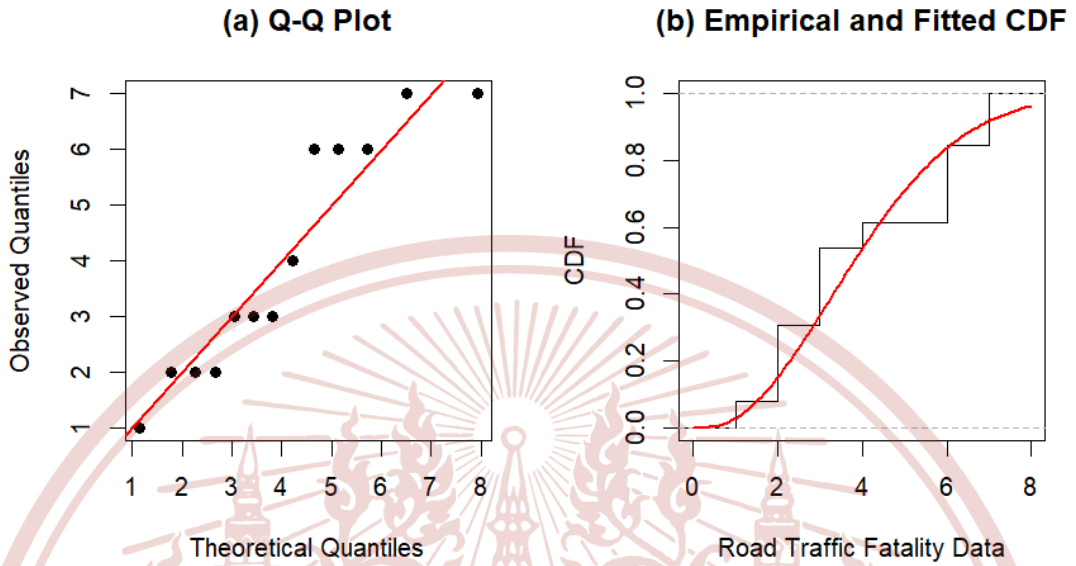


FIGURE 2-5 (a) Q-Q plot of Road Traffic Fatality Data and (b) CDF of Road Traffic Fatality Data during the Songkran Festival in Southern Thailand, 2024

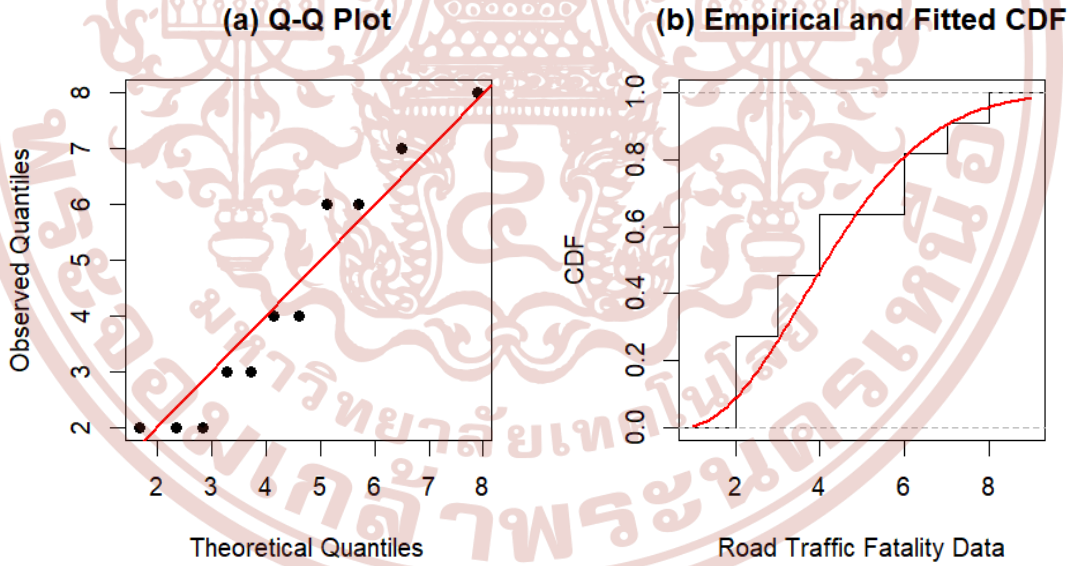


FIGURE 2-6 (a) Q-Q plot of Road Traffic Fatality Data and (b) CDF of Road Traffic Fatality Data during the Songkran Festival in Southern Thailand, 2025

TABLE 2-19 Confidence Intervals for the Difference in Mean Road Traffic Fatalities Using Various Methods under the Zero-Inflated Two-Parameter Rayleigh Distribution during the Songkran Festival in Southern Thailand, 2024–2025.

Method	Lower	Upper	Length
PB	-1.42825	2.078264	3.506514
BS	0.269991	2.389715	2.119724
GCI	-1.69777	2.192666	3.890439
MOVER	-1.64454	2.235432	3.879967
AN	-2.81678	3.456087	6.272865
Bayes	-1.47332	1.611625	3.08494
HPD	-1.48023	1.599352	**3.079581**

TABLE 2-20 Confidence Intervals for the ratio in Mean Road Traffic Fatalities Using Various Methods under the Zero-Inflated Two-Parameter Rayleigh Distribution during the Songkran Festival in Southern Thailand, 2024–2025.

Method	Lower	Upper	Length
PB	0.684892	1.922864	1.237972
BS	1.248869	1.809433	0.560564
GCI	0.621492	1.965381	1.343889
MOVER	0.637013	1.969493	1.332481
AN	0.158052	2.02809	1.870038
Bayes	0.664939	1.592669	0.92773
HPD	0.620266	1.517805	**0.89754**

The results demonstrate that the BS technique yields the smallest predicted length; nevertheless, simulation outcomes reveal that its coverage probability is below the nominal level. For limited sample sets, techniques that provide coverage probabilities around or beyond the 0.95 nominal confidence level—specifically GCI, MOVER, NA, MCMC, and HPD—exhibit greater reliability. Simulation data indicates that the HPD method is the best suitable technique for building confidence intervals for both the difference and ratio of means in the context of the zero-inflated two-parameter Rayleigh distribution.

2.6 Conclusion

This chapter presents several methods for constructing confidence intervals for the mean, difference of means, and ratio of means under the ZITR distribution. Owing to the complexity of direct theoretical comparisons among the proposed methods, this study employed Monte Carlo simulation together with analyses based on real-world datasets to evaluate their performance. The effectiveness of each method was assessed using coverage probabilities (CPs) and expected lengths (ELs) as the primary evaluation criteria based on the simulation results.

The mean estimate showed that the PB, GCI, AN, Bayesian MCMC, and Bayesian HPD methods provided coverage probabilities that were higher than the nominal confidence level. However, STD method was not appropriate for the sample sizes under consideration, as they produced too long expected lengths while sufficiently containing the parameter of interest. In some cases, the Bayesian Highest Posterior Density (HPD) method outperformed the other methods by producing coverage probabilities (CPs) that sufficiently covered the parameter of interest while producing shorter expected lengths (ELs). In instances of poor coverage performance, the Bayesian MCMC method exhibited the second best efficiency among the considered methods.

The simulation results for the parameters of the difference and ratio of means under the Zero-Inflated Two-Parameter Rayleigh (ZITR) distribution indicated the superiority of the GCI, normal approximation, Bayesian MCMC, and Bayesian HPD methods for all sample sizes, while the Percentile Bootstrap method improved with the increase in the sample size. The most reliable results among all the approaches were obtained by the Bayesian HPD method, which maintained the coverage probabilities close to the nominal confidence level of 0.95 and had relatively shorter confidence intervals. The Bayesian MCMC and GCI methods had the second and third best performance, respectively.

The Bayesian methods, particularly the Bayesian HPD method, demonstrated superior statistical performance in many scenarios. However, their computational cost should also be considered. Both the Bayesian MCMC and Bayesian HPD methods required relatively long computational times compared with the non-Bayesian methods. This is mainly because the Bayesian analysis requires MCMC sampling to obtain the posterior distributions, while the construction of the HPD intervals requires additional processing based on the posterior samples. Although the additional computational burden may be acceptable for moderate sample sizes, it may become more substantial when the sample size increases or when repeated interval estimation is required. Therefore, the selection of a confidence interval method should consider not only coverage probability and expected length but also computational efficiency.

For the GCI method, its performance depends strongly on the construction of an appropriate generalized pivotal quantity (GPQ). Although the GPQ used in this study provided satisfactory results, alternative GPQ constructions may potentially improve the coverage probabilities and reduce the expected lengths of the resulting confidence intervals. Therefore, future research could focus on investigating alternative GPQ constructions or developing new GPQs specifically tailored to the ZITR distribution. Such developments may further improve the performance of the GCI method and make it a computationally efficient alternative to Bayesian methods.

Chapter 3
CONFIDENCE INTERVALS FOR VARIANCES OF THE ZERO-INFLATED TWO-PARAMETER RAYLEIGH DISTRIBUTIONS

In this chapter, we discuss the construction of confidence intervals for variance, difference of variances, and ratio of variances under the Zero-Inflated Two Parameter Rayleigh (ZITR) distribution. The proposed methods for estimating a single variance parameter are Bayesian Markov Chain Monte Carlo (MCMC), Bayesian Highest Posterior Density (HPD), standard confidence interval (STD), approximate normal (AN), percentile bootstrap (PB), bootstrap with standard error (BS), and generalized confidence interval (GCI) methods. The proposed methods for constructing confidence intervals for difference and ratio of variances of the ZITR distribution are MCMC, HPD, GCI, method of variance estimates recovery (MOVER), AN, PB, and BS methods.

3.1 Introduction

Variance is an important measure of data dispersion and plays a crucial role in statistical inference and evidence-based decision making. In studies involving road traffic fatalities and other datasets with excess zeros, variability arises from both structural zeros and the distribution of positive observations, resulting in a more complex variance structure than that of the mean (Lambert (1992), Hall (2000)). In many zero-inflated and mixture models, the variance depends nonlinearly on the model parameters, making interval estimation particularly challenging, especially for small sample sizes (McLachlan and Peel (2000)). From a practical perspective, large variation in fatality counts may reflect unequal risk exposure, spatial heterogeneity, or unobserved factors affecting transportation safety (Lord and Mannering, 2010). Therefore, accurate estimation and comparison of variances, including the difference and ratio of variances between two populations, are essential for reliable statistical analysis and policy evaluation.

Several studies have proposed confidence interval methods for variance and variance comparisons under various probability distributions. Araveeporn (2022) developed a Bayesian approach for constructing confidence intervals for the variance of a normal distribution, demonstrating the usefulness of Bayesian methods for variance estimation. Khumpasee et al. (2025) investigated confidence intervals for the variance and standard deviation of the delta-inverse Gaussian distribution, with an application to traffic mortality counts. These studies highlight the importance of developing reliable interval estimation procedures for variance parameters under different distributional settings. For variance comparisons, Bebu and Mathew (2008) studied comparisons of means and variances under the bivariate log-normal distribution, while Herbert et al. (2011) developed analytical confidence intervals for the difference between two independent variances. Wongyai and Suwan (2019), Maneerat et al. (2020), Khooriphan et al. (2023), Ratasukharom et al. (2024), and Puggard et al. (2022) further extended interval estimation methods for skewed and zero-inflated distributions. However, to the best of our knowledge, no study has investigated confidence intervals for the variance, difference between variances, or ratio of variances under the Zero-Inflated Two-Parameter Rayleigh distribution, leaving an important gap in the literature.

The construction of confidence intervals for the variance, the difference between variances, and the ratio of variances under the Zero-Inflated Two-Parameter Rayleigh

(ZITR) distribution has not previously been investigated in the literature. Therefore, this chapter presents methods for constructing confidence intervals for these parameters. For the estimation of confidence intervals for the single variance parameter of the ZITR distribution, the proposed methods include Bayesian approaches based on MCMC and HPD intervals, together with the standard confidence interval, AN, PB, BS, and GCI methods. For constructing confidence intervals for the difference and ratio of variances under the ZITR distribution, the proposed methods consist of Bayesian MCMC, HPD intervals, AN, GCI, MOVER, PB, and BS methods. The proposed methods were further applied to real-world road traffic fatality data to provide practical illustrations for constructing confidence intervals for the variance, as well as the difference and ratio of variances.

3.2 The confidence intervals for the variance of ZITR distribution

Let $X = (X_1, X_2, \ldots, X_n)$ denote a random sample drawn from the Zero-Inflated Two-Parameter Rayleigh (ZITR) distribution, denoted by $X \sim ZITR(\mu, \lambda, \delta)$, where $P(X_i = 0) = \delta$ and $n_0 \sim \text{Binomial}(n, \delta)$. Suppose that a random sample of size n is observed, consisting of n_0 zero observations and $n_1 = n - n_0$ positive observations. In this section, the focus is on the variance parameter, where the population variance of X is given by

$$\xi = \frac{4(1-\delta) - (1-\delta)^2 \left(\Gamma\left(\frac{1}{2}\right) \right)^2}{4\lambda} + \frac{\mu}{\sqrt{\lambda}} \Gamma\left(\frac{1}{2}\right) \delta(1-\delta) + \mu^2 \delta(1-\delta). \qquad (3\text{-}1)$$

The following section presents the method for constructing confidence intervals for ξ

3.2.1 Bayesian MCMC

The Bayesian framework offers a methodical approach to parameter estimation by integrating prior knowledge with empirical data (Gelman et al., 1995). Inference is performed with Bayes' theorem, in which the prior distribution is revised via the likelihood function to get the posterior distribution. The prior distribution signifies uncertainty about the model parameters before data observation, whereas the posterior distribution denotes the revised uncertainty subsequent to the integration of sample information. Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ represent an independent random sample from the ZITR distribution. This work utilizes Bayesian inference to estimate the variance parameter, ξ, using the joint posterior distribution of the model parameters. Due to the absence of a closed-form formulation for the resultant posterior distribution, Markov chain Monte Carlo (MCMC) methods are used to provide posterior samples for the estimation of ξ.

The scale parameter λ is given a Gamma prior, the location parameter μ a Uniform prior Dey et al. (2014), and the zero-inflation parameter δ a Beta prior. These minimally informative priors are used to integrate existing prior knowledge and to guarantee a well-structured posterior distribution. All Bayesian calculations are executed using the R2OpenBUGS package inside the R statistical framework. Utilizing the posterior samples derived from the MCMC technique, two-sided Bayesian credible intervals for the variance parameter ξ are established as

$$CI_{\xi(\mathrm{MCMC})}=\left(L_{\xi(\mathrm{MCMC})},U_{\xi(\mathrm{MCMC})}\right)=\left(\xi_{\mathrm{MCMC}}(\alpha/2),\xi_{\mathrm{MCMC}}(1-\alpha/2)\right). \quad (3\text{-}2)$$

where $\xi_{\mathrm{MCMC}}(\alpha/2)$ represents the $100(\alpha/2)\%$ percentile of ξ_{MCMC}

3.2.2 Bayesian HPD

The Highest Posterior Density (HPD) interval is the segment of the posterior distribution where the density at any location inside the interval exceeds that of any point outside, while maintaining the minimum feasible length. This research uses HPD credible intervals to quantify the uncertainty associated with the variance parameter, represented by ξ. The intervals are derived using the HDInterval package in R.

The two-sided Bayesian credible interval based on HPD for ξ is therefore provided by

$$CI_{\xi(\mathrm{HPD})}=\left(L_{\xi(\mathrm{HPD})},U_{\xi(\mathrm{HPD})}\right). \quad (3\text{-}3)$$

where $L_{\xi(\mathrm{HPD})}$ and $U_{\xi(\mathrm{HPD})}$ represent the lower and upper bounds of ξ_{HPD} credible interval

3.2.3 Standard confidence interval (STD)

The standard confidence interval is a widely used method for estimating unknown values. Dey et al. (2014) proposed a large-sample confidence interval approach for the two-parameter Rayleigh distribution.

Let $\mathbf{x}=(x_1,x_2,\ldots,x_n)$ represent a random sample of size n extracted from this distribution. Under the regularity constraints of maximum likelihood estimation, the estimators $\hat{\delta}$, $\hat{\lambda}$, and $\hat{\mu}$ exhibit asymptotic normality, and their variance-covariance structure may be approximated by the inverse of the Fisher information matrix.

The estimated $(1-\alpha)\%$ confidence intervals for the parameters δ, λ, and μ are given in Equations (2-7)–(2-9).

For sufficiently high sample sizes, $\hat{\delta}$, $\hat{\lambda}$, and $\hat{\mu}$ are essentially normally distributed, with expectations corresponding to the real parameter values and variances derived from the maximum likelihood estimation method. Thus, the confidence range for the variance parameter ξ may be established using a large sample as follows:

$$CI_{\xi(STD)}=\left(L_{\xi(STD)},\mathrm{U}_{\xi(STD)}\right)=\left(\begin{array}{l}\dfrac{4(1-L_\delta)-(1-L_\delta)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4L_\lambda}+\dfrac{L_\mu}{\sqrt{L_\lambda}}\Gamma\left(\frac{1}{2}\right)L_\delta(1-L_\delta)+{L_\mu}^2L_\delta(1-L_\delta),\\ \dfrac{4(1-U_\delta)-(1-U_\delta)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4U_\lambda}+\dfrac{U_\mu}{\sqrt{U_\lambda}}\Gamma\left(\frac{1}{2}\right)U_\delta(1-U_\delta)+{U_\mu}^2U_\delta(1-U_\delta)\end{array}\right) \quad (3\text{-}4)$$

In this context, (L_δ,U_δ), (L_λ,U_λ), and (L_μ,U_μ) represent the lower and upper confidence limits for the parameters δ, λ, and μ, respectively.

3.2.4 Approximate normal (AN)

The approximate normal method considered in this study is based on the delta method, which provides a large-sample approximation for the distribution of a

function of asymptotically normal estimators. Let $\hat{\delta}$, $\hat{\mu}$, and $\hat{\lambda}$ denote the maximum likelihood estimators (MLEs) of the parameters δ, μ, and λ, respectively. For the zero-inflated two-parameter Rayleigh distribution, the variance of X is given by

$$\xi = \frac{4(1-\delta)-(1-\delta)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda} + \frac{\mu}{\sqrt{\lambda}}\Gamma\left(\frac{1}{2}\right)\delta(1-\delta) + \mu^2\delta(1-\delta).$$

The corresponding plug-in estimator of ξ is defined as

$$\hat{\xi} = g(\hat{\delta}, \hat{\mu}, \hat{\lambda}),$$

where $\hat{\xi} = \frac{4(1-\hat{\delta})-(1-\hat{\delta})^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\hat{\lambda}} + \frac{\hat{\mu}}{\sqrt{\hat{\lambda}}}\Gamma\left(\frac{1}{2}\right)\hat{\delta}(1-\hat{\delta}) + \hat{\mu}^2\hat{\delta}(1-\hat{\delta}).$

Using a first-order Taylor series expansion of $g(\hat{\delta}, \hat{\mu}, \hat{\lambda})$ around the true parameter values (δ, μ, λ), we obtain

$$\hat{\xi} \approx g(\delta, \mu, \lambda) + \frac{\partial g}{\partial \delta}(\hat{\delta}-\delta) + \frac{\partial g}{\partial \mu}(\hat{\mu}-\mu) + \frac{\partial g}{\partial \lambda}(\hat{\lambda}-\lambda).$$

Since $g(\delta, \mu, \lambda) = \xi$, the variance of $\hat{\xi}$ can be approximated using the delta method as

$$\mathrm{Var}(\hat{\xi}) \approx \left(\frac{\partial g(\delta,\mu,\lambda)}{\partial \delta}\right)^2 \mathrm{Var}(\hat{\delta}-\delta) + \left(\frac{\partial g(\delta,\mu,\lambda)}{\partial \mu}\right)^2 \mathrm{Var}(\hat{\mu}-\mu) + \left(\frac{\partial g(\delta,\mu,\lambda)}{\partial \lambda}\right)^2 \mathrm{Var}(\hat{\lambda}-\lambda).$$

The partial derivatives of $g(\delta, \mu, \lambda)$ are obtained as follows

$$\frac{\partial}{\partial \delta} g(\delta,\mu,\lambda) = \frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta)-4}{4\lambda} + \frac{\mu}{\sqrt{\lambda}}\Gamma\left(\frac{1}{2}\right)[1-2\delta] + \mu^2[1-2\delta],$$

$$\frac{\partial}{\partial \mu} g(\delta,\lambda,\mu) = \frac{\delta(1-\delta)}{\sqrt{\lambda}}\Gamma\left(\frac{1}{2}\right) + 2\mu\delta(1-\delta), \text{ and}$$

$$\frac{\partial}{\partial\lambda}g(\delta,\lambda,\mu)=\frac{(1-\delta)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta)}{4\lambda^2}-\frac{\mu\delta(1-\delta)}{2\lambda^{\frac{3}{2}}}\Gamma\left(\frac{1}{2}\right).$$

Therefore,

$$V\left(\hat{\xi}\right)=V\left(\begin{array}{l}\xi+\dfrac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta)-4}{4\lambda}+\dfrac{\mu}{\sqrt{\lambda}}\Gamma\left(\frac{1}{2}\right)[1-2\delta]+\mu^2[1-2\delta]\left(\hat{\delta}-\delta\right)\\ +\left(\dfrac{\Gamma\left(\frac{1}{2}\right)\delta(1-\delta)}{\sqrt{\lambda}}+2\mu\delta(1-\delta)\right)(\hat{\mu}-\mu)\\ +\left(\dfrac{(1-\delta)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta)}{4\lambda^2}-\dfrac{\mu\Gamma\left(\frac{1}{2}\right)\delta(1-\delta)}{2\lambda^{\frac{3}{2}}}\right)\left(\hat{\lambda}-\lambda\right)\end{array}\right)$$

$$V\left(\hat{\xi}\right)=\left(\begin{array}{l}\left(\dfrac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta)-4}{4\lambda}+\dfrac{\mu}{\sqrt{\lambda}}\Gamma\left(\frac{1}{2}\right)[1-2\delta]+\mu^2[1-2\delta]\right)^2V\left(\hat{\delta}-\delta\right)\\ +\left(\dfrac{\Gamma\left(\frac{1}{2}\right)\delta(1-\delta)}{\sqrt{\lambda}}+2\mu\delta(1-\delta)\right)^2V(\hat{\mu}-\mu)\\ +\left(\dfrac{(1-\delta)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta)}{4\lambda^2}-\dfrac{\mu\Gamma\left(\frac{1}{2}\right)\delta(1-\delta)}{2\lambda^{\frac{3}{2}}}\right)^2V\left(\hat{\lambda}-\lambda\right)\end{array}\right).$$

Dey et al. (2014) provide the variance and distributional characteristics as follows: $V\left(\hat{\lambda}-\lambda\right)=\frac{\lambda^2}{n}$ and $V\left(\hat{\mu}-\mu\right)\approx\frac{1}{2\mu+\frac{1}{n}\sum_{i=1}^{n}\left(x_i-\mu\right)^{-2}}$.

Therefore,

$$V\left(\hat{\xi}\right)=\left(\begin{array}{l}\left(\dfrac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta)-4}{4\lambda}+\dfrac{\mu}{\sqrt{\lambda}}\Gamma\left(\frac{1}{2}\right)\left[1-2\delta\right]+\mu^2\left[1-2\delta\right]\right)^2\dfrac{\delta(1-\delta)}{n}\\ +\left(\dfrac{\Gamma\left(\frac{1}{2}\right)\delta(1-\delta)}{\sqrt{\lambda}}+2\mu\delta(1-\delta)\right)^2\dfrac{1}{2\mu+\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^{-2}}\\ +\left(\dfrac{(1-\delta)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta)}{4\lambda^2}-\dfrac{\mu\Gamma\left(\frac{1}{2}\right)\delta(1-\delta)}{2\lambda^{\frac{3}{2}}}\right)^2\dfrac{\lambda^2}{n}\end{array}\right).$$

The estimates of $V\left(\hat{\xi}\right)$ defined as

$$\hat{V}\left(\hat{\xi}\right)=\left(\begin{array}{l}\left(\dfrac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\hat{\delta})-4}{4\hat{\lambda}}+\dfrac{\hat{\mu}}{\sqrt{\hat{\lambda}}}\Gamma\left(\frac{1}{2}\right)\left[1-2\hat{\delta}\right]+\mu^2\left[1-2\hat{\delta}\right]\right)^2\dfrac{\hat{\delta}(1-\hat{\delta})}{n}\\ +\left(\dfrac{\Gamma\left(\frac{1}{2}\right)\hat{\delta}(1-\hat{\delta})}{\sqrt{\hat{\lambda}}}+2\hat{\mu}\hat{\delta}(1-\hat{\delta})\right)^2\dfrac{1}{2\hat{\mu}+\frac{1}{n_{(1)}}\sum_{i=1}^{n_{(1)}}(x_i-\hat{\mu})^{-2}}\\ +\left(\dfrac{(1-\hat{\delta})^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\hat{\delta})}{4\hat{\lambda}^2}-\dfrac{\hat{\mu}\Gamma\left(\frac{1}{2}\right)\hat{\delta}(1-\hat{\delta})}{2\hat{\lambda}^{\frac{3}{2}}}\right)^2\dfrac{\hat{\lambda}^2}{n_{(1)}}\end{array}\right). \quad (3\text{-}5)$$

Consequently, the estimated normal confidence interval for ξ is formulated as

$$CI_{\xi(AN)}=\left(\hat{\xi}-z_{\alpha/2}\sqrt{\hat{V}(\hat{\xi})},\ \hat{\xi}+z_{\alpha/2}\sqrt{\hat{V}(\hat{\xi})}\right), \quad (3\text{-}6)$$

Here, $z_{\alpha/2}$ represents the higher $\alpha/2$ quantile of the standard normal distribution, whereas $\hat{V}(\hat{\xi})$ signifies the variance estimator derived using the delta technique.

3.2.5 Percentile Bootstrap

The percentile bootstrap technique generates a confidence range derived from the empirical quantiles of the bootstrap samples. Upon arranging the bootstrap estimates in ascending order, the interval bounds are derived from the associated empirical quantile locations. The resultant confidence interval for the variance parameter ξ is provided by

$$CI_{\xi(PB)} = \left(L_{\xi(PB)}, U_{\xi(PB)}\right) = \left(\hat{\xi}^{*b}_{(\alpha/2)}, \hat{\xi}^{*b}_{(1-\alpha/2)}\right). \tag{3-7}$$

In this context, $\hat{\xi}^{*b}_{(\alpha/2)}$ and $\hat{\xi}^{*b}_{(1-\alpha/2)}$ represent the empirical $\alpha/2$ and $1-\alpha/2$ quantiles of the bootstrap replicates, respectively.

3.2.6 Bootstrap with standard error (BS)

The bootstrap standard error method offers an alternate technique for constructing conventional confidence intervals. It obviates the need for a definitive analytical variance expression by directly assessing variability via frequent resampling. This method uses the bootstrap distribution to estimate the standard error, which then underpins interval estimation. This technique is especially advantageous when the sample size is constrained or when the theoretical variance is analytically unmanageable.

Let $\hat{\xi}^{*b}$, where $b = 1, 2, \ldots, B$, represent the bootstrap estimates derived from B resamples. The standard error is calculated as the sample standard deviation of the bootstrap replicates. The associated two-sided $100(1-\alpha)\%$ confidence interval derived from the bootstrap standard error is expressed as

$$CI_{\xi(BS)} = \left(L_{\xi(BS)}, U_{\xi(BS)}\right) = \left(\bar{\xi} - Z_{\alpha/2}SE, \bar{\xi} + Z_{\alpha/2}SE\right), \tag{3-8}$$

where $SE = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\xi}^{*b} - \bar{\xi}\right)^2}$ is the bootstrap standard error of $\hat{\xi}$, and $\bar{\xi} = \frac{1}{B}\sum_{b=1}^{B}\hat{\xi}^{*b}$ denotes the bootstrap mean estimate.

3.2.7 The generalized confidence interval (GCI)

Section 2.2 presents the generalized pivotal quantities (GPQs) for the parameters λ and μ, as suggested by Krishnamoorthy et al., which may be articulated as follows:

$$Q_a = Q_\mu,$$

and

$$Q_\lambda = \frac{1}{2Q_b^{\,2}}$$

where $Q_a = \hat{a} - \hat{b}\frac{\hat{a}^*}{\hat{b}^*}$ and $Q_b = \frac{\hat{b}}{\hat{b}^*}$.

Furthermore, Wu and Hsieh (2014) construct the pivotal quantities based on VST to construct intervals, which is

$$Q_\delta = \sin^2\left(\arcsin\sqrt{\hat{\delta}} - \frac{T}{2\sqrt{n}}\right),$$

where $T = 2\sqrt{n}\left(\arcsin\sqrt{\hat{\delta}} - \arcsin\sqrt{\delta}\right) \sim N(0,1)$.

Hence, considering the three pivotal Q_a, Q_λ, and Q_δ, which are independent of the parameter values, GPQ for the variance of *ZITR* distribution is given by

$$Q_\xi = \frac{4(1-Q_\delta) - (1-Q_\delta)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4Q_\lambda} + \frac{Q_\mu}{\sqrt{Q_\lambda}}\Gamma\left(\frac{1}{2}\right)Q_\delta(1-Q_\delta) + Q_\mu^2 Q_\delta(1-Q_\delta). \quad (3\text{-}9)$$

The associated GCI for ξ is

$$CI_{\xi(GCI)} = \left(L_{\xi(GCI)}, U_{\xi(GCI)}\right) = \left(Q_\xi(\alpha/2), Q_\xi(1-\alpha/2)\right). \quad (3\text{-}10)$$

where $Q_\xi(\alpha/2)$ represent the $100(\alpha/2)\%$ percentile of Q_ξ

3.3 The confidence intervals for difference and ratio variances of ZITR distribution

This section examines confidence intervals for the difference and ratio of two variances, using the methodologies of MCMC, HPD, GCI, MOVER, AN, PB, and BS interval approaches. Let $X_i = (X_{i1}, \ldots, X_{in_i})$, where $i = 1, 2$, signify two independent random samples drawn from the zero-inflated two-parameter Rayleigh distribution, referred to as $ZITR(\mu_i, \lambda_i, \delta_i)$. In this context, μ_i, λ_i, and δ_i represent the location parameter, the scale parameter, and the probability of zero, respectively. The variance of $x_i = (x_{i1}, \ldots, x_{in_i})$ is expressed as

$$\mathrm{Var}(X_i) = \xi_i = \frac{4(1-\delta_i) - (1-\delta_i)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda_i} + \frac{\mu_i}{\sqrt{\lambda_i}}\Gamma\left(\frac{1}{2}\right)\delta_i(1-\delta_i) + \mu_i^2\delta_i(1-\delta_i). \quad (3\text{-}11)$$

Let x_1 and x_2 represent two independent samples of sizes n_1 and n_2, extracted from populations characterized by characteristics $(\mu_1, \lambda_1, \delta_1)$ and $(\mu_2, \lambda_2, \delta_2)$, respectively. The parameters of interest are the variance difference and variance ratio, defined as

$$\theta_{\mathrm{var}} = \xi_1 - \xi_2 = \frac{4(1-\delta_1) - (1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda_1} + \frac{\mu_1}{\sqrt{\lambda_1}}\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1) + \mu_1^2\delta_1(1-\delta_1) \quad (3\text{-}12)$$

$$- \frac{4(1-\delta_2) - (1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda_2} + \frac{\mu_2}{\sqrt{\lambda_2}}\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2) + \mu_2^2\delta_2(1-\delta_2),$$

and

$$\tau_{\text{var}} = \xi_1/\xi_2 = \frac{\frac{4(1-\delta_1)-(1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda_1} + \frac{\mu_1}{\sqrt{\lambda_1}}\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1) + \mu_1^2\delta_1(1-\delta_1)}{\frac{4(1-\delta_2)-(1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda_2} + \frac{\mu_2}{\sqrt{\lambda_2}}\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2) + \mu_2^2\delta_2(1-\delta_2)} \quad (3\text{-}13)$$

respectively. The following section outlines the method for constructing confidence intervals for θ_{var} and τ_{var}.

3.3.1 Bayesian MCMC

The zero-inflated two-parameter Rayleigh distribution is assumed for two independent samples, $X_{ij} = (X_{i1},\ldots,X_{in_i})$ for $i = 1,2$ and $j = 1,2,\ldots,n_i$. This work focuses on Bayesian inference for two measures of variability, which are described in terms of the model parameters: the variance ratio τ and the variance difference θ. Simulation-based inference is necessary since there is no closed-form formulation for the posterior distribution. After obtaining posterior samples using the MCMC approach, the values θ and τ are repeatedly assessed to build their empirical posterior distributions. The prior distributions for the model parameters are as follows: a uniform prior for the location parameter μ, a gamma prior for the scale parameter λ, and a beta prior for δ.

Consequently, the Bayesian credible intervals for θ_{var} and τ_{var}, defined as follows, are constructed using the posterior samples derived from the MCMC procedure:

$$CI_{\theta_{\text{var}}(\text{MCMC})} = \left(L_{\theta_{\text{var}}(\text{MCMC})}, U_{\theta_{\text{var}}(\text{MCMC})}\right) = \left(\theta_{\text{var(MCMC)}}(\alpha/2), \theta_{\text{var(MCMC)}}(1-\alpha/2)\right), \quad (3\text{-}14)$$

and

$$CI_{\tau_{\text{var}}(\text{MCMC})} = \left(L_{\tau_{\text{var}}(\text{MCMC})}, U_{\tau_{\text{var}}(\text{MCMC})}\right) = \left(\tau_{\text{var(MCMC)}}(\alpha/2), \tau_{\text{var(MCMC)}}(1-\alpha/2)\right). \quad (3\text{-}15)$$

3.3.2 Bayesian HPD

The studies use HPD credible intervals to evaluate the uncertainty associated with the variance difference and variance ratio, represented by θ_{var} and τ_{var}, respectively. These intervals are generated using posterior samples acquired by the HDInterval package inside the R environment. According to the simulated posterior samples, the two-sided HPD credible intervals for θ_{var} and τ_{var} are provided by

$$CI_{\theta_{\text{var}}(HPD)} = \left(L_{\theta_{\text{var}}(HPD)}, U_{\theta_{\text{var}}(HPD)}\right), \quad (3\text{-}16)$$

and

$$CI_{\tau_{\text{var}}(HPD)} = \left(L_{\tau_{\text{var}}(HPD)}, U_{\tau_{\text{var}}(HPD)}\right). \quad (3\text{-}17)$$

3.3.3 Generalized confidence interval (GCI)

The GPQ for ξ_i can be defined using for random variable x_i, where $i = 1,2$ as follow

$$Q_{\xi_i} = \frac{4\left(1-Q_{\delta_i}\right)-\left(1-Q_{\delta_i}\right)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4Q_{\lambda_i}} + \frac{Q_{\mu_i}}{\sqrt{Q_{\lambda_i}}}\Gamma\left(\frac{1}{2}\right)Q_{\delta_i}\left(1-Q_{\delta_i}\right)+Q_{\mu_i}^2 Q_{\delta_i}\left(1-Q_{\delta_i}\right). \quad (3\text{-}18)$$

where Q_{δ_i}, Q_{μ_i} and Q_{λ_i} be pivotal quantities of δ_i, μ_i and λ_i respectively. The estimators of $\hat{\theta}_{mean}$ and $\hat{\tau}_{mean}$ obtained using the generalized confidence interval (GCI) method are denoted by Q_θ and Q_τ respectively, defined as

$$Q_{\theta_{\text{var}}} = Q_{\xi_1} - Q_{\xi_2}, \quad (3\text{-}19)$$

and

$$Q_{\tau_{\text{var}}} = \frac{Q_{\xi_1}}{Q_{\xi_2}}. \quad (3\text{-}20)$$

The empirical distributions of $Q_{\theta_{\text{var}}}$ and $Q_{\tau_{\text{var}}}$, which do not include unknown parameters, may be derived by Monte Carlo simulation. The $(1-\alpha)$ generalized confidence intervals are generated using the empirical quantiles of $Q_{\theta_{\text{var}}}$ and $Q_{\tau_{\text{var}}}$. In particular,

$$CI_{\theta_{\text{var}}(\text{GCI})} = \left(Q_{\theta_{\text{var}}}(\alpha/2), Q_{\theta_{\text{var}}}(1-\alpha/2)\right), \quad (3\text{-}21)$$

and

$$CI_{\tau_{\text{var}}(\text{GCI})} = \left(Q_{\tau_{\text{var}}}(\alpha/2), Q_{\tau_{\text{var}}}(1-\alpha/2)\right). \quad (3\text{-}22)$$

Where $Q_{\theta_{\text{var}}}(\alpha/2)$ and $Q_{\tau_{\text{var}}}(\alpha/2)$ represent the $100(\alpha/2)$th percentiles of $Q_{\theta_{\text{var}}}$ and $Q_{\tau_{\text{var}}}$, respectively.

3.3.4 Method of Variance Estimates Recovery (MOVER)

Donner and Zou (2012) introduced a technique for generating confidence intervals for the difference and the ratio of two distribution parameters. This method is especially advantageous when obtaining analytical expressions for the variances of the estimators proves challenging, thereby providing a pragmatic alternative for interval estimation of both metrics. In this investigation, the confidence intervals for the variance difference and variance ratio, denoted by θ_{var} and τ_{var}, are expressed as

$$CI_{\theta_{\text{var}}(\text{MOVER})} = \left(L_{\theta_{\text{var}}(\text{MOVER})}, U_{\theta_{\text{var}}(\text{MOVER})}\right), \quad (3\text{-}23)$$

and

$$CI_{\tau_{\text{var}}(\text{MOVER})} = \left(L_{\tau_{\text{var}}(\text{MOVER})}, U_{\tau_{\text{var}}(\text{MOVER})}\right). \quad (3\text{-}24)$$

The lower and upper bounds for θ_{var} are given by

$$L_{\theta_{\text{var}}(\text{MOVER})} = \left(\hat{\xi}_1 - \hat{\xi}_2\right) - \sqrt{\left(\hat{\xi}_1 - l_1\right)^2 - \left(u_2 - \hat{\xi}_2\right)^2},$$

and

$$U_{\theta_{\text{var}}(\text{MOVER})} = \left(\hat{\xi}_1 - \hat{\xi}_2\right) + \sqrt{\left(u_1 - \hat{\xi}_1\right)^2 - \left(\hat{\xi}_2 - l_2\right)^2}.$$

The corresponding bounds for the variance ratio τ_{var} are given by

$$L_{\tau_{\text{var}}(\text{MOVER})} = \frac{\hat{\xi}_1\hat{\xi}_2 - \sqrt{\left(\hat{\xi}_1\hat{\xi}_2\right)^2 - l_1u_2\left(2\hat{\xi}_1 - l_1\right)\left(2\hat{\xi}_2 - u_2\right)}}{u_2\left(2\hat{\xi}_2 - u_2\right)},$$

and

$$U_{\tau_{\text{var}}(\text{MOVER})} = \frac{\hat{\xi}_1\hat{\xi}_2 + \sqrt{\left(\hat{\xi}_1\hat{\xi}_2\right)^2 - u_1l_2\left(2\hat{\xi}_1 - u_1\right)\left(2\hat{\xi}_2 - l_2\right)}}{l_2\left(2\hat{\xi}_2 - l_2\right)}.$$

In this context, (l_1, u_1) and (l_2, u_2) represent the confidence intervals for the respective variance parameters, derived through the generalized confidence interval (GCI) methodology.

3.3.5 Approximate Normal (AN)

The derivation of the variance approximation for $\hat{\theta}$ and $\hat{\tau}$ via the delta approach. Let $\hat{\xi}_i = g(\hat{\delta}_i, \hat{\mu}_i, \hat{\lambda}_i)$, where the variance of the zero-inflated two-parameter Rayleigh distribution is defined as $g(\delta_i, \mu_i, \lambda_i) = \frac{4(1-\delta_i) - (1-\delta_i)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda_i} + \frac{\mu_i}{\sqrt{\lambda_i}}\Gamma\left(\frac{1}{2}\right)\delta_i(1-\delta_i) + \mu_i^2\delta_i(1-\delta_i)$.

The delta method is an essential instrument within the normal approximation framework, often used to derive the asymptotic distribution of a function of estimators. Let $g(U_1, U_2, U_3, U_4, U_5, U_6)$ be a continuously differentiable scalar function defined in relation to the parameters of interest, and let $G = g(\hat{U}_1, \hat{U}_2, \hat{U}_3, \hat{U}_4, \hat{U}_5, \hat{U}_6)$ represent its estimator. To get the asymptotic distribution of G, a first-order Taylor expansion of $g(\cdot)$ is performed around the true parameter vector $(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6)$, which corresponds to $(U_1, U_2, U_3, U_4, U_5, U_6)$. Under typical regularity assumptions, as the sample size rises, this expansion indicates that G is asymptotically regularly distributed. This outcome facilitates the derivation of approximation formulae for the mean and variance of G, which are then used to formulate confidence intervals for θ and τ.

$$g\left(U_1, U_2, U_3, U_4, U_5, U_6\right) \approx g\left(\left(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6\right)\right) + \sum_{i=1}^{6} \frac{\partial g\left(\left(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6\right)\right)}{\partial u_i}\left(U_i - \omega_i\right).$$

In this setting, define the vector of basic statistics as $U_1 = \hat{\delta}_1$, $U_2 = \hat{\mu}_1$, $U_3 = \hat{\lambda}_1$, $U_4 = \hat{\delta}_2$, $U_5 = \hat{\mu}_2$, $U_6 = \hat{\lambda}_2$, where $\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2$ are the maximum likelihood estimators (MLEs) of $\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2$, respectively. Substituting these estimators into the function $g(\cdot)$ yields the estimator of θ_{var} as

$$\hat{\theta}_{\text{var}} = g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1)$$

$$= \left(\frac{4(1-\hat{\delta}_1) - (1-\hat{\delta}_1)^2 \left(\Gamma\left(\frac{1}{2}\right) \right)^2}{4\hat{\lambda}_1} + \frac{\hat{\mu}_1 \Gamma\left(\frac{1}{2}\right) \hat{\delta}_1 (1-\hat{\delta}_1)}{\sqrt{\hat{\lambda}_1}} + \hat{\mu}_1^2 \hat{\delta}_1 (1-\hat{\delta}_1) \right)$$

$$- \left(\frac{4\left(1-\hat{\delta}_2\right) - \left(1-\hat{\delta}_2\right)^2 \left(\Gamma\left(\frac{1}{2}\right) \right)^2}{4\hat{\lambda}_2} + \frac{\hat{\mu}_2 \Gamma\left(\frac{1}{2}\right) \hat{\delta}_2 (1-\hat{\delta}_2)}{\sqrt{\hat{\lambda}_2}} + \hat{\mu}_2{}^2 \hat{\delta}_2 \left(1-\hat{\delta}_2\right) \right),$$

and the estimator of τ_{var} as

$$\hat{\tau}_{\text{var}} = g(\hat{\delta}_1, \hat{\mu}_1, \hat{\lambda}_1, \hat{\delta}_2, \hat{\mu}_2, \hat{\lambda}_2) = \frac{\frac{4(1-\hat{\delta}_1) - (1-\hat{\delta}_1)^2 \left(\Gamma\left(\frac{1}{2}\right) \right)^2}{4\hat{\lambda}_1} + \frac{\hat{\mu}_1 \Gamma\left(\frac{1}{2}\right) \hat{\delta}_1 (1-\hat{\delta}_1)}{\sqrt{\hat{\lambda}_1}} + \hat{\mu}_1^2 \hat{\delta}_1 (1-\hat{\delta}_1)}{\frac{4(1-\hat{\delta}_2) - (1-\hat{\delta}_2)^2 \left(\Gamma\left(\frac{1}{2}\right) \right)^2}{4\hat{\lambda}_2} + \frac{\hat{\mu}_2 \Gamma\left(\frac{1}{2}\right) \hat{\delta}_2 (1-\hat{\delta}_2)}{\sqrt{\hat{\lambda}_2}} + \hat{\mu}_2^2 \hat{\delta}_2 (1-\hat{\delta}_2)}.$$

The first-order partial derivatives of $\hat{\theta}_{\text{var}}$ concerning the components of the parameter vector are derived as follows:

$$\frac{\partial g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2)}{\partial \delta_1} = \frac{2\left(\Gamma\left(\frac{1}{2}\right) \right)^2 (1-\delta_1) - 4}{4\lambda_1} + \frac{\mu_1}{\sqrt{\lambda_1}} \Gamma\left(\frac{1}{2}\right) [1-2\delta_1] + \mu_1{}^2 [1-2\delta_1]$$

$$\frac{\partial g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2)}{\partial \mu_1} = \frac{\Gamma\left(\frac{1}{2}\right) \delta_1 (1-\delta_1)}{\sqrt{\lambda_1}} + 2\mu_1 \delta_1 (1-\delta_1)$$

$$\frac{\partial g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2)}{\partial \lambda_1} = \frac{(1-\delta_1)^2 \left(\Gamma\left(\frac{1}{2}\right) \right)^2 - 4(1-\delta_1)}{4\lambda_1{}^2} - \frac{\mu_1 \Gamma\left(\frac{1}{2}\right) \delta_1 (1-\delta_1)}{2\lambda_1{}^{\frac{3}{2}}}$$

$$\frac{\partial g(\delta_1, \mu_1, \lambda_1, \delta_2, \mu_2, \lambda_2)}{\partial \delta_2} = -\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right) \right)^2 (1-\delta_2) - 4}{4\lambda_2} + \frac{\mu_2}{\sqrt{\lambda_2}} \Gamma\left(\frac{1}{2}\right) [1-2\delta_2] + \mu_2{}^2 [1-2\delta_2] \right)$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \mu_2} = -\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{\sqrt{\lambda_2}} + 2\mu_2\delta_2(1-\delta_2) \right)$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \lambda_2} = -\left(\frac{(1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2 - 4(1-\delta_2)}{4{\lambda_2}^2} - \frac{\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{2{\lambda_2}^{\frac{3}{2}}} \right)$$

Therefore,

$$\hat{\theta}_{\text{var}} \approx g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2) + \frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \delta_1}(\hat{\delta}_1-\delta_1)$$
$$+ \frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \mu_1}(\hat{\mu}_1-\mu_1) + \frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \lambda_1}(\hat{\lambda}_1-\lambda_1)$$
$$+ \frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \delta_2}(\hat{\delta}_2-\delta_2) + \frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \mu_2}(\hat{\mu}_2-\mu_2)$$
$$+ \frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \lambda_2}(\hat{\lambda}_2-\lambda_2)$$

$$\hat{\theta}_{\text{var}} \approx g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)+\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_1)-4}{4\lambda_1}+\frac{\mu_1\Gamma\left(\frac{1}{2}\right)(1-2\delta_1)}{\sqrt{\lambda_1}}+\mu_1^2(1-2\delta_1)\right)(\hat{\delta}_1-\delta_1)$$

$$+\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{\sqrt{\lambda_1}}+2\mu_1\delta_1(1-\delta_1)\right)(\hat{\mu}_1-\mu_1)$$

$$+\left(\frac{(1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta_1)}{4\lambda_1^2}-\frac{\mu_1\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{2\lambda_1^{3/2}}\right)(\hat{\lambda}_1-\lambda_1)$$

$$-\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_2)-4}{4\lambda_2}+\frac{\mu_2\Gamma\left(\frac{1}{2}\right)(1-2\delta_2)}{\sqrt{\lambda_2}}+\mu_2^2(1-2\delta_2)\right)(\hat{\delta}_2-\delta_2)$$

$$-\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{\sqrt{\lambda_2}}+2\mu_2\delta_2(1-\delta_2)\right)(\hat{\mu}_2-\mu_2)$$

$$-\left(\frac{(1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta_2)}{4\lambda_2^2}-\frac{\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{2\lambda_2^{3/2}}\right)(\hat{\lambda}_2-\lambda_2)$$

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$$V(\hat{\theta}_{\text{var}}) = \left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_1)-4}{4\lambda_1} + \frac{\mu_1\Gamma\left(\frac{1}{2}\right)(1-2\delta_1)}{\sqrt{\lambda_1}} + \mu_1^2(1-2\delta_1) \right)^2 V(\hat{\delta}_1-\delta_1)$$

$$+ \left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{\sqrt{\lambda_1}} + 2\mu_1\delta_1(1-\delta_1) \right)^2 V(\hat{\mu}_1-\mu_1)$$

$$+ \left(\frac{(1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2 - 4(1-\delta_1)}{4\lambda_1^2} - \frac{\mu_1\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{2\lambda_1^{3/2}} \right)^2 V(\hat{\lambda}_1-\lambda_1)$$

$$+ \left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_2)-4}{4\lambda_2} + \frac{\mu_2\Gamma\left(\frac{1}{2}\right)(1-2\delta_2)}{\sqrt{\lambda_2}} + \mu_2^2(1-2\delta_2) \right)^2 V(\hat{\delta}_2-\delta_2)$$

$$+ \left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{\sqrt{\lambda_2}} + 2\mu_2\delta_2(1-\delta_2) \right)^2 V(\hat{\mu}_2-\mu_2)$$

$$+ \left(\frac{(1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2 - 4(1-\delta_2)}{4\lambda_2^2} - \frac{\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{2\lambda_2^{3/2}} \right)^2 V(\hat{\lambda}_2-\lambda_2)$$

$$V(\hat{\theta}_{\text{var}}) = \left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_1)-4}{4\lambda_1} + \frac{\mu_1\Gamma\left(\frac{1}{2}\right)(1-2\delta_1)}{\sqrt{\lambda_1}} + \mu_1^2(1-2\delta_1) \right)^2 \frac{\delta_1(1-\delta_1)}{n_1}$$

$$+ \left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{\sqrt{\lambda_1}} + 2\mu_1\delta_1(1-\delta_1) \right)^2 \frac{1}{2\mu_1 + \frac{1}{n_{1(1)}}\sum_{j=1}^{n_{1(1)}}(x_{1j(1)}-\mu_1)^{-2}}$$

$$+ \left(\frac{(1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2 - 4(1-\delta_1)}{4\lambda_1^2} - \frac{\mu_1\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{2\lambda_1^{3/2}} \right)^2 \frac{\lambda_1^2}{n_{1(1)}}$$

$$+ \left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_2)-4}{4\lambda_2} + \frac{\mu_2\Gamma\left(\frac{1}{2}\right)(1-2\delta_2)}{\sqrt{\lambda_2}} + \mu_2^2(1-2\delta_2) \right)^2 \frac{\delta_2(1-\delta_2)}{n_2}$$

$$+ \left(\frac{\sqrt{\pi}\delta_2(1-\delta_2)}{\sqrt{\lambda_2}} + 2\mu_2\delta_2(1-\delta_2) \right)^2 \frac{1}{2\mu_2 + \frac{1}{n_{2(1)}}\sum_{j=1}^{n_{2(1)}}(x_{2j(1)}-\mu_2)^{-2}}$$

$$+ \left(\frac{(1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2 - 4(1-\delta_2)}{4\lambda_2^2} - \frac{\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{2\lambda_2^{3/2}} \right)^2 \frac{\lambda_2^2}{n_{2(1)}}.$$

The estimations of $\hat{V}\left(\hat{\theta}_{\text{var}}\right)$ are delineated as

$$\hat{V}(\hat{\theta}_{\text{var}})=\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\hat{\delta}_1)-4}{4\hat{\lambda}_1}+\frac{\hat{\mu}_1\Gamma\left(\frac{1}{2}\right)(1-2\hat{\delta}_1)}{\sqrt{\hat{\lambda}_1}}+\hat{\mu}_1^2(1-2\hat{\delta}_1)\right)^2\frac{\hat{\delta}_1(1-\hat{\delta}_1)}{n_1}$$

$$+\left(\frac{\Gamma\left(\frac{1}{2}\right)\hat{\delta}_1(1-\hat{\delta}_1)}{\sqrt{\hat{\lambda}_1}}+2\hat{\mu}_1\hat{\delta}_1(1-\hat{\delta}_1)\right)^2\frac{1}{2\hat{\mu}_1+\frac{1}{n_{1(1)}}\sum_{j=1}^{n_{1(1)}}(x_{1j(1)}-\hat{\mu}_1)^{-2}}$$

$$+\left(\frac{(1-\hat{\delta}_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\hat{\delta}_1)}{4\hat{\lambda}_1^2}-\frac{\hat{\mu}_1\Gamma\left(\frac{1}{2}\right)\hat{\delta}_1(1-\hat{\delta}_1)}{2\hat{\lambda}_1^{3/2}}\right)^2\frac{\hat{\lambda}_1^2}{n_{1(1)}}.$$

$$+\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\hat{\delta}_2)-4}{4\hat{\lambda}_2}+\frac{\hat{\mu}_2\Gamma\left(\frac{1}{2}\right)(1-2\hat{\delta}_2)}{\sqrt{\hat{\lambda}_2}}+\hat{\mu}_2^2(1-2\hat{\delta}_2)\right)^2\frac{\hat{\delta}_2(1-\hat{\delta}_2)}{n_2}$$

$$+\left(\frac{\Gamma\left(\frac{1}{2}\right)\hat{\delta}_2(1-\hat{\delta}_2)}{\sqrt{\hat{\lambda}_2}}+2\hat{\mu}_2\hat{\delta}_2(1-\hat{\delta}_2)\right)^2\frac{1}{2\hat{\mu}_2+\frac{1}{n_{2(1)}}\sum_{j=1}^{n_{2(1)}}(x_{2j(1)}-\hat{\mu}_2)^{-2}} \qquad (3\text{-}25)$$

$$+\left(\frac{(1-\hat{\delta}_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\hat{\delta}_2)}{4\hat{\lambda}_2^2}-\frac{\hat{\mu}_2\Gamma\left(\frac{1}{2}\right)\hat{\delta}_2(1-\hat{\delta}_2)}{2\hat{\lambda}_2^{3/2}}\right)^2\frac{\hat{\lambda}_2^2}{n_{2(1)}}.$$

The first-order partial derivatives concerning the elements of the parameter vector are derived as follows:

$$g\left(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2\right)=\frac{4\lambda_2\left(4(1-\delta_1)-(1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2+4\sqrt{\lambda_1}\mu_1\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)+4\lambda_1\mu_1^2\delta_1(1-\delta_1)\right)}{4\lambda_1\left(4(1-\delta_2)-(1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2+4\sqrt{\lambda_2}\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)+4\lambda_2\mu_2^2\delta_2(1-\delta_2)\right)}$$

Consider

$$g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)=\frac{4(1-\delta_1)-(1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2+4\sqrt{\lambda_1}\mu_1\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)+4\lambda_1\mu_1^2\delta_1(1-\delta_1)}{4\lambda_1\xi_2}$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\delta_1}=\frac{2\pi(1-\delta_1)-4+4\sqrt{\lambda_1}\mu_1\Gamma\left(\frac{1}{2}\right)(1-2\delta_1)+4\lambda_1\mu_1^2(1-2\delta_1)}{4\lambda_1\xi_2}$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\mu_1}=\frac{4\sqrt{\lambda_1}\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)+8\mu_1\delta_1\lambda_1(1-\delta_1)}{4\lambda_1\xi_2}$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\lambda_1}=\frac{-(1-\delta_1)\left(4-(1-\delta_1)\left(\Gamma\left(\frac{1}{2}\right)\right)^2+2\sqrt{\lambda_1}\mu_1\Gamma\left(\frac{1}{2}\right)\delta_1\right)}{4\xi_2\lambda_1^{\,2}}$$

and consider

$$g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)=4\lambda_2\xi_1\left(4(1-\delta_2)-(1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2+4\sqrt{\lambda_2}\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)+4\lambda_2\mu_2^2\delta_2(1-\delta_2)\right)^{-1}$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\delta_2}=\frac{-\xi_1\left(2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_2)-4+4\sqrt{\lambda_2}\mu_2\Gamma\left(\frac{1}{2}\right)[1-2\delta_2]+4\lambda_2\mu_2{}^2[1-2\delta_2]\right)}{4\lambda_2\xi_2{}^2}$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\mu_2}=\left(\frac{-\xi_1\left(4\sqrt{\lambda_2}\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)+8\lambda_2\mu_2\delta_2(1-\delta_2)\right)}{4\lambda_2\xi_2^2}\right)$$

$$\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\lambda_2}=\frac{-\xi_1\left(\frac{2\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{\sqrt{\lambda_2}}+4\mu_2^2\delta_2(1-\delta_2)\right)}{4\lambda_2\xi_2^2}+\frac{\xi_1}{\lambda_2\xi_2}.$$

Therefore

$$\hat{\tau}_{\text{var}} \approx g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \delta_1}(\hat{\delta}_1-\delta_1)$$
$$+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \mu_1}(\hat{\mu}_1-\mu_1)+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \lambda_1}(\hat{\lambda}_1-\lambda_1)$$
$$+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \delta_2}(\hat{\delta}_2-\delta_2)+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \mu_2}(\hat{\mu}_2-\mu_2)$$
$$+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial \lambda_2}(\hat{\lambda}_2-\lambda_2)$$

$$\hat{\tau} \approx g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)+\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_1)-4+4\sqrt{\lambda_1}\mu_1\Gamma\left(\frac{1}{2}\right)(1-2\delta_1)+4\lambda_1\mu_1^2(1-2\delta_1)}{4\lambda_1\hat{\xi}_2}(\hat{\delta}_1-\delta_1)$$
$$+\frac{4\sqrt{\lambda_1}\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)+8\mu_1\delta_1\lambda_1(1-\delta_1)}{4\lambda_1\hat{\xi}_2}(\hat{\mu}_1-\mu_1)+\frac{-(1-\delta_1)\left(4-(1-\delta_1)\left(\Gamma\left(\frac{1}{2}\right)\right)^2+2\sqrt{\lambda_1}\mu_1\sqrt{\pi}\delta_1\right)}{4\xi_2\lambda_1^2}(\hat{\lambda}_1-\lambda_1)$$
$$+\frac{-\hat{\xi}_1\left(2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_2)-4+4\sqrt{\lambda_2}\mu_2\Gamma\left(\frac{1}{2}\right)(1-2\delta_2)+4\lambda_2\mu_2^2(1-2\delta_2)\right)}{4\lambda_2\xi_2^2}(\hat{\delta}_2-\delta_2)$$
$$+\left(\frac{-\xi_1\left(4\sqrt{\lambda_2}\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)+8\lambda_2\mu_2\delta_2(1-\delta_2)\right)}{4\lambda_2\xi_2^2}\right)(\hat{\mu}_2-\mu_2)$$
$$+\left(\frac{-\xi_1\left(\frac{2\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{\sqrt{\hat{\lambda}_2}}+4\mu_2^2\hat{\delta}_2(1-\delta_2)\right)}{4\lambda_2\xi_2^2}+\frac{\xi_1}{\lambda_2\xi_2}\right)(\hat{\lambda}_2-\lambda_2)$$

$$V(\hat{\tau}_{\text{var}}) = \left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_1) - 4 + 4\sqrt{\lambda_1}\mu_1\Gamma\left(\frac{1}{2}\right)(1-2\delta_1) + 4\lambda_1\mu_1^2(1-2\delta_1)}{4\lambda_1\xi_2} \right)^2 V(\hat{\delta}_1 - \delta_1)$$

$$+ \left(\frac{4\sqrt{\lambda_1}\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1) + 8\mu_1\delta_1\lambda_1(1-\delta_1)}{4\lambda_1\xi_2} \right)^2 V(\hat{\mu}_1 - \mu_1)$$

$$+ \left(\frac{-(1-\hat{\delta}_1)\left(4 - (1-\delta_1)\left(\Gamma\left(\frac{1}{2}\right)\right)^2 + 2\sqrt{\lambda_1}\hat{\mu}_1\Gamma\left(\frac{1}{2}\right)\hat{\delta}_1\right)}{4\xi_2\lambda_1^2} \right)^2 V(\hat{\lambda}_1 - \lambda_1)$$

$$+ \left(\frac{-\xi_1\left(2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_2) - 4 + 4\sqrt{\lambda_2}\mu_2\Gamma\left(\frac{1}{2}\right)(1-2\delta_2) + 4\lambda_2\mu_2^2(1-2\delta_2)\right)}{4\lambda_2\xi_2^2} \right)^2 V(\hat{\delta}_2 - \delta_2)$$

$$+ \left(\frac{-\hat{\xi}_1\left(4\sqrt{\lambda_2}\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2) + 8\lambda_2\mu_2\delta_2(1-\delta_2)\right)}{4\lambda_2\xi_2^2} \right)^2 V(\hat{\mu}_2 - \mu_2)$$

$$+ \left(\frac{-\xi_1\left(\frac{2\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{\sqrt{\lambda_2}} + 4\mu_2^2\delta_2(1-\delta_2)\right)}{4\lambda_2\xi_2^2} + \frac{\xi_1}{\lambda_2\xi_2} \right)^2 V(\hat{\lambda}_2 - \lambda_2)$$

$$V(\hat{\tau}_{\text{var}}) = \left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_1) - 4 + 4\sqrt{\lambda_1}\mu_1\Gamma\left(\frac{1}{2}\right)(1-2\delta_1) + 4\lambda_1\mu_1^2(1-2\delta_1)}{4\lambda_1\xi_2} \right)^2 \frac{\delta_1(1-\delta_1)}{n_1}$$

$$+ \left(\frac{4\sqrt{\lambda_1}\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1) + 8\mu_1\delta_1\lambda_1(1-\delta_1)}{4\lambda_1\xi_2} \right)^2 \left(\frac{1}{2\mu_1 + \frac{1}{n_{1(1)}}\sum_{j=1}^{n_{1(1)}}(x_{1j}-\mu_1)^{-2}} \right)$$

$$+ \left(\frac{-(1-\delta_1)\left(4-(1-\delta_1)\left(\Gamma\left(\frac{1}{2}\right)\right)^2 + 2\sqrt{\lambda_1}\mu_1\Gamma\left(\frac{1}{2}\right)\delta_1\right)}{4\xi_2\lambda_1^2} \right)^2 \frac{\lambda_1^2}{n_{1(1)}}$$

$$+ \left(\frac{-\xi_1\left(2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_2) - 4 + 4\sqrt{\lambda_2}\mu_2\Gamma\left(\frac{1}{2}\right)(1-2\delta_2) + 4\lambda_2\mu_2^2(1-2\delta_2)\right)}{4\lambda_2\xi_2^2} \right)^2 \frac{\delta_2(1-\delta_2)}{n_2}$$

$$+ \left(\frac{-\xi_1\left(4\sqrt{\lambda_2}\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2) + 8\lambda_2\mu_2\delta_2(1-\delta_2)\right)}{4\lambda_2\xi_2^2} \right)^2 \left(\frac{1}{2\mu_2 + \frac{1}{n_{2(1)}}\sum_{j=1}^{n_{2(1)}}(x_{2j}-\mu_2)^{-2}} \right)$$

$$+ \left(\frac{-\xi_1\left(\frac{2\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{\sqrt{\lambda_2}} + 4\mu_2^2\delta_2(1-\delta_2)\right)}{4\lambda_2\xi_2^2} + \frac{\xi_1}{\lambda_2\xi_2} \right)^2 \frac{\lambda_2^2}{n_{2(1)}}.$$

The estimations of $\hat{V}\left(\hat{\tau}\right)$ are delineated as

$$\hat{V}(\hat{\tau}_{\text{var}}) = \left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\hat{\delta}_1) - 4 + 4\sqrt{\hat{\lambda}_1}\hat{\mu}_1\Gamma\left(\frac{1}{2}\right)(1-2\hat{\delta}_1) + 4\hat{\lambda}_1\hat{\mu}_1^2(1-2\hat{\delta}_1)}{4\hat{\lambda}_1\hat{\xi}_2} \right)^2 \frac{\hat{\delta}_1(1-\hat{\delta}_1)}{n_1}$$

$$+ \left(\frac{4\sqrt{\hat{\lambda}_1}\Gamma\left(\frac{1}{2}\right)\hat{\delta}_1(1-\hat{\delta}_1) + 8\hat{\mu}_1\hat{\delta}_1\hat{\lambda}_1(1-\hat{\delta}_1)}{4\hat{\lambda}_1\hat{\xi}_2} \right)^2 \left(\frac{1}{2\hat{\mu}_1 + \frac{1}{n_{1(1)}}\sum_{j=1}^{n_{1(1)}}(x_{1j(1)} - \hat{\mu}_1)^{-2}} \right)$$

$$+ \left(\frac{-(1-\hat{\delta}_1)\left(4 - (1-\hat{\delta}_1)\left(\Gamma\left(\frac{1}{2}\right)\right)^2 + 2\sqrt{\hat{\lambda}_1}\hat{\mu}_1\Gamma\left(\frac{1}{2}\right)\hat{\delta}_1\right)}{4\hat{\xi}_2\hat{\lambda}_1^2} \right)^2 \frac{\hat{\lambda}_1^2}{n_{1(1)}}$$

$$+ \left(\frac{-\hat{\xi}_1\left(2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\hat{\delta}_2) - 4 + 4\sqrt{\hat{\lambda}_2}\hat{\mu}_2\Gamma\left(\frac{1}{2}\right)(1-2\hat{\delta}_2) + 4\hat{\lambda}_2\hat{\mu}_2^2(1-2\hat{\delta}_2)\right)}{4\hat{\lambda}_2\hat{\xi}_2^2} \right)^2 \frac{\hat{\delta}_2(1-\hat{\delta}_2)}{n_2}$$

$$+ \left(\frac{-\hat{\xi}_1\left(4\sqrt{\hat{\lambda}_2}\Gamma\left(\frac{1}{2}\right)\hat{\delta}_2(1-\hat{\delta}_2) + 8\hat{\lambda}_2\hat{\mu}_2\hat{\delta}_2(1-\hat{\delta}_2)\right)}{4\hat{\lambda}_2\hat{\xi}_2^2} \right)^2 \left(\frac{1}{2\hat{\mu}_2 + \frac{1}{n_{2(1)}}\sum_{j=1}^{n_{2(1)}}(x_{2j(1)} - \hat{\mu}_2)^{-2}} \right)$$

$$+ \left(-\hat{\xi}_1\left(\frac{2\hat{\mu}_2\Gamma\left(\frac{1}{2}\right)\hat{\delta}_2(1-\hat{\delta}_2)}{\sqrt{\hat{\lambda}_2}} + 4\hat{\mu}_2^2\hat{\delta}_2(1-\hat{\delta}_2) \right) \frac{1}{4\hat{\lambda}_2\hat{\xi}_2^2} + \frac{\hat{\xi}_1}{\hat{\lambda}_2\hat{\xi}_2} \right)^2 \frac{\lambda_2^2}{n_{2(1)}} \qquad (3\text{-}26)$$

respectively. The asymptotic standard normal distribution is represented as $Z = \dfrac{\hat{\theta}_{\text{var}} - \theta_{\text{var}}}{\sqrt{\hat{V}\left(\hat{\theta}_{\text{var}}\right)}}$ and $Z = \dfrac{\hat{\tau}_{\text{var}} - \tau_{\text{var}}}{\sqrt{\hat{V}\left(\hat{\tau}_{\text{var}}\right)}}$. Consequently, the confidence interval for the roughly normal distribution, derived using the delta approach for the difference and ratio of variances, is formulated by Equations (3-27) and (3-28):

$$CI_{\theta_{\text{var}}(AN)} = \left(L_{\theta_{\text{var}}(AN)}, U_{\theta_{\text{var}}(AN)}\right) = \left(\hat{\theta}_{\text{var}} - z_{1-\alpha/2}\sqrt{\hat{V}\left(\hat{\theta}_{\text{var}}\right)}, \hat{\theta}_{mean} + z_{1-\alpha/2}\sqrt{\hat{V}\left(\hat{\theta}_{\text{var}}\right)}\right) \qquad (3\text{-}27)$$

and

$$CI_{\tau_{\text{var}}(AN)} = \left(L_{\tau_{\text{var}}(AN)}, U_{\tau_{\text{var}}(AN)}\right) = \left(\hat{\tau}_{\text{var}} - z_{1-\alpha/2}\sqrt{\hat{V}\left(\hat{\tau}_{\text{var}}\right)}, \hat{\tau}_{\text{var}} + z_{1-\alpha/2}\sqrt{\hat{V}\left(\hat{\tau}_{\text{var}}\right)}\right), \qquad (3\text{-}28)$$

where

$$\hat{\theta}_{\text{var}} = \left(\frac{4(1-\hat{\delta}_1) - (1-\hat{\delta}_1)^2 \left(\Gamma\left(\frac{1}{2}\right) \right)^2}{4\hat{\lambda}_1} + \frac{\hat{\mu}_1 \Gamma\left(\frac{1}{2}\right) \hat{\delta}_1 (1-\hat{\delta}_1)}{\sqrt{\hat{\lambda}_1}} + \hat{\mu}_1^2 \hat{\delta}_1 (1-\hat{\delta}_1) \right)$$

$$- \left(\frac{4\left(1-\hat{\delta}_2\right) - \left(1-\hat{\delta}_2\right)^2 \left(\Gamma\left(\frac{1}{2}\right) \right)^2}{4\hat{\lambda}_2} + \frac{\hat{\mu}_2 \Gamma\left(\frac{1}{2}\right) \hat{\delta}_2 (1-\hat{\delta}_2)}{\sqrt{\hat{\lambda}_2}} + \hat{\mu}_2{}^2 \hat{\delta}_2 \left(1-\hat{\delta}_2\right) \right),$$

and

$$\hat{\tau}_{mean} = \frac{4\hat{\lambda}_2 \left(4(1-\hat{\delta}_1) - (1-\hat{\delta}_1)^2 \left(\Gamma\left(\frac{1}{2}\right) \right)^2 + 4\sqrt{\hat{\lambda}_1} \hat{\mu}_1 \Gamma\left(\frac{1}{2}\right) \hat{\delta}_1 (1-\hat{\delta}_1) + 4\hat{\lambda}_1 \hat{\mu}_1^2 \hat{\delta}_1 (1-\hat{\delta}_1) \right)}{4\hat{\lambda}_1 \left(4(1-\hat{\delta}_2) - (1-\hat{\delta}_2)^2 \left(\Gamma\left(\frac{1}{2}\right) \right)^2 + 4\sqrt{\hat{\lambda}_2} \hat{\mu}_2 \Gamma\left(\frac{1}{2}\right) \hat{\delta}_2 (1-\hat{\delta}_2) + 4\hat{\lambda}_2 \hat{\mu}_2^2 \hat{\delta}_2 (1-\hat{\delta}_2) \right)},$$

respectively. The explicit formulations of $\hat{V}(\hat{\theta})$ and $\hat{V}(\hat{\tau})$ are derived using the delta technique and are provided in equation (3-25) and (3-26)

3.3.6 Percentile Bootstrap (PB)

The percentile bootstrap is a resampling method used to estimate the sampling distribution of a statistic and to derive confidence intervals without applying stringent parametric assumptions. Let $X_{ij} = \left(X_{i1}, X_{i2}, \ldots, X_{in_i} \right)$ represent a random sample from *ZITR* .A bootstrap sample $X_i^{*b} = \left(X_{i1}^{*b}, X_{i2}^{*b}, \ldots, X_{in_i}^{*b} \right)$ is generated by sampling with replacement from the original dataset, and this process is repeated B times. For every bootstrap sample, the relevant statistics are calculated. Let θ represent the disparity between two variances and τ signify the ratio of two variances. Let $\hat{\theta}$ and $\hat{\tau}$ represent the estimators derived from the original sample. For the b th bootstrap sample, denote the associated bootstrap estimates as $\hat{\theta}^{*b}$ and $\hat{\tau}^{*b}$ for $b = 1, 2, \ldots, B$. The empirical distributions of these bootstrap repetitions are used to ascertain the confidence bounds. The two-sided percentile bootstrap confidence interval for the variance difference is defined as

$$CI_{\theta_{\text{var}}(PB)} = \left(L_{\theta_{\text{var}}(PB)}, U_{\theta_{\text{var}}(PB)} \right) = \left(\hat{\theta}^*_{\text{var}}\left(\frac{\alpha}{2}\right), \hat{\theta}^*_{\text{var}}\left(1-\frac{\alpha}{2}\right) \right), \tag{3-29}$$

Here, $\hat{\theta}^*_{\text{var}}\left(\frac{\alpha}{2}\right)$ and $\hat{\theta}^*_{\text{var}}\left(1-\frac{\alpha}{2}\right)$ represent the empirical $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$ quantiles of the ordered bootstrap estimates $\hat{\theta}^{*b}$.

The two-sided percentile bootstrap confidence interval for the variance ratio is expressed as

$$CI_{\tau_{\text{var}}(PB)} = \left(L_{\tau_{\text{var}}(PB)}, U_{\tau_{\text{var}}(PB)} \right) = \left(\hat{\tau}^*_{\text{var}}\left(\frac{\alpha}{2}\right), \hat{\tau}^*_{\text{var}}\left(1-\frac{\alpha}{2}\right) \right), \tag{3-30}$$

where $\hat{\tau}^*_{\text{var}}\left(\frac{\alpha}{2}\right)$ and $\hat{\tau}^*_{\text{var}}\left(1-\frac{\alpha}{2}\right)$ represent the respective empirical quantiles of the bootstrap distribution.

3.3.7 The Bootstrap with Standard Error (BS)

The bootstrap technique for standard error estimation generates confidence intervals by approximating the variability of an estimator by periodic resampling from the observed data. This method circumvents the direct mathematical calculation of the variance of the parameter of interest, in contrast to traditional approaches. The dispersion of bootstrap replicates is used to estimate the standard error, offering a pragmatic option when the sampling distribution is intricate or analytically unmanageable. Let θ and τ represent the difference and the ratio of variances, respectively, under ZITR. Let $\hat{\theta}^{*b}$ and $\hat{\tau}^{*b}$, where $b = 1, 2, \ldots, B$, represent the bootstrap estimates derived from B resamples. The bootstrap means are delineated as

$$\bar{\theta}^{*}_{\text{var}} = \frac{1}{B}\sum_{b=1}^{B}\hat{\theta}^{*b}_{\text{var}}, \text{ and } \bar{\tau}^{*}_{\text{var}} = \frac{1}{B}\sum_{b=1}^{B}\hat{\tau}^{*b}_{\text{var}}.$$

The standard errors are derived from the sample standard deviations of the bootstrap distributions expressed as:

$$SE(\hat{\theta}_{\text{var}}) = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\theta}^{*b}_{\text{var}} - \bar{\theta}^{*}_{\text{var}}\right)^2}, \text{and } SE(\hat{\tau}_{\text{var}}) = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\tau}^{*b}_{\text{var}} - \bar{\tau}^{*}_{\text{var}}\right)^2}.$$

The two-sided $100(1-\alpha)\%$ confidence interval for the variance difference is expressed as

$$CI_{\theta_{\text{var}}(BS)} = \left(L_{\theta_{\text{var}}(BS)}, U_{\theta_{\text{var}}(BS)}\right) = \left(\bar{\theta}^{*}_{\text{var}} - Z_{\alpha/2}SE(\hat{\theta}_{\text{var}}), \bar{\theta}^{*}_{\text{var}} + Z_{\alpha/2}SE(\hat{\theta}_{\text{var}})\right), \quad (3\text{-}31)$$

and the confidence interval for the ratio of variances is

$$CI_{\tau_{\text{var}}(BS)} = \left(L_{\tau_{\text{var}}(BS)}, U_{\tau_{\text{var}}(BS)}\right) = \left(\bar{\tau}^{*}_{\text{var}} - Z_{\alpha/2}SE(\hat{\tau}^{*}_{\text{var}}), \bar{\tau}^{*}_{\text{var}} + Z_{\alpha/2}SE(\hat{\tau}^{*}_{\text{var}})\right), \quad (3\text{-}32)$$

where $Z_{\alpha/2}$ represents the higher $\alpha/2$ quantile of the ordinary normal distribution.

3.4 Simulation study and results

This study evaluates the performance of the proposed methods for constructing confidence intervals for the single variance, as well as the difference and ratio of variances between two ZITR distributions. For estimating the confidence interval of the variance, the method MCMC, HPD, STD, AN, PB, BS, and GCI are employed. For estimating the confidence intervals of the difference and ratio of variances, the methods MCMC, HPD, GCI, MOVER, AN, PB, and BS are used. The evaluation process was conducted by simulating data using the R software with the Monte Carlo simulation technique. The performance of the methods was assessed based on coverage probability (CP) and expected Length (EL). Methods with CP values close to or greater than the nominal confidence level of 0.95 were considered appropriate. Among these methods, the one with the shortest EL was regarded as the most efficient. For the single variance, the data were generated for $X \sim ZITR(\mu, \lambda, \delta)$. with sample sized n = 20, 50, 70, 100 and 200 and probability of zero $\delta = 0.1, 0.2, 0.3$ and 0.4, where we set $\lambda = 0.2, 0.5$, and 1, and μ=1, and 2. We used M = 1000 replications for the Monte Carlo simulation. For the bootstrap method, each situation involved B = 1000 replications, along with q = 2500 pivotal quantities, and we generated T = 20,000 realizations of Bayesian using Gibbs sampling with a burn-in of C = 5000.

For difference and ratio variances, the data generate for two independent ZITR distribution: $X_i \sim ZITR(\mu_i, \lambda_i, \delta_i), i = 1, 2.$ Sample sizes were considered

$(n_1, n_2) = (20,20), (20,50), (20,70), (20,100), (50,50), (50,70), (50,100), (70,70), (70,100), (100,100)$, and $(200,200)$. The probability of zeros were set at $(\delta_1, \delta_2) = (0.1,0.1), (0.1,0.3)$ and $(0.3,0.3)$. The scale parameter $(\lambda_1, \lambda_2) = (0.2,0.2)$,$(0.2,1)$, and $(1,1)$. The location parameter fixed $(\mu_1, \mu_2) = (0.5,0.5)$. The number of generated M = 1000 replications for the Monte Carlo simulation, 1000 resampling for bootstrap method, 2500 pivotal quantities were utilized for the GCI, and we generated T = 20,000 realizations of Bayesian using Gibbs sampling with a burn-in of C = 5000.

The simulation study evaluating the performance of confidence interval estimation methods for the single variance, as presented in Tables 3-1 to 3-6 and Figure 3-1. The simulation results indicate that the AN technique consistently yielded CP values under 0.95 across all parameter combinations and sample sizes. Despite AN producing the briefest interval lengths, its inadequate coverage suggests that the real parameter is not sufficiently represented at the required frequency. A same trend was seen with the BS technique, which also displayed sub-nominal CP values in several cases. Thus, both AN and BS are deemed unreliable within the analyzed parameters. The STD approach often attained acceptable CP values in the majority of scenarios. Nonetheless, its intervals were somewhat broader than those of the other methods, resulting in diminished estimate accuracy. Consequently, although its coverage performance is satisfactory, STD is less efficient regarding EL. The performance of the PB and GCI approaches enhanced with an increase in sample size. Both approaches yielded CP values of 0.95 for small samples, demonstrating modest EL and suggesting satisfactory stability and resilience. The MCMC and HPD methodologies had the most commendable overall efficacy. Both methodologies regularly attained CP levels at or above the nominal level while preserving very short interval durations in relation to most traditional techniques. HPD demonstrated a small benefit by generating somewhat shorter intervals while maintaining coverage accuracy. In summary, HPD offers the most advantageous equilibrium between CP and EL, following by MCMC, PB, and GCI. Conversely, AN and BS are not advisable owing to their insufficient coverage efficacy.

TABLE 3-1 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Variance of the ZITR Distribution with $\mu = 1$, $\lambda = 0.2$, and $\delta = 0.1, 0.2, 0.3, 0.4$.

λ	δ	Coverage probability							
		n	PB	BS	GCI	STD	AN	MCMC	HPD
0.2	0.1	20	0.861	0.746	0.941	0.499	0.645	0.963	0.966
			(1.8979)	(1.8988)	(2.3130)	(1.3060)	(1.1159)	(2.0663)	**(1.9906)**
		50	0.910	0.794	0.937	0.816	0.764	0.950	0.956
			(1.3015)	(1.3058)	(1.3836)	(1.6112)	(0.8949)	(1.3283)	**(1.3061)**
		70	0.903	0.778	0.948	0.820	0.772	0.956	0.958
			(1.3011)	(1.2815)	(1.3810)	(1.5901)	(0.9019)	(1.3254)	**(1.3031)**
		100	0.929	0.813	0.943	0.963	0.806	0.954	0.949
			(0.9523)	(0.9493)	(0.9752)	(1.6364)	(0.6833)	**(0.9517)**	(0.9428)
		200	0.945	0.813	0.953	1.000	0.847	0.964	0.959
			(0.6750)	(0.6751)	(0.6835)	(1.6016)	(0.4920)	(0.6741)	**(0.6701)**
	0.2	20	0.868	0.763	0.948	0.674	0.640	0.955	0.935
			(2.1138)	(2.1263)	(2.5782)	(1.8506)	(1.2103)	**(2.2004)**	(2.1201)
		50	0.941	0.823	0.968	0.957	0.740	0.970	0.963
			(1.4347)	(1.4535)	(1.5375)	(2.3773)	(0.8825)	(1.4444)	**(1.4236)**
		70	0.929	0.821	0.946	0.991	0.750	0.949	0.946
			(1.2281)	(1.2427)	(1.2809)	**(2.4529)**	(0.7740)	(1.2254)	(1.2130)
		100	0.938	0.840	0.950	0.999	0.786	0.954	0.954
			(1.0306)	(1.0303)	(1.0674)	(2.5224)	(0.6525)	(1.0343)	**(1.0262)**

TABLE 3-1 (Con)

λ	δ	n	Coverage probability						
			PB	BS	GCI	STD	AN	MCMC	HPD
		200	0.944	0.817	0.959	1.000	0.784	0.956	0.952
			(0.7330)	(0.7325)	(0.7443)	(2.5782)	(0.4684)	(0.7324)	**(0.7290)**
	0.3	20	0.851	0.778	0.964	0.628	0.547	0.947	0.928
			(2.1682)	(2.1972)	**(2.8395)**	(1.5856)	(1.0464)	(2.3830)	(2.2802)
		50	0.919	0.795	0.961	0.980	0.552	0.949	0.941
			(1.4277)	(1.4224)	**(1.5694)**	(2.5559)	(0.6268)	(1.4921)	(1.4656)
		70	0.925	0.818	0.947	0.997	0.570	0.944	0.938
			(1.2172)	(1.2247)	(1.3065)	**(2.7488)**	(0.5315)	(1.2632)	(1.2468)
		100	0.834	0.754	0.955	0.796	0.557	0.960	0.955
			(1.0654)	(1.0625)	(1.4279)	(1.4717)	(0.5294)	(1.2818)	**(1.2251)**
		200	0.940	0.814	0.949	1.000	0.587	0.946	0.946
			(0.7329)	(0.7316)	(0.7517)	**(3.1172)**	(0.3147)	(0.7459)	(0.7417)
	0.4	20	0.828	0.751	0.951	0.476	0.451	0.920	0.904
			(2.1595)	(2.2054)	**(3.0410)**	(0.7362)	(0.8962)	(2.5048)	(2.3743)
		50	0.916	0.808	0.950	1.000	0.429	0.950	0.938
			(0.5762)	(0.5770)	(0.6226)	(2.4278)	(0.1895)	**(0.6160)**	(0.6074)
		70	0.896	0.762	0.946	0.987	0.290	0.932	0.929
			(1.2202)	(1.1978)	(1.3398)	**(2.6466)**	(0.2951)	(1.2999)	(1.2759)
		100	0.920	0.824	0.956	1.000	0.300	0.950	0.942
			(1.0375)	(1.0324)	(1.1040)	(2.9081)	(0.2140)	**(1.0870)**	(1.0716)
		200	0.951	0.842	0.959	1.000	0.251	0.960	0.955
			(0.7365)	(0.7396)	(0.7606)	(3.2537)	(0.1242)	(0.7583)	**(0.7520)**

TABLE 3-2 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the variance of the ZITR Distribution with $\mu = 1$, $\lambda = 0.5$, and $\delta = 0.1, 0.2, 0.3, 0.4$.

λ	δ	n	Coverage probability						
			PB	BS	GCI	STD	AN	MCMC	HPD
0.5	0.1	20	0.849	0.728	0.930	0.655	0.658	0.956	0.974
			(0.9699)	(0.9759)	(1.1068)	(1.2919)	(0.6540)	(1.0553)	**(1.0227)**
		50	0.912	0.776	0.948	0.972	0.832	0.960	0.966
			(0.6823)	(0.6709)	(0.7024)	(1.4090)	(0.5424)	(0.6821)	**(0.6723)**
		70	0.927	0.831	0.953	0.994	0.842	0.958	0.963
			(0.5873)	(0.5822)	(0.5966)	(1.4051)	(0.4717)	(0.5799)	**(0.5735)**
		100	0.937	0.822	0.947	0.999	0.865	0.953	0.959
			(0.4977)	(0.5019)	(0.5008)	(1.3528)	(0.4088)	(0.4902)	**(0.4860)**
		200	0.931	0.826	0.941	1.000	0.873	0.947	0.950
			(0.3560)	(0.3574)	(0.3579)	(1.3170)	(0.2954)	(0.3523)	**(0.3503)**
	0.2	20	0.916	0.800	0.956	0.883	0.729	0.970	0.973
			(1.1140)	(1.1200)	(1.2691)	(1.8971)	(0.7436)	(1.1384)	**(1.1061)**
		50	0.926	0.797	0.948	1.000	0.805	0.959	0.955
			(0.7388)	(0.7398)	(0.7653)	(2.0768)	(0.5391)	(0.7244)	**(0.7174)**
		70	0.935	0.804	0.947	1.000	0.830	0.953	0.954
			(0.6328)	(0.6219)	(0.6455)	(2.0972)	(0.4657)	(0.6182)	**(0.6142)**
		100	0.865	0.800	0.956	0.874	0.655	0.963	0.952
			(1.0948)	(1.0932)	(1.3307)	(1.8503)	(0.6441)	(1.2010)	**(1.1613)**
		200	0.949	0.824	0.953	1.000	0.855	0.958	0.954
			(0.3797)	(0.3894)	(0.3820)	(2.0971)	(0.2861)	(0.3752)	**(0.3739)**
	0.3	20	0.925	0.815	0.960	1.000	0.681	0.959	0.948
			(0.7129)	(0.7172)	(0.7603)	(2.3382)	(0.3924)	**(0.7351)**	(0.7265)
		50	0.924	0.829	0.945	1.000	0.670	0.948	0.948
			(0.6008)	(0.6009)	(0.6312)	**(2.4208)**	(0.3273)	(0.6176)	(0.6123)
		70	0.938	0.827	0.949	1.000	0.855	0.956	0.953
			(0.5341)	(0.5330)	(0.5433)	(2.1146)	(0.3971)	(0.5260)	**(0.5234)**
		100	0.834	0.754	0.955	0.796	0.557	0.960	0.955
			(1.0654)	(1.0625)	(1.4279)	(1.4717)	(0.5294)	(1.2818)	**(1.2251)**

TABLE 3-2 (Con)

λ	δ	Coverage probability							
		n	PB	BS	GCI	STD	AN	MCMC	HPD
		200	0.944	0.832	0.951	1.000	0.698	0.952	0.952
			(0.3598)	(0.3568)	(0.3650)	(2.5467)	(0.2026)	(0.3626)	**(0.3612)**
	0.4	20	0.834	0.754	0.955	0.796	0.557	0.960	0.955
			(1.0654)	(1.0625)	(1.4279)	(1.4717)	(0.5294)	(1.2818)	**(1.2251)**
		50	0.909	0.801	0.956	0.998	0.481	0.955	0.951
			(0.6799)	(0.6739)	(0.7564)	(2.2694)	(0.2517)	(0.7431)	**(0.7298)**
		70	0.916	0.808	0.950	1.000	0.429	0.950	0.938
			(0.5762)	(0.5770)	(0.6226)	(2.4278)	(0.1895)	**(0.6160)**	(0.6074)
		100	0.922	0.807	0.951	1.000	0.399	0.946	0.944
			(0.4832)	(0.4847)	**(0.5110)**	(2.5533)	(0.1415)	(0.5100)	(0.5046)
		200	0.938	0.839	0.958	1.000	0.376	0.956	0.957
			(0.3432)	(0.3446)	(0.3536)	(2.7162)	(0.0871)	(0.3542)	**(0.3519)**

TABLE 3-3 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Variance of the ZITR Distribution with $\mu = 1$, $\lambda = 1$, and $\delta = 0.1, 0.2, 0.3, 0.4$.

λ	δ	Coverage probability							
		n	PB	BS	GCI	STD	AN	MCMC	HPD
1	0.1	20	0.880	0.749	0.928	0.824	0.701	0.958	0.978
			(0.6414)	(0.6585)	(0.6903)	(1.2581)	(0.4766)	(0.6848)	**(0.6677)**
		50	0.917	0.791	0.943	0.999	0.847	0.950	0.965
			(0.4572)	(0.4633)	(0.4603)	(1.2794)	(0.3930)	(0.4498)	**(0.4439)**
		70	0.923	0.803	0.945	0.999	0.867	0.962	0.964
			(0.3924)	(0.3874)	(0.3935)	(1.2457)	(0.3416)	(0.3847)	**(0.3806)**
		100	0.926	0.802	0.943	1.000	0.874	0.949	0.947
			(0.3331)	(0.3319)	(0.3332)	**(1.2119)**	(0.2950)	(0.3264)	(0.3236)
		200	0.936	0.850	0.942	1.000	0.900	0.948	0.948
			(0.2395)	(0.2394)	(0.2391)	(1.1533)	(0.2136)	(0.2357)	(0.2344)
	0.2	20	0.891	0.791	0.932	0.966	0.784	0.979	0.975
			(0.7222)	(0.7276)	(0.7737)	(1.7351)	(0.5501)	(0.7285)	**(0.7134)**
		50	0.929	0.832	0.943	1.000	0.853	0.966	0.958
			(0.4915)	(0.4905)	(0.4956)	(1.8770)	(0.3957)	(0.4726)	**(0.4696)**
		70	0.944	0.825	0.963	1.000	0.886	0.968	0.968
			(0.4175)	(0.4168)	(0.4194)	(1.8816)	(0.3405)	(0.4038)	**(0.4019)**
		100	0.943	0.839	0.956	1.000	0.884	0.961	0.961
			(0.3522)	(0.3523)	(0.3530)	(1.8585)	(0.2918)	(0.3426)	**(0.3413)**
		200	0.945	0.831	0.946	1.000	0.891	0.951	0.949
			(0.2516)	(0.2495)	(0.2513)	(1.8234)	(0.2091)	**(0.2468)**	(0.2461)
	0.3	20	0.891	0.798	0.954	0.982	0.714	0.978	0.968
			(0.6980)	(0.7072)	(0.7966)	(1.8133)	(0.4666)	(0.7679)	**(0.7485)**
		50	0.938	0.828	0.962	0.998	0.767	0.966	0.966
			(0.4469)	(0.4464)	(0.4676)	(2.1173)	(0.2908)	(0.4621)	**(0.4586)**
		70	0.951	0.819	0.958	1.000	0.777	0.961	0.960
			(0.3844)	(0.3804)	(0.3954)	(2.1672)	(0.2523)	(0.3917)	**(0.3898)**
		100	0.926	0.812	0.936	1.000	0.774	0.946	0.947
			(0.3218)	(0.3206)	(0.3270)	**(2.1900)**	(0.2143)	(0.3248)	(0.3236)
		200	0.944	0.855	0.950	1.000	0.789	0.951	0.953
			(0.2260)	(0.2266)	(0.2279)	(2.2186)	(0.1503)	(0.2272)	**(0.2266)**
	0.4	20	0.856	0.797	0.952	0.944	0.613	0.975	0.968
			(0.6742)	(0.6660)	(0.8476)	(1.5323)	(0.3757)	(0.8328)	**(0.8027)**
		50	0.903	0.808	0.955	0.998	0.531	0.966	0.963
			(0.4045)	(0.4033)	(0.4447)	(2.0875)	(0.1774)	(0.4534)	**(0.4475)**
		70	0.919	0.805	0.952	0.997	0.550	0.960	0.959
			(0.3423)	(0.3378)	(0.3657)	(2.1889)	(0.1461)	(0.3727)	**(0.3690)**
		100	0.938	0.802	0.958	1.000	0.492	0.959	0.963
			(0.2858)	(0.2840)	(0.3009)	(2.2783)	(0.1072)	(0.3067)	**(0.3044)**
		200	0.935	0.821	0.952	1.000	0.469	0.953	0.955
			(0.2017)	(0.1993)	(0.2067)	(2.3745)	(0.0685)	(0.2093)	**(0.2082)**

TABLE 3-4 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Variance of the ZITR Distribution with $\mu = 2$, $\lambda = 0.2$, and $\delta = 0.1, 0.2, 0.3, 0.4$.

λ	δ	Coverage probability							
		n	PB	BS	GCI	STD	AN	MCMC	HPD
0.2	0.1	20	0.859	0.729	0.912	0.761	0.673	0.964	0.980
			(2.8849)	(2.9370)	(3.1883)	(2.2497)	(2.0698)	(2.9759)	**(2.8993)**
		50	0.931	0.794	0.943	0.926	0.843	0.963	0.962
			(2.0689)	(2.0640)	(2.0972)	(2.2595)	(1.7329)	(2.0102)	**(1.9837)**
		70	0.916	0.786	0.938	0.949	0.855	0.966	0.957
			(1.7713)	(1.7539)	(1.7804)	(2.1430)	(1.5097)	(1.7189)	**(1.7005)**
		100	0.925	0.823	0.942	0.977	0.879	0.964	0.960
			(1.5036)	(1.5209)	(1.5067)	(2.0473)	(1.3033)	(1.4641)	**(1.4517)**
		200	0.949	0.826	0.949	0.999	0.905	0.956	0.959
			(1.0804)	(1.0826)	(1.0803)	(1.8878)	(0.9466)	(1.0616)	**(1.0558)**
	0.2	20	0.899	0.788	0.939	0.809	0.754	0.965	0.954
			(3.3314)	(3.2904)	(3.6246)	(2.7560)	(2.4076)	(3.1570)	**(3.0912)**
		50	0.925	0.828	0.937	0.962	0.827	0.958	0.950
			(2.2247)	(2.2392)	(2.2684)	(2.8789)	(1.7550)	(2.1157)	**(2.1003)**
		70	0.941	0.824	0.957	0.991	0.861	0.963	0.951
			(1.9054)	(1.8914)	(1.9219)	(2.8590)	(1.5140)	(1.8202)	**(1.8106)**
		100	0.933	0.824	0.946	0.997	0.869	0.951	0.950
			(1.6047)	(1.5769)	(1.6110)	(2.8390)	(1.2872)	(1.5471)	**(1.5409)**
		200	0.954	0.834	0.960	1.000	0.877	0.958	0.960
			(1.1414)	(1.1518)	(1.1433)	(2.7660)	(0.9187)	(1.1182)	**(1.1149)**
	0.3	20	0.878	0.777	0.935	0.736	0.703	0.919	0.909
			(3.2336)	(3.2204)	(3.7589)	(2.1345)	(2.0983)	(3.2686)	(3.1909)
		50	0.924	0.805	0.950	0.963	0.716	0.935	0.932
			(2.0646)	(2.0409)	**(2.1801)**	(2.7857)	(1.2920)	(2.0750)	(2.0581)
		70	0.930	0.830	0.946	0.987	0.741	0.941	0.947
			(1.7484)	(1.7513)	(1.8131)	**(2.8958)**	(1.1022)	(1.7540)	(1.7436)
		100	0.928	0.833	0.945	1.000	0.741	0.937	0.934
			(1.4700)	(1.4868)	(1.5078)	(2.9929)	(0.9322)	(1.4754)	(1.4687)
		200	0.937	0.822	0.943	1.000	0.754	0.943	0.945
			(1.0390)	(1.0351)	(1.0511)	(3.1106)	(0.6596)	(1.0424)	(1.0390)
	0.4	20	0.828	0.754	0.950	0.488	0.566	0.912	0.892
			(3.0123)	(2.9869)	**(3.8492)**	(0.7599)	(1.6487)	(3.3283)	(3.2190)
		50	0.915	0.826	0.948	0.908	0.534	0.933	0.928
			(1.9235)	(1.9425)	(2.1284)	(2.1727)	(0.8112)	(2.0547)	(2.0261)
		70	0.919	0.820	0.940	0.974	0.463	0.930	0.920
			(1.6208)	(1.6402)	(1.7372)	**(2.4692)**	(0.6131)	(1.7017)	(1.6831)
		100	0.924	0.806	0.938	0.999	0.462	0.933	0.928
			(1.3495)	(1.3512)	(1.4167)	**(2.7239)**	(0.4820)	(1.4043)	(1.3927)
		200	0.932	0.844	0.945	1.000	0.402	0.945	0.940
			(0.9600)	(0.9607)	(0.9785)	**(3.0472)**	(0.2971)	(0.9794)	(0.9739)

TABLE 3-5 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Variance of the ZITR Distribution with $\mu = 2$, $\lambda = 0.5$, and $\delta = 0.1, 0.2, 0.3, 0.4$.

λ	δ	n	Coverage probability						
			PB	BS	GCI	STD	AN	MCMC	HPD
0.5	0.1	20	0.875	0.738	0.893	0.854	0.701	0.964	0.975
			(1.8155)	(1.8878)	(1.8935)	(2.1515)	(1.4573)	(1.8181)	**(1.7812)**
		50	0.928	0.777	0.954	0.975	0.869	0.972	0.971
			(1.3199)	(1.3122)	(1.3076)	(1.9722)	(1.1840)	(1.2588)	**(1.2431)**
		70	0.928	0.801	0.941	0.984	0.892	0.950	0.951
			(1.1402)	(1.1349)	(1.1297)	(1.8823)	(1.0456)	(1.0901)	**(1.0786)**
		100	0.944	0.823	0.949	0.999	0.901	0.967	0.962
			(0.9684)	(0.9547)	(0.9613)	(1.7823)	(0.8998)	(0.9334)	**(0.9256)**
		200	0.945	0.826	0.952	1.000	0.927	0.943	0.944
			(0.6961)	(0.7028)	**(0.6925)**	(1.6201)	(0.6548)	(0.6801)	(0.6764)
	0.2	20	0.908	0.805	0.929	0.940	0.797	0.973	0.960
			(2.0494)	(2.1072)	(2.0951)	(2.6627)	(1.6925)	(1.8736)	**(1.8508)**
		50	0.934	0.816	0.946	0.996	0.880	0.963	0.955
			(1.3983)	(1.4045)	(1.3855)	(2.6295)	(1.2188)	(1.2934)	**(1.2881)**
		70	0.949	0.821	0.956	0.999	0.904	0.966	0.963
			(1.2027)	(1.2024)	(1.1919)	(2.5609)	(1.0581)	(1.1268)	**(1.1229)**
		100	0.938	0.835	0.947	1.000	0.898	0.954	0.949
			(1.0128)	(1.0220)	(1.0043)	(2.4992)	(0.9009)	**(0.9621)**	(0.9593)
		200	0.942	0.832	0.944	1.000	0.911	0.948	0.945
			(0.7198)	(0.7299)	(0.7184)	**(2.3956)**	(0.6449)	(0.7015)	(0.6996)
	0.3	20	0.915	0.817	0.953	0.916	0.761	0.953	0.948
			(1.9194)	(1.8504)	(2.0821)	(2.3758)	(1.4204)	**(1.8883)**	(1.8646)
		50	0.926	0.817	0.938	0.997	0.801	0.947	0.947
			(1.2320)	(1.2144)	(1.2610)	**(2.6843)**	(0.9203)	(1.2148)	(1.2104)
		70	0.931	0.853	0.943	1.000	0.820	0.945	0.946
			(1.0500)	(1.0592)	(1.0655)	**(2.7380)**	(0.7984)	(1.0374)	(1.0344)
		100	0.930	0.823	0.942	1.000	0.814	0.936	0.939
			(0.8734)	(0.8822)	(0.8770)	**(2.7444)**	(0.6549)	(0.8628)	(0.8604)
		200	0.940	0.835	0.944	1.000	0.839	0.942	0.943
			(0.6197)	(0.6196)	(0.6202)	**(2.7527)**	(0.4750)	(0.6143)	(0.6126)
	0.4	20	0.864	0.820	0.950	0.818	0.695	0.943	0.936
			(1.7554)	(1.7532)	**(2.0628)**	(1.6846)	(1.1322)	(1.8957)	(1.8606)
		50	0.922	0.831	0.958	0.994	0.630	0.951	0.947
			(1.0510)	(1.0462)	(1.1241)	(2.3771)	(0.5539)	**(1.1188)**	(1.1109)
		70	0.918	0.806	0.947	1.000	0.612	0.938	0.940
			(0.8741)	(0.8610)	(0.9205)	**(2.5316)**	(0.4495)	(0.9246)	(0.9194)
		100	0.945	0.839	0.967	1.000	0.586	0.954	0.954
			(0.7225)	(0.7316)	(0.7478)	(2.6285)	(0.3410)	(0.7561)	**(0.7526)**
		200	0.927	0.823	0.948	1.000	0.567	0.941	0.944
			(0.5051)	(0.5041)	(0.5140)	**(2.7961)**	(0.2298)	(0.5187)	(0.5169)

TABLE 3-6 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Variance of the ZITR Distribution with $\mu = 2$, $\lambda = 1$, and $\delta = 0.1, 0.2, 0.3, 0.4$.

λ	δ	Coverage probability							
		n	PB	BS	GCI	STD	AN	MCMC	HPD
1	0.1	20	0.873	0.732	0.877	0.877	0.712	0.970	0.972
			(1.3806)	(1.4219)	(1.4088)	(1.9926)	(1.1702)	(1.3823)	**(1.3578)**
		50	0.927	0.791	0.951	0.985	0.888	0.960	0.958
			(1.0275)	(1.0155)	(1.0095)	(1.8088)	(0.9621)	(0.9749)	**(0.9631)**
		70	0.926	0.806	0.943	0.995	0.900	0.961	0.960
			(0.8863)	(0.8950)	(0.8747)	(1.7129)	(0.8398)	(0.8464)	**(0.8376)**
		100	0.930	0.820	0.947	0.993	0.909	0.960	0.957
			(0.7528)	(0.7538)	(0.7432)	(1.6093)	(0.7218)	(0.7233)	**(0.7169)**
		200	0.945	0.819	0.946	1.000	0.928	0.951	0.950
			(0.5424)	(0.5433)	(0.5395)	(1.4595)	(0.5237)	(0.5294)	**(0.5264)**
	0.2	20	0.914	0.808	0.924	0.983	0.828	0.975	0.966
			(1.5761)	(1.6209)	(1.5458)	(2.5388)	(1.3683)	(1.4105)	**(1.3992)**
		50	0.936	0.805	0.938	0.998	0.886	0.958	0.951
			(1.0736)	(1.0687)	(1.0536)	(2.4105)	(0.9869)	(0.9862)	**(0.9825)**
		70	0.950	0.852	0.953	1.000	0.909	0.963	0.958
			(0.9220)	(0.9215)	(0.9082)	(2.3577)	(0.8534)	(0.8597)	**(0.8568)**
		100	0.932	0.813	0.938	1.000	0.910	0.950	0.940
			(0.7754)	(0.7748)	(0.7670)	(2.2897)	(0.7229)	**(0.7364)**	(0.7342)
		200	0.947	0.832	0.951	1.000	0.922	0.960	0.956
			(0.5544)	(0.5503)	(0.5525)	(2.1739)	(0.5200)	(0.5404)	**(0.5389)**
	0.3	20	0.931	0.815	0.958	0.981	0.807	0.966	0.967
			(1.4330)	(1.4195)	(1.4773)	(2.4033)	(1.1628)	(1.3893)	**(1.3798)**
		50	0.942	0.839	0.949	1.000	0.839	0.953	0.952
			(0.9107)	(0.9180)	(0.9097)	(2.5480)	(0.7522)	(0.8889)	**(0.8860)**
		70	0.937	0.833	0.945	1.000	0.842	0.944	0.948
			(0.7715)	(0.7836)	(0.7703)	**(2.5539)**	(0.6401)	(0.7573)	(0.7547)
		100	0.941	0.823	0.942	1.000	0.873	0.946	0.948
			(0.6439)	(0.6442)	(0.6435)	**(2.5584)**	(0.5419)	(0.6365)	(0.6342)
		200	0.948	0.842	0.947	1.000	0.896	0.949	0.952
			(0.4579)	(0.4575)	(0.4567)	(2.5246)	(0.3920)	(0.4537)	**(0.4521)**
	0.4	20	0.872	0.824	0.950	0.934	0.767	0.962	0.961
			(1.2459)	(1.2434)	(1.3844)	(1.9092)	(0.9018)	(1.3571)	**(1.3420)**
		50	0.926	0.832	0.946	0.999	0.727	0.949	0.951
			(0.7228)	(0.7156)	(0.7565)	(2.3802)	(0.4832)	(0.7765)	**(0.7733)**
		70	0.921	0.833	0.936	1.000	0.696	0.933	0.944
			(0.6026)	(0.5960)	(0.6192)	**(2.4572)**	(0.3761)	(0.6377)	(0.6352)
		100	0.933	0.855	0.941	0.999	0.671	0.937	0.943
			(0.4893)	(0.4937)	(0.5026)	**(2.5204)**	(0.2922)	(0.5185)	(0.5166)
		200	0.936	0.833	0.944	1.000	0.679	0.940	0.945
			(0.3390)	(0.3404)	(0.3436)	**(2.6081)**	(0.1964)	(0.3512)	(0.3501)

TABLE 3-7 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference in Variances of the ZITR Distribution with $(\mu_1, \mu_2) = (0.5, 0.5)$, $(\lambda_1, \lambda_2) = (0.2, 0.2)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.2:0.2	0.1:0.1	20:20	0.890	0.798	0.946	0.946	0.941	0.967	0.974
			(1.8697)	(1.8541)	(3.0073)	(2.9821)	(2.3000)	(2.0196)	**(1.9967)**
		20:50	0.898	0.804	0.948	0.949	0.944	0.962	0.968
			(1.8930)	(1.9068)	(2.3617)	(2.3497)	(2.3570)	(2.0281)	**(2.0047)**
		20:70	0.901	0.797	0.946	0.948	0.930	0.964	0.968
			(1.7750)	(1.7854)	(2.2490)	(2.2385)	(2.2481)	(1.9353)	**(1.9037)**
		20:100	0.902	0.799	0.954	0.952	0.933	0.968	0.983
			(1.7485)	(1.7496)	(2.2235)	(2.2150)	(2.2643)	(1.8852)	**(1.8448)**
		50:50	0.945	0.828	0.940	0.943	0.984	0.947	0.954
			(1.5002)	(1.4928)	(1.6585)	(1.6527)	(2.1003)	(1.5352)	**(1.5297)**
		50:70	0.942	0.823	0.954	0.950	0.982	0.961	0.968
			(1.3963)	(1.3828)	(1.5251)	(1.5184)	(2.0359)	(1.4463)	**(1.4406)**
		50:100	0.951	0.854	0.964	0.963	0.991	0.966	0.966
			(1.2959)	(1.3012)	(1.4141)	(1.4098)	(1.9428)	(1.3483)	**(1.3408)**
		70:70	0.942	0.841	0.942	0.944	0.991	0.949	0.949
			(1.2791)	(1.2774)	(1.3755)	(1.3720)	(1.9651)	(1.2992)	**(1.2950)**
		70:100	0.938	0.829	0.946	0.947	0.997	0.953	0.957
			(1.1784)	(1.1851)	(1.2548)	(1.2517)	(1.8903)	(1.2129)	**(1.2087)**
		100:100	0.949	0.823	0.951	0.951	0.999	0.956	0.962
			(1.0776)	(1.0635)	(1.1328)	(1.1314)	(1.8550)	(1.0880)	**(1.0848)**
		200:200	0.945	0.846	0.940	0.936	1.000	0.952	0.952
			(0.7674)	(0.7709)	(0.7872)	(0.7871)	(1.6979)	(0.7674)	**(0.7653)**
	0.1:0.3	20:20	0.940	0.848	0.955	0.955	0.980	0.940	0.947
			(2.4178)	(2.4070)	(3.3615)	**(3.3347)**	(3.3853)	(2.6985)	(2.6750)
		20:50	0.936	0.846	0.958	0.956	0.999	0.959	0.973
			(2.0098)	(2.0110)	(2.5138)	(2.5001)	(3.2709)	(2.1640)	**(2.1452)**
		20:70	0.901	0.799	0.953	0.949	0.999	0.953	0.959
			(1.8845)	(1.8643)	(2.3650)	(2.3527)	(3.3024)	(2.0308)	**(2.0057)**
		20:100	0.905	0.801	0.957	0.957	0.999	0.961	0.973
			(1.7812)	(1.7721)	(2.2542)	(2.2462)	(3.2407)	(1.9317)	**(1.8986)**
		50:50	0.935	0.845	0.950	0.950	0.998	0.936	0.942
			(1.6166)	(1.6279)	(1.8148)	**(1.8098)**	(3.1311)	(1.6959)	(1.6884)
		50:70	0.947	0.826	0.952	0.951	1.000	0.951	0.956
			(1.4871)	(1.5049)	(1.6438)	(1.6399)	(3.0830)	(1.5561)	**(1.5506)**
		50:100	0.954	0.838	0.963	0.964	1.000	0.964	0.966
			(1.3727)	(1.3593)	(1.4853)	(1.4833)	(3.0340)	(1.4179)	**(1.4118)**
		70:70	0.938	0.847	0.945	0.944	0.999	0.946	0.950
			(1.3662)	(1.3828)	(1.4883)	(1.4862)	(3.0070)	(1.4262)	**(1.4207)**
		70:100	0.945	0.823	0.950	0.954	1.000	0.954	0.958
			(1.2547)	(1.2460)	(1.3443)	(1.3414)	(2.9930)	(1.2987)	**(1.2946)**
		100:100	0.948	0.827	0.946	0.950	1.000	0.947	0.948
			(1.1594)	(1.1792)	(1.2242)	**(1.2222)**	(2.9635)	(1.1914)	(1.1874)
		200:200	0.945	0.854	0.954	0.952	1.000	0.953	0.952
			(0.8234)	(0.8340)	(0.8461)	(0.8451)	(2.8313)	(0.8350)	**(0.8326)**
	0.3:0.3	20:20	0.942	0.823	0.954	0.954	0.995	0.951	0.966
			(2.5712)	(2.6018)	(3.6391)	(3.6100)	(3.9938)	(2.8824)	**(2.8585)**
		20:50	0.911	0.812	0.954	0.951	0.997	0.951	0.958
			(2.1774)	(2.1700)	(2.8987)	(2.8862)	(3.9748)	(2.4190)	**(2.3811)**
		20:70	0.910	0.806	0.962	0.961	0.994	0.950	0.953
			(2.0972)	(2.0901)	(2.8033)	(2.7886)	(3.9818)	(2.3236)	**(2.2722)**
		20:100	0.887	0.786	0.959	0.956	0.996	0.949	0.942
			(1.9726)	(1.9846)	(2.6467)	**(2.6367)**	(3.9361)	(2.1958)	(2.1379)
		50:50	0.945	0.830	0.952	0.953	1.000	0.948	0.959
			(1.7255)	(1.7371)	(1.9664)	(1.9593)	(3.8686)	(1.8176)	**(1.8101)**
		50:70	0.956	0.832	0.959	0.959	1.000	0.957	0.963
			(1.6112)	(1.6173)	(1.7946)	(1.7892)	(3.8690)	(1.6868)	**(1.6791)**

TABLE 3-7 (CON)

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		50:100	0.950	0.835	0.962	0.965	1.000	0.960	0.960
			(1.4968)	(1.5082)	(1.6585)	(1.6529)	(3.8057)	(1.5651)	**(1.5546)**
		70:70	0.950	0.836	0.950	0.950	1.000	0.946	0.951
			(1.4698)	(1.4649)	(1.6093)	(1.6058)	(3.8379)	(1.5294)	**(1.5239)**
		70:100	0.944	0.834	0.947	0.945	1.000	0.947	0.949
			(1.3465)	(1.3127)	(1.4567)	(1.4530)	**(3.7813)**	(1.4008)	(1.3950)
		100:100	0.940	0.826	0.946	0.948	1.000	0.942	0.947
			(1.2338)	(1.2004)	(1.3157)	(1.3150)	**(3.7572)**	(1.2739)	(1.2698)
		200:200	0.943	0.825	0.945	0.947	1.000	0.947	0.946
			(0.8781)	(0.8749)	(0.9071)	(0.9070)	**(3.6713)**	(0.8929)	(0.8903)

TABLE 3-8 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference in Variances of the ZITR Distribution with $(\mu_1, \mu_2) = (0.5, 0.5)$, $(\lambda_1, \lambda_2) = (0.2, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.2:1	0.1:0.1	20:20	0.889	0.769	0.970	0.969	0.824	0.971	0.962
			(1.6072)	(1.6154)	(2.1014)	(2.0984)	(1.8003)	(1.7370)	**(1.6835)**
		20:50	0.863	0.754	0.954	0.956	0.822	0.965	0.963
			(1.5663)	(1.5309)	(2.0524)	(2.0507)	(1.8456)	(1.7120)	**(1.6485)**
		20:70	0.856	0.753	0.956	0.956	0.798	0.969	0.969
			(1.5327)	(1.5728)	(2.0163)	(2.0158)	(1.7670)	(1.6914)	**(1.6261)**
		20:100	0.844	0.727	0.944	0.943	0.811	0.948	0.956
			(1.5382)	(1.5377)	(2.0320)	(2.0307)	(1.7944)	(1.6938)	**(1.6266)**
		50:50	0.920	0.804	0.949	0.948	0.917	0.961	0.956
			(1.0796)	(1.0909)	(1.1927)	(1.1925)	(1.5153)	(1.1146)	**(1.0960)**
		50:70	0.903	0.796	0.940	0.939	0.910	0.945	0.945
			(1.0840)	(1.0927)	(1.1857)	(1.1850)	(1.5193)	(1.1195)	(1.1004)
		50:100	0.917	0.794	0.951	0.951	0.914	0.957	0.959
			(1.0723)	(1.0681)	(1.1775)	(1.1772)	(1.5083)	(1.1110)	**(1.0913)**
		70:70	0.938	0.816	0.954	0.952	0.932	0.961	0.964
			(0.9312)	(0.9397)	(0.9922)	(0.9914)	(1.4186)	(0.9468)	**(0.9349)**
		70:100	0.922	0.791	0.953	0.955	0.931	0.958	0.956
			(0.9177)	(0.8985)	(0.9769)	(0.9773)	(1.4014)	(0.9417)	**(0.9292)**
		100:100	0.929	0.825	0.953	0.952	0.968	0.952	0.952
			(0.7887)	(0.7909)	(0.8244)	(0.8245)	(1.3567)	(0.7960)	**(0.7881)**
		200:200	0.951	0.838	0.949	0.950	0.991	0.952	0.951
			(0.5576)	(0.5583)	(0.5710)	(0.5711)	(1.2434)	(0.5605)	**(0.5570)**
	0.1:0.3	20:20	0.871	0.770	0.946	0.945	0.868	0.961	0.970
			(1.6143)	(1.6232)	(2.1198)	(2.1189)	(1.9919)	(1.7795)	**(1.7345)**
		20:50	0.871	0.768	0.959	0.958	0.865	0.972	0.970
			(1.5640)	(1.5808)	(2.0479)	(2.0456)	(1.9636)	(1.7123)	**(1.6517)**
		20:70	0.831	0.732	0.943	0.943	0.842	0.961	0.965
			(1.5391)	(1.5354)	(2.0349)	(2.0339)	(1.9740)	(1.7016)	**(1.6355)**
		20:100	0.831	0.732	0.943	0.943	0.842	0.961	0.965
			(1.5391)	(1.5354)	(2.0349)	(2.0339)	(1.9740)	(1.7016)	**(1.6355)**
		50:50	0.909	0.797	0.939	0.938	0.956	0.949	0.954
			(1.0986)	(1.0974)	(1.2042)	(1.2040)	(1.7119)	(1.1392)	**(1.1226)**
		50:70	0.919	0.816	0.954	0.953	0.943	0.964	0.960
			(1.0698)	(1.0772)	(1.1816)	(1.1818)	(1.6622)	(1.1173)	**(1.0993)**
		50:100	0.915	0.781	0.955	0.955	0.954	0.960	0.965
			(1.0737)	(1.0550)	(1.1805)	(1.1802)	(1.6732)	(1.1141)	**(1.0950)**
		70:70	0.926	0.802	0.952	0.951	0.971	0.953	0.956
			(0.9317)	(0.9317)	(0.9991)	(0.9987)	(1.6229)	(0.9634)	**(0.9520)**
		70:100	0.929	0.811	0.951	0.951	0.973	0.952	0.954
			(0.9225)	(0.9264)	(0.9839)	(0.9848)	(1.6016)	(0.9482)	**(0.9360)**
		100:100	0.935	0.818	0.951	0.952	0.988	0.950	0.956
			(0.7909)	(0.7876)	(0.8310)	(0.8310)	(1.5297)	(0.8098)	**(0.8023)**

TABLE 3-8 (CON)

$\lambda_1:\lambda_2$	$\delta_1:\delta_2$	Coverage probability							
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		200:200	0.934	0.805	0.945	0.947	0.997	0.950	0.949
			(0.5605)	(0.5575)	(0.5753)	(0.5752)	(1.4328)	**(0.5692)**	(0.5659)
	0.3:0.3	20:20	0.866	0.773	0.956	0.955	0.942	0.943	0.931
			(1.8684)	(1.8636)	(2.5855)	**(2.5839)**	(2.9839)	(2.0867)	(2.0101)
		20:50	0.854	0.750	0.954	0.953	0.923	0.941	0.927
			(1.7983)	(1.8050)	(2.5109)	**(2.5082)**	(2.9599)	(2.0243)	(1.9313)
		20:70	0.833	0.738	0.952	0.952	0.927	0.938	0.918
			(1.7813)	(1.8093)	(2.4759)	**(2.4749)**	(2.9853)	(2.0017)	(1.9077)
		20:100	0.836	0.726	0.957	0.958	0.915	0.934	0.906
			(1.7724)	(1.7688)	(2.4441)	**(2.4436)**	(2.9267)	(1.9735)	(1.8775)
		50:50	0.917	0.816	0.964	0.962	0.998	0.955	0.944
			(1.2268)	(1.2387)	(1.3997)	(1.4001)	(2.8355)	**(1.3043)**	(1.2795)
		50:70	0.920	0.836	0.959	0.958	0.998	0.951	0.944
			(1.2336)	(1.2308)	(1.3995)	(1.3995)	(2.8659)	**(1.3087)**	(1.2821)
		50:100	0.913	0.799	0.946	0.947	0.993	0.949	0.941
			(1.2325)	(1.2344)	(1.3867)	(1.3864)	(2.8508)	(1.2947)	(1.2672)
		70:70	0.937	0.827	0.964	0.965	0.999	0.956	0.946
			(1.0665)	(1.0738)	(1.1614)	(1.1610)	(2.8198)	**(1.1102)**	(1.0935)
		70:100	0.917	0.798	0.935	0.932	0.999	0.933	0.932
			(1.0540)	(1.0538)	(1.1392)	(1.1392)	**(2.7861)**	(1.0906)	(1.0737)
		100:100	0.928	0.817	0.950	0.952	1.000	0.945	0.935
			(0.8914)	(0.8848)	**(0.9427)**	(0.9428)	(2.7395)	(0.9153)	(0.9053)
		200:200	0.935	0.822	0.943	0.941	1.000	0.944	0.938
			(0.6354)	(0.6392)	(0.6559)	(0.6555)	**(2.6865)**	(0.6490)	(0.6446)

TABLE 3-9 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference in Variances of the ZITR Distribution with $(\mu_1, \mu_2) = (0.5, 0.5)$, $(\lambda_1, \lambda_2) = (1,1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\lambda_1:\lambda_2$	$\delta_1:\delta_2$	Coverage probability							
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
1:1	0.1:0.1	20:20	0.933	0.819	0.951	0.952	0.978	0.977	0.987
			(0.5729)	(0.5738)	(0.6970)	(0.6925)	(0.7100)	(0.6563)	**(0.6529)**
		20:50	0.911	0.814	0.938	0.939	0.951	0.971	0.980
			(0.4973)	(0.4939)	(0.5742)	(0.5713)	(0.6853)	(0.5615)	**(0.5547)**
		20:70	0.903	0.815	0.940	0.941	0.942	0.957	0.975
			(0.4749)	(0.4788)	(0.5515)	(0.5489)	(0.6633)	(0.5393)	**(0.5306)**
		20:100	0.887	0.778	0.931	0.931	0.923	0.954	0.974
			(0.4510)	(0.4421)	(0.5260)	(0.5241)	(0.6499)	(0.5165)	**(0.5061)**
		50:50	0.942	0.844	0.949	0.949	0.996	0.964	0.969
			(0.3964)	(0.3993)	(0.4182)	(0.4167)	(0.6319)	(0.3998)	**(0.3986)**
		50:70	0.931	0.843	0.944	0.944	0.993	0.959	0.965
			(0.3687)	(0.3692)	(0.3871)	(0.3859)	(0.6079)	(0.3828)	**(0.3814)**
		50:100	0.963	0.831	0.964	0.965	0.992	0.977	0.981
			(0.3456)	(0.3374)	(0.3605)	(0.3598)	(0.5920)	(0.3563)	**(0.3546)**
		70:70	0.937	0.831	0.939	0.940	0.997	0.954	0.960
			(0.3373)	(0.3369)	(0.3493)	(0.3482)	(0.5851)	(0.3349)	**(0.3340)**
		70:100	0.939	0.830	0.946	0.946	0.992	0.957	0.961
			(0.3123)	(0.3072)	(0.3214)	(0.3207)	(0.5664)	(0.3176)	**(0.3165)**
		100:100	0.945	0.821	0.952	0.951	0.998	0.956	0.957
			(0.2847)	(0.2868)	(0.2911)	(0.2906)	(0.5471)	(0.2795)	**(0.2787)**
		200:200	0.957	0.845	0.959	0.959	1.000	0.959	0.959
			(0.2030)	(0.1997)	(0.2053)	(0.2050)	(0.5065)	(0.1987)	**(0.1981)**
	0.1:0.3	20:20	0.934	0.811	0.948	0.945	0.982	0.954	0.960
			(0.6168)	(0.6115)	(0.7795)	(0.7770)	(1.0019)	(0.7436)	**(0.7377)**
		20:50	0.904	0.804	0.943	0.941	1.000	0.947	0.971
			(0.5093)	(0.5089)	(0.5961)	(0.5944)	(0.9918)	(0.5853)	**(0.5798)**

TABLE 3-9 (CON)

$\lambda_1:\lambda_2$	$\delta_1:\delta_2$	Coverage probability							
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		20:70	0.898	0.796	0.939	0.942	1.000	0.950	0.970
			(0.4745)	(0.4767)	(0.5545)	(0.5523)	(0.9641)	(0.5475)	**(0.5402)**
		20:100	0.898	0.808	0.952	0.951	1.000	0.953	0.969
			(0.4602)	(0.4639)	(0.5357)	(0.5336)	(0.9753)	(0.5251)	**(0.5158)**
		50:50	0.951	0.840	0.957	0.954	0.997	0.963	0.966
			(0.4104)	(0.4093)	(0.4416)	(0.4410)	(0.9406)	(0.4382)	**(0.4366)**
		50:70	0.940	0.826	0.949	0.951	1.000	0.944	0.950
			(0.3801)	(0.3762)	(0.4035)	(0.4026)	(0.9400)	(0.4000)	**(0.3986)**
		50:100	0.951	0.843	0.958	0.955	1.000	0.955	0.964
			(0.3528)	(0.3606)	(0.3700)	(0.3698)	(0.9309)	(0.3673)	**(0.3657)**
		70:70	0.940	0.832	0.942	0.942	0.999	0.949	0.950
			(0.3486)	(0.3553)	(0.3684)	(0.3684)	(0.9190)	(0.3661)	**(0.3649)**
		70:100	0.941	0.830	0.944	0.944	1.000	0.942	0.949
			(0.3231)	(0.3200)	(0.3355)	(0.3351)	**(0.9146)**	(0.3326)	(0.3316)
		100:100	0.955	0.843	0.957	0.955	1.000	0.956	0.960
			(0.2953)	(0.2923)	(0.3048)	(0.3044)	(0.9023)	(0.3027)	**(0.3019)**
		200:200	0.949	0.834	0.953	0.954	1.000	0.953	0.956
			(0.2096)	(0.2085)	(0.2132)	(0.2132)	(0.8647)	(0.2115)	**(0.2110)**
		20:20	0.949	0.828	0.957	0.950	0.998	0.977	0.986
			(0.6528)	(0.6365)	(0.8505)	(0.8479)	(1.2132)	(0.7976)	**(0.7920)**
		20:50	0.923	0.831	0.951	0.949	0.998	0.971	0.982
			(0.5440)	(0.5467)	(0.6740)	(0.6736)	(1.1939)	(0.6509)	**(0.6399)**
		20:70	0.924	0.806	0.955	0.955	0.998	0.968	0.974
			(0.5309)	(0.5319)	(0.6531)	(0.6529)	(1.1951)	(0.6269)	**(0.6127)**
		20:100	0.904	0.799	0.951	0.948	0.998	0.976	0.977
			(0.5044)	(0.5025)	(0.6288)	(0.6285)	(1.1852)	(0.6025)	**(0.5860)**
		50:50	0.952	0.848	0.958	0.958	1.000	0.963	0.970
			(0.4260)	(0.4186)	(0.4663)	(0.4667)	(1.1804)	(0.4605)	**(0.4588)**
	0.3:0.3	50:70	0.940	0.825	0.947	0.946	1.000	0.958	0.961
			(0.3951)	(0.3883)	(0.4285)	(0.4284)	(1.1800)	(0.4271)	**(0.4253)**
		50:100	0.935	0.838	0.945	0.944	1.000	0.959	0.962
			(0.3715)	(0.3746)	(0.3989)	(0.3985)	(1.1704)	(0.3975)	**(0.3952)**
		70:70	0.928	0.819	0.931	0.937	1.000	0.938	0.947
			(0.3660)	(0.3642)	(0.3898)	(0.3895)	**(1.1735)**	(0.3853)	(0.3840)
		70:100	0.949	0.842	0.958	0.953	1.000	0.960	0.967
			(0.3334)	(0.3318)	(0.3529)	(0.3532)	(1.1563)	(0.3523)	**(0.3509)**
		100:100	0.945	0.840	0.951	0.947	1.000	0.954	0.962
			(0.3043)	(0.3029)	(0.3182)	(0.3183)	(1.1504)	(0.3151)	**(0.3142)**
		200:200	0.937	0.839	0.935	0.935	1.000	0.936	0.939
			(0.2173)	(0.2183)	(0.2218)	(0.2216)	**(1.1189)**	(0.2191)	(0.2185)

TABLE 3-10 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Ratio in Variances of the ZITR Distribution with $(\mu_1, \mu_2) = (0.5, 0.5)$, $(\lambda_1, \lambda_2) = (0.2, 0.2)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\lambda_1:\lambda_2$	$\delta_1:\delta_2$	Coverage probability							
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.2:0.2	0.1:0.1	20:20	0.926	0.865	0.946	0.946	0.951	0.959	0.953
			(2.3133)	(2.3674)	(2.1306)	(2.1305)	(2.7186)	(1.6255)	**(1.5268)**
		20:50	0.898	0.796	0.948	0.949	0.937	0.962	0.958
			(1.3898)	(1.4129)	(1.5884)	(1.5990)	(2.1751)	(1.3310)	**(1.2629)**
		20:70	0.901	0.783	0.946	0.948	0.940	0.964	0.962
			(1.2672)	(1.2824)	(1.5239)	(1.5336)	(2.0341)	(1.2861)	**(1.2230)**
		20:100	0.902	0.786	0.954	0.952	0.933	0.968	0.977
			(1.2108)	(1.2162)	(1.4929)	(1.5029)	(2.0237)	(1.2490)	**(1.1911)**
		50:50	0.945	0.834	0.940	0.943	0.979	0.947	0.947
			(1.1545)	(1.1566)	(1.1223)	(1.1223)	**(2.1047)**	(1.0119)	(0.9807)

TABLE 3-10 (CON)

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		50:70	0.942	0.818	0.954	0.950	0.984	0.961	0.958
			(1.0176)	(1.0196)	(1.0247)	(1.0248)	(1.9632)	(0.9540)	**(0.9269)**
		50:100	0.951	0.834	0.964	0.963	0.981	0.966	0.964
			(0.9148)	(0.9218)	(0.9517)	(0.9536)	(1.8834)	(0.8960)	**(0.8726)**
		70:70	0.942	0.846	0.942	0.944	0.991	0.949	0.950
			(0.9386)	(0.9417)	(0.9254)	(0.9250)	(1.9042)	(0.8542)	**(0.8340)**
		70:100	0.938	0.836	0.946	0.947	0.985	0.953	0.951
			(0.8307)	(0.8367)	(0.8383)	(0.8386)	(1.8718)	(0.7987)	**(0.7814)**
		100:100	0.949	0.826	0.951	0.951	0.994	0.956	0.956
			(0.7578)	(0.7492)	(0.7491)	(0.7499)	(1.8641)	(0.7085)	**(0.6958)**
		200:200	0.946	0.852	0.940	0.936	0.998	0.952	0.950
			(0.5254)	(0.5289)	(0.5234)	(0.5234)	(1.7388)	(0.5048)	**(0.4995)**
	0.1:0.3	20:20	0.930	0.846	0.954	0.953	0.949	0.940	0.964
			(1.3857)	(1.4147)	(1.4766)	(1.4764)	(2.9845)	(1.3116)	**(1.2380)**
		20:50	0.918	0.806	0.957	0.955	0.918	0.963	0.975
			(0.9756)	(0.9839)	(1.1757)	(1.1816)	(2.3373)	(1.0405)	**(0.9903)**
		20:70	0.884	0.756	0.949	0.952	0.918	0.956	0.956
			(0.8926)	(0.8888)	(1.1090)	(1.1135)	(2.2047)	(0.9647)	**(0.9200)**
		20:100	0.890	0.781	0.960	0.958	0.885	0.965	0.966
			(0.8464)	(0.8505)	(1.0697)	(1.0750)	(2.0340)	(0.9222)	**(0.8811)**
		50:50	0.942	0.841	0.951	0.956	0.913	0.940	0.955
			(0.7626)	(0.7700)	(0.7857)	(0.7853)	(2.1805)	(0.7703)	**(0.7493)**
		50:70	0.948	0.827	0.955	0.953	0.896	0.955	0.967
			(0.6888)	(0.7011)	(0.7266)	(0.7266)	(2.0487)	(0.7084)	**(0.6905)**
		50:100	0.943	0.822	0.960	0.961	0.891	0.961	0.965
			(0.6252)	(0.6223)	(0.6624)	(0.6635)	(1.9431)	(0.6433)	**(0.6280)**
		70:70	0.938	0.847	0.945	0.947	0.904	0.949	0.952
			(0.6203)	(0.6291)	(0.6367)	(0.6366)	(1.9793)	(0.6320)	**(0.6188)**
		70:100	0.940	0.815	0.950	0.947	0.900	0.948	0.960
			(0.5704)	(0.5687)	(0.5915)	(0.5912)	(1.8855)	(0.5836)	**(0.5723)**
		100:100	0.945	0.834	0.948	0.948	0.914	0.946	0.955
			(0.5202)	(0.5314)	(0.5276)	(0.5274)	(1.8782)	(0.5262)	**(0.5181)**
		200:200	0.945	0.843	0.950	0.948	0.927	0.950	0.953
			(0.3561)	(0.3597)	(0.3590)	(0.3588)	(1.7522)	(0.3588)	**(0.3556)**
	0.3:0.3	20:20	0.942	0.855	0.954	0.954	0.997	0.951	0.959
			(1.7159)	(1.8086)	(1.8804)	(1.8868)	(5.4840)	(1.5829)	**(1.4886)**
		20:50	0.911	0.804	0.954	0.951	0.996	0.951	0.936
			(1.1647)	(1.1688)	(1.4468)	(1.4569)	(4.2976)	**(1.2259)**	(1.1600)
		20:70	0.910	0.796	0.962	0.961	0.993	0.950	0.936
			(1.0934)	(1.0926)	(1.4018)	(1.4100)	(3.9692)	**(1.1728)**	(1.1098)
		20:100	0.887	0.774	0.959	0.956	0.991	0.949	0.923
			(0.9936)	(0.9986)	**(1.3038)**	(1.3108)	(3.7809)	(1.0853)	(1.0288)
		50:50	0.945	0.843	0.952	0.953	0.999	0.948	0.949
			(0.9436)	(0.9550)	**(0.9745)**	(0.9754)	(4.1058)	(0.9270)	(0.9023)
		50:70	0.956	0.829	0.959	0.959	1.000	0.957	0.950
			(0.8332)	(0.8396)	(0.8740)	(0.8749)	(3.9598)	(0.8361)	**(0.8149)**
		50:100	0.950	0.821	0.962	0.965	0.999	0.960	0.948
			(0.7652)	(0.7739)	(0.8186)	(0.8196)	(3.9290)	**(0.7797)**	(0.7604)
		70:70	0.950	0.831	0.950	0.950	1.000	0.946	0.948
			(0.7763)	(0.7745)	**(0.7944)**	(0.7950)	(3.9938)	(0.7699)	(0.7545)
		70:100	0.943	0.834	0.947	0.945	1.000	0.947	0.945
			(0.6913)	(0.6776)	(0.7171)	(0.7178)	**(4.0195)**	(0.6978)	(0.6844)
		100:100	0.940	0.824	0.946	0.948	1.000	0.942	0.947
			(0.6385)	(0.6214)	(0.6497)	(0.6504)	**(3.9838)**	(0.6373)	(0.6277)
		200:200	0.943	0.824	0.945	0.947	1.000	0.947	0.952
			(0.4434)	(0.4432)	(0.4478)	(0.4481)	(3.9113)	(0.4437)	**(0.4398)**

TABLE 3-11 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Ratio in Variances of the ZITR Distribution with $(\mu_1, \mu_2) = (0.5, 0.5)$, $(\lambda_1, \lambda_2) = (0.2, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\lambda_1 : \lambda_2$	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.2:1	0.1:0.1	20:20	0.928	0.861	0.957	0.961	0.948	0.964	0.932
			(10.8528)	(11.3711)	(9.4251)	(9.4855)	(13.1279)	**(6.0397)**	(5.6461)
		20:50	0.908	0.791	0.951	0.950	0.914	0.963	0.948
			(6.3288)	(6.2282)	**(7.1640)**	(7.2365)	(10.5652)	(5.5712)	(5.2687)
		20:70	0.921	0.793	0.964	0.961	0.898	0.972	0.963
			(5.5040)	(5.5999)	(6.5613)	(6.6304)	(10.1635)	(5.2871)	**(5.0180)**
		20:100	0.893	0.755	0.944	0.943	0.906	0.958	0.952
			(5.1249)	(5.1118)	(6.3247)	(6.3765)	(9.9624)	(5.1465)	**(4.8981)**
		50:50	0.946	0.850	0.952	0.953	0.957	0.959	0.949
			(5.2476)	(5.3728)	(5.0183)	(5.0320)	(10.0571)	**(4.1581)**	(4.0117)
		50:70	0.924	0.824	0.926	0.928	0.943	0.949	0.938
			(4.5621)	(4.6110)	(4.5536)	(4.5619)	(9.6611)	(4.0171)	(3.8913)
		50:100	0.950	0.830	0.958	0.962	0.939	0.967	0.964
			(4.0225)	(3.9867)	(4.1405)	(4.1567)	(9.2330)	(3.7579)	**(3.6509)**
		70:70	0.948	0.841	0.946	0.948	0.947	0.954	0.939
			(4.2014)	(4.2289)	(4.0677)	(4.0768)	(9.5088)	**(3.5541)**	(3.4575)
		70:100	0.931	0.800	0.937	0.941	0.940	0.945	0.935
			(3.6473)	(3.5783)	(3.6428)	(3.6549)	(9.2202)	(3.3462)	(3.2651)
		100:100	0.947	0.845	0.949	0.951	0.962	0.947	0.940
			(3.4006)	(3.3852)	(3.3289)	**(3.3341)**	(9.1634)	(2.9890)	(2.9282)
		200:200	0.948	0.821	0.956	0.955	0.975	0.951	0.946
			(2.2864)	(2.3125)	(2.2730)	(2.2731)	(8.6120)	**(2.1377)**	(2.1115)
	0.1:0.3	20:20	0.935	0.832	0.957	0.957	0.879	0.978	0.973
			(5.0182)	(5.2784)	(5.3722)	(5.4109)	(14.7933)	(4.2908)	**(4.0446)**
		20:50	0.904	0.799	0.957	0.960	0.782	0.966	0.962
			(3.4222)	(3.4679)	(4.2240)	(4.2513)	(10.7319)	(3.5811)	**(3.4077)**
		20:70	0.866	0.739	0.948	0.946	0.735	0.963	0.966
			(3.0434)	(3.0608)	(3.9111)	(3.9306)	(10.1196)	(3.3134)	**(3.1647)**
		20:100	0.866	0.739	0.948	0.946	0.748	0.963	0.966
			(3.0434)	(3.0608)	(3.9111)	(3.9306)	(10.4168)	(3.3134)	**(3.1647)**
		50:50	0.931	0.812	0.942	0.941	0.710	0.949	0.956
			(2.7451)	(2.7514)	(2.8492)	(2.8598)	(10.7623)	(2.6963)	**(2.6206)**
		50:70	0.936	0.820	0.950	0.951	0.656	0.955	0.962
			(2.4365)	(2.4512)	(2.5848)	(2.5920)	(10.1551)	(2.4647)	**(2.4013)**
		50:100	0.932	0.791	0.951	0.952	0.600	0.956	0.962
			(2.2801)	(2.2360)	(2.4420)	(2.4500)	(9.6562)	(2.3301)	**(2.2737)**
		70:70	0.933	0.831	0.944	0.943	0.624	0.953	0.950
			(2.2229)	(2.2209)	(2.2963)	(2.3046)	(9.9628)	(2.2295)	**(2.1815)**
		70:100	0.945	0.837	0.954	0.954	0.580	0.956	0.958
			(2.0371)	(2.0571)	(2.1207)	(2.1283)	(9.3351)	(2.0600)	**(2.0194)**
		100:100	0.955	0.828	0.962	0.959	0.565	0.958	0.964
			(1.8378)	(1.8172)	(1.8811)	(1.8852)	(9.4529)	(1.8455)	**(1.8161)**
		200:200	0.946	0.817	0.946	0.947	0.473	0.948	0.952
			(1.2671)	(1.2552)	(1.2847)	(1.2852)	(8.6903)	(1.2731)	**(1.2615)**
	0.3:0.3	20:20	0.946	0.853	0.955	0.953	0.992	0.956	0.932
			(6.5064)	(6.6377)	(6.8758)	(6.9468)	(25.9735)	**(5.1730)**	(4.8602)
		20:50	0.923	0.798	0.961	0.962	0.970	0.957	0.934
			(4.0976)	(4.1151)	(5.2007)	(5.2547)	(20.8871)	**(4.2381)**	(4.0062)
		20:70	0.887	0.788	0.945	0.949	0.974	0.945	0.926
			(3.8271)	(3.8944)	(4.9617)	(5.0113)	**(20.2661)**	(4.0633)	(3.8476)
		20:100	0.881	0.764	0.963	0.961	0.967	0.939	0.923
			(3.6190)	(3.6102)	**(4.7642)**	(4.7933)	(19.5078)	(3.8821)	(3.6784)
		50:50	0.953	0.848	0.954	0.952	0.998	0.962	0.953
			(3.3220)	(3.3759)	(3.4823)	(3.5036)	(21.5867)	(3.2074)	**(3.1162)**
		50:70	0.949	0.848	0.957	0.954	0.998	0.956	0.952
			(3.0196)	(3.0292)	(3.2182)	(3.2357)	(20.5206)	(3.0144)	**(2.9338)**

TABLE 3-11 (CON)

$\lambda_1:\lambda_2$	$\delta_1:\delta_2$	Coverage probability							
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		50:100	0.946	0.825	0.950	0.946	0.996	0.950	0.948
			(2.7632)	(2.7902)	(2.9849)	(2.9950)	(19.7240)	**(2.7987)**	(2.7274)
		70:70	0.950	0.852	0.951	0.953	1.000	0.957	0.955
			(2.7462)	(2.7670)	(2.8367)	(2.8453)	(20.6301)	(2.6922)	**(2.6343)**
		70:100	0.941	0.817	0.946	0.943	1.000	0.946	0.939
			(2.4960)	(2.4848)	(2.5920)	(2.6027)	**(20.0404)**	(2.4873)	(2.4371)
		100:100	0.946	0.833	0.951	0.951	1.000	0.952	0.953
			(2.2390)	(2.2363)	(2.2827)	(2.2888)	(19.9660)	(2.2034)	**(2.1693)**
		200:200	0.951	0.829	0.957	0.954	1.000	0.954	0.955
			(1.5544)	(1.5749)	(1.5745)	(1.5762)	(19.4650)	(1.5466)	**(1.5325)**

TABLE 3-12 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Ratio in Variances of the ZITR Distribution with $(\mu_1, \mu_2) = (0.5, 0.5)$, $(\lambda_1, \lambda_2) = (1,1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$.

$\lambda_1:\lambda_2$	$\delta_1:\delta_2$	Coverage probability							
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
1:1	0.1:0.1	20:20	0.933	0.854	0.951	0.952	0.986	0.977	0.981
			(2.6333)	(2.7518)	(2.1920)	(2.1874)	(5.9435)	(1.6237)	**(1.5199)**
		20:50	0.911	0.809	0.938	0.939	0.986	0.971	0.979
			(1.5811)	(1.5915)	(1.6794)	(1.6884)	(4.9467)	(1.5299)	**(1.4505)**
		20:70	0.904	0.793	0.940	0.941	0.989	0.957	0.979
			(1.4454)	(1.4681)	(1.5982)	(1.6083)	(4.6874)	(1.4968)	**(1.4247)**
		20:100	0.887	0.757	0.931	0.931	0.988	0.954	0.978
			(1.3143)	(1.2939)	(1.4932)	(1.5016)	(4.9223)	(1.4292)	**(1.3654)**
		50:50	0.942	0.867	0.949	0.949	0.999	0.964	0.963
			(1.3117)	(1.3327)	(1.2226)	(1.2225)	(5.0304)	(1.0576)	**(1.0227)**
		50:70	0.931	0.856	0.944	0.944	0.999	0.959	0.965
			(1.1552)	(1.1638)	(1.1256)	(1.1258)	(4.9718)	(1.0439)	**(1.0122)**
		50:100	0.963	0.843	0.964	0.965	1.000	0.977	0.983
			(1.0186)	(0.9993)	(1.0194)	(1.0218)	(4.9308)	(0.9717)	**(0.9462)**
		70:70	0.937	0.844	0.939	0.940	1.000	0.954	0.947
			(1.0787)	(1.0837)	(1.0243)	(1.0231)	(4.9728)	**(0.9113)**	(0.8876)
		70:100	0.939	0.821	0.946	0.946	1.000	0.957	0.956
			(0.9357)	(0.9248)	(0.9163)	(0.9168)	(4.8705)	(0.8670)	**(0.8469)**
		100:100	0.945	0.841	0.952	0.951	1.000	0.956	0.954
			(0.8530)	(0.8676)	(0.8225)	(0.8223)	(4.7330)	(0.7500)	**(0.7353)**
		200:200	0.957	0.850	0.959	0.959	1.000	0.959	0.957
			(0.5835)	(0.5757)	(0.5741)	(0.5739)	(4.6716)	(0.5417)	**(0.5353)**
	0.1:0.3	20:20	0.924	0.832	0.947	0.951	0.986	0.952	0.977
			(1.2818)	(1.3376)	(1.2876)	(1.2909)	(6.6105)	(1.1869)	**(1.1211)**
		20:50	0.883	0.776	0.948	0.946	0.974	0.942	0.976
			(0.8929)	(0.8943)	(1.0203)	(1.0256)	(5.3003)	(1.0126)	**(0.9667)**
		20:70	0.876	0.759	0.937	0.939	0.977	0.945	0.974
			(0.8196)	(0.8278)	(0.9538)	(0.9576)	(5.1103)	(0.9570)	**(0.9155)**
		20:100	0.881	0.771	0.949	0.950	0.977	0.942	0.975
			(0.7963)	(0.8069)	(0.9307)	(0.9336)	(5.0450)	(0.9236)	**(0.8861)**
		50:50	0.939	0.837	0.952	0.954	0.994	0.960	0.968
			(0.6848)	(0.6872)	(0.6898)	(0.6904)	(5.1755)	(0.6876)	**(0.6692)**
		50:70	0.936	0.818	0.940	0.942	0.999	0.932	0.958
			(0.6383)	(0.6393)	(0.6534)	(0.6540)	(5.0899)	(0.6537)	**(0.6379)**
		50:100	0.941	0.836	0.962	0.964	0.992	0.966	0.972
			(0.5895)	(0.6048)	(0.6073)	(0.6089)	(4.8915)	(0.6082)	**(0.5950)**
		70:70	0.933	0.828	0.938	0.938	0.995	0.948	0.956
			(0.5637)	(0.5739)	(0.5694)	(0.5702)	(5.0456)	(0.5696)	**(0.5581)**
		70:100	0.932	0.832	0.941	0.948	0.999	0.940	0.952
			(0.5275)	(0.5289)	(0.5357)	(0.5358)	(4.7709)	(0.5345)	**(0.5251)**

TABLE 3-12 (CON)

$\lambda_1:\lambda_2$	$\delta_1:\delta_2$	Coverage probability							
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		100:100	0.947	0.825	0.951	0.952	1.000	0.949	0.961
			(0.4678)	(0.4647)	(0.4688)	(0.4688)	(4.8638)	(0.4682)	**(0.4612)**
		200:200	0.961	0.830	0.956	0.959	1.000	0.958	0.958
			(0.3255)	(0.3254)	(0.3263)	(0.3263)	(4.7848)	(0.3245)	**(0.3217)**
	0.3:0.3	20:20	0.949	0.864	0.957	0.950	1.000	0.977	0.971
			(1.7118)	(1.7341)	(1.6976)	(1.7088)	(13.3458)	(1.4657)	**(1.3846)**
		20:50	0.923	0.825	0.951	0.949	0.999	0.972	0.971
			(1.0692)	(1.0718)	(1.2533)	(1.2651)	(11.1990)	(1.2013)	**(1.1429)**
		20:70	0.924	0.802	0.955	0.953	1.000	0.968	0.972
			(1.0154)	(1.0231)	(1.2076)	(1.2179)	(10.6296)	(1.1576)	**(1.1031)**
		20:100	0.904	0.791	0.951	0.948	1.000	0.976	0.979
			(0.9443)	(0.9427)	(1.1531)	(1.1618)	(10.4533)	(1.1061)	**(1.0554)**
		50:50	0.951	0.847	0.958	0.958	1.000	0.963	0.965
			(0.8668)	(0.8544)	(0.8769)	(0.8811)	(11.6174)	(0.8502)	**(0.8298)**
		50:70	0.940	0.824	0.947	0.945	1.000	0.958	0.964
			(0.7603)	(0.7500)	(0.7864)	(0.7890)	(11.3871)	(0.7789)	**(0.7618)**
		50:100	0.935	0.830	0.945	0.943	1.000	0.959	0.954
			(0.7007)	(0.7074)	(0.7333)	(0.7344)	(10.9346)	(0.7284)	**(0.7135)**
		70:70	0.928	0.830	0.931	0.936	1.000	0.938	0.948
			(0.7095)	(0.7123)	(0.7152)	(0.7158)	**(11.4203)**	(0.6996)	(0.6870)
		70:100	0.949	0.849	0.958	0.953	1.000	0.960	0.960
			(0.6343)	(0.6329)	(0.6506)	(0.6530)	(11.1265)	(0.6470)	**(0.6365)**
		100:100	0.945	0.848	0.951	0.949	1.000	0.954	0.957
			(0.5813)	(0.5809)	(0.5875)	(0.5882)	(11.0721)	(0.5780)	**(0.5703)**
		200:200	0.937	0.843	0.935	0.935	1.000	0.936	0.937
			(0.4009)	(0.4043)	(0.4025)	(0.4024)	**(10.8743)**	(0.3968)	(0.3936)

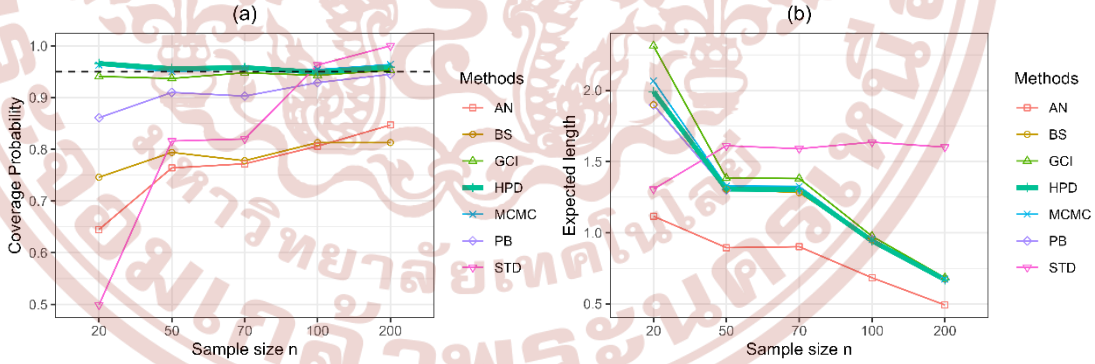


FIGURE 3-1 Graphs comparing the performance of the proposed methods for the single variance parameter in terms of (a) coverage probability and (b) expected length.

The evaluation of the 95% confidence intervals for the variance difference under the ZITR distribution focused on coverage probability (CP) and expected length (EL), as detailed in Tables 3-7 to 3-9 and Figure 3-2. The findings demonstrate that the Bayesian HPD technique achieves an effective equilibrium between coverage and interval length, preserving nominal coverage while producing comparatively concise intervals. The Bayesian MCMC, Bayesian HPD, GCI, and MOVER methods achieved coverage probabilities close to or above the nominal level of 0.95 across most of the scenarios. Conversely, the PB and AN techniques exhibited enhanced coverage with

increasing sample sizes, demonstrating superior performance in bigger samples. Regarding interval accuracy, as shown by predicted length, the PB and BS approaches yielded the shortest intervals. Nonetheless, their coverage probability decreased below the nominal threshold in many instances, and hence, they were not pursued further. Among the approaches that satisfied the coverage condition, the Bayesian HPD method consistently produced the shortest anticipated lengths, demonstrating superior efficiency.

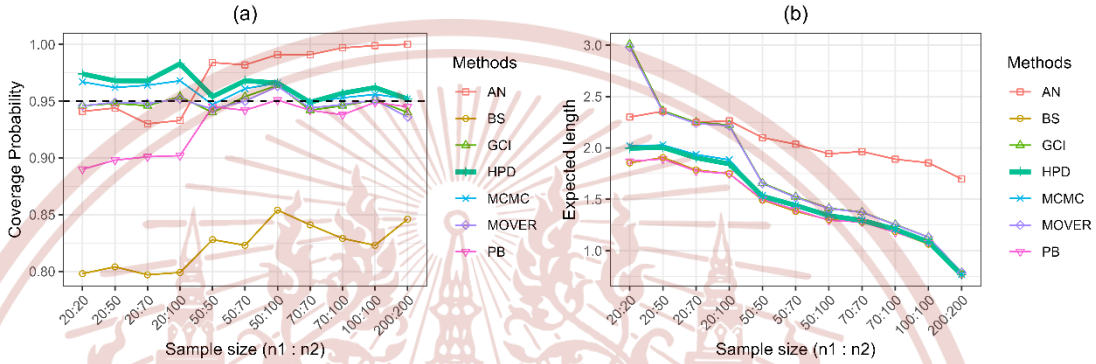


FIGURE 3-2 Graphs comparing the performance of the proposed methods for the difference between variances parameter in terms of (a) coverage probability and (b) expected length.

The coverage probabilities and anticipated interval lengths of the 95% confidence intervals for the ratio of variances under the ZITR distribution are shown in Tables 3-10 to 3-12 and Figure 3-3 , with the associated graphical representations depicted in Figures 3-3. Analogous patterns were seen for both the variance difference and the variance ratio, and the primary results persisted across the two contexts. The Bayesian MCMC, Bayesian HPD, GCI, and MOVER methodologies sustained coverage probability around or above 0.95 in the majority of instances. The PB and BS approaches yielded shorter intervals; nonetheless, their coverage performance was deficient in several contexts, leading to their exclusion from further evaluation. With the augmentation of sample sizes, the PB and AN methodologies exhibited enhanced coverage performance. A uniform pattern in performance was seen across all cases. The Bayesian HPD technique exhibited the most advantageous equilibrium between coverage probability and interval length. Conversely, although the PB and BS approaches often yielded shorter intervals, their trustworthiness was undermined by under-coverage. The performance disparity may be ascribed to the Bayesian HPD method's superior ability to address posterior uncertainty, resulting in interval estimates that are both well-calibrated and efficient.

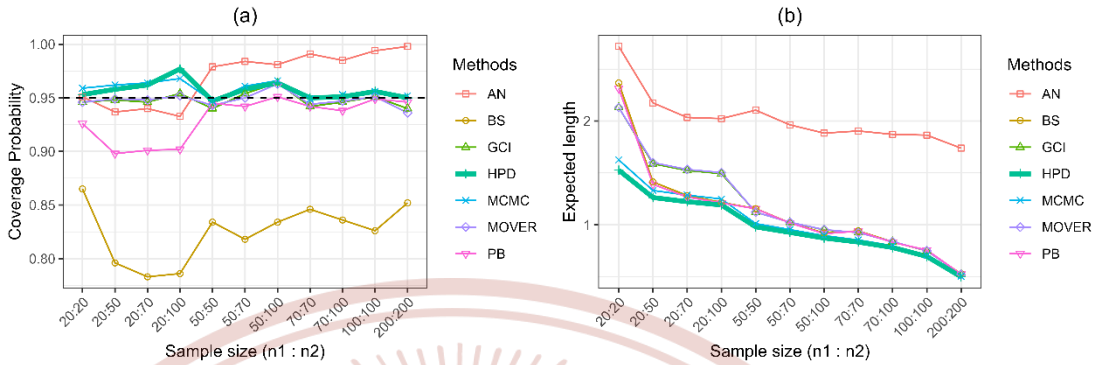


FIGURE 3-3 Graphs comparing the performance of the proposed methods for the ratio variances parameters in terms of (a) coverage probability and (b) expected length.

3.5 An empirical application

This study used road traffic accident fatality data to test and compare the efficacy of the suggested techniques for calculating variance, variance differences, and variance ratios under the ZITR distribution.

3.5.1 Application of variance for ZITR

This study applies various confidence interval methods for the variance of the ZITR distribution, including the MCMC, HPD, STD, AN, PB, BS, and GCI approaches, to road traffic fatality data collected at the district level in Chiang Mai Province, Thailand, in March 2024, to demonstrate their effectiveness. The data were obtained from the Thai Road Safety Collaboration (ThaiRSC) database and consist of $n = 25$ observations, as presented in Table 3-13.

TABLE 3-13 Road traffic fatality data by district in Chiang Mai Province for March 2024

Province	Observation									
Chiangmai	5	5	1	0	1	2	3	1	3	1
	2	2	0	0	0	3	1	0	0	3
	4	2	3	1	3					

The statistics provide a zero-inflation percentage δ of 0.24. The dataset has a mean of around 1.8819 and a variation of approximately 2.158317, suggesting that the variability surpasses the degree indicated by the mean. Moreover, the favorable findings demonstrate a right-skewed distribution, indicating variability in accident severity across districts. These characteristics suggest that the data deviate from the assumptions commonly adopted in traditional count-data models, particularly due to the presence of excess zeros and substantial variability. Thus, this dataset serves as a pertinent real-world illustration for assessing the efficacy of confidence interval methodologies for the variance parameter, since accurate variance estimate is crucial for measuring variation across districts.

Before calculating the confidence intervals, the distribution of the positive observations was analyzed. Six probability distributions commonly used in actuarial science, namely the Rayleigh, Normal, Weibull, Lognormal, Exponential, and Gamma distributions, were considered for modeling the positive observations. The

appropriateness of the competing models was evaluated using the Akaike information criterion (AIC) and the Bayesian information criterion (BIC). According to Table 3-14, the two-parameter Rayleigh distribution had the lowest AIC and BIC values among the competing models, indicating the best overall fit. Furthermore, Figure 3-4 presents the Q-Q plot and the comparison between the empirical and fitted CDFs, providing additional graphical evidence of the adequacy of the selected two-parameter Rayleigh distribution. The selected model was then used to apply the proposed confidence interval techniques for estimating the variance and to evaluate their performance in a real-data application.

TABLE 3-14 AIC and BIC values for road traffic fatality data in Chiang Mai Province, March 2024

Distribution	Rayleigh	Normal	Weibull	Lognormal	Exponential	Gamma
AIC	**63.7144**	66.99392	64.14151	63.94212	72.59969	63.77111
BIC	**65.60328**	68.8828	66.03038	65.83099	74.54413	65.65999

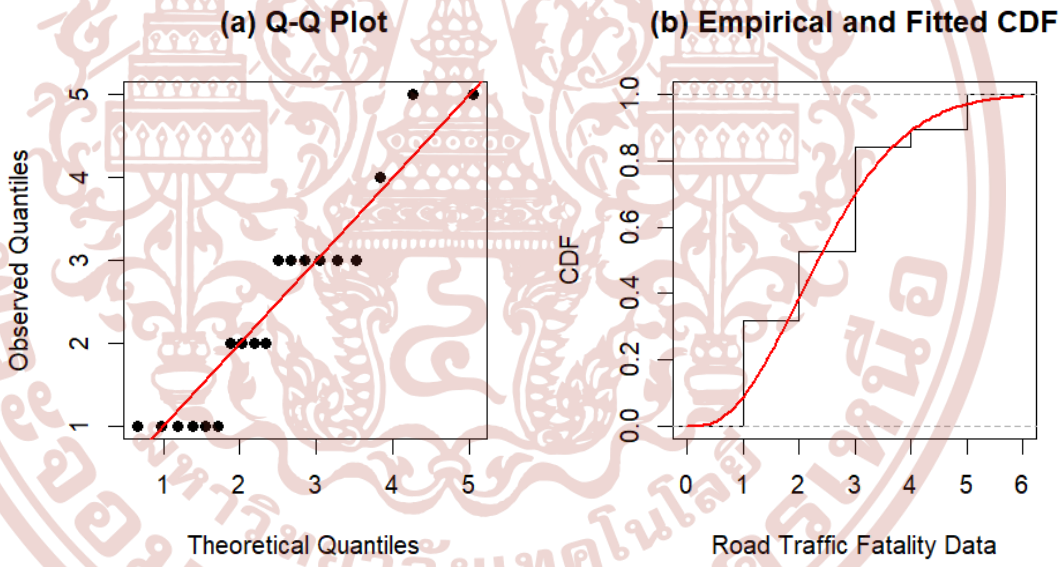


FIGURE 3-4 (a) Q-Q plot of Road Traffic Fatality Data and (b) CDF of Road Traffic Fatality Data by district in Chiang Mai Province for March 2024.

The empirical confidence intervals for the variance parameter are shown in Table 3-15. Significant disparities in interval widths are seen across the competing approaches. The STD technique yields the broadest interval (Length = 4.1218), indicating a considerable degree of uncertainty. Conversely, the AN approach produces the most constrained interval (Length = 0.6094), indicating much reduced estimated variability. The MCMC and HPD approaches, together with PB and BS, provide intervals of similar size, with lengths roughly between 1.87 and 2.04. The GCI technique yields a broader interval (Length = 2.5393) in comparison to these alternatives. Excluding STD and AN, the other approaches have comparable interval lengths, indicating a general consistency in variance estimate for this dataset. The simulation findings endorse the HPD technique, since it reliably attains coverage

probabilities at or near the nominal level while preserving rather short interval durations. The STD and AN approaches are not recommended due to their tendency to demonstrate inadequate coverage or instability in small to moderate sample sizes, thereby compromising inferential reliability.

TABLE 3-15 Confidence intervals for the variance of the ZITR distribution based on road traffic fatality data from Chiang Mai Province in March 2024

Method	Lower	Upper	Length
MCMC	1.3332	3.3022	1.969
HPD	1.2349	3.105	**1.8701**
STD	2.3174	6.4392	4.1218
AN	1.8536	2.463	0.6094
PB	1.2078	3.2472	2.0394
BS	1.23	3.2536	2.0236
GCI	1.4524	3.9916	2.5392

3.5.2 Application of difference and ratio variances for ZITR

To illustrate the practical applicability and effectiveness of the proposed confidence interval methods for the difference between and ratio of variances under the ZITR distribution, including MCMC, HPD, GCI, MOVER, AN, PB, and BS, road traffic fatality data recorded at the district level in Chanthaburi and Narathiwat Provinces, Thailand, during January 2025 were analyzed. The data were sourced from the Thai Road Safety Collaboration (ThaiRSC) database and served as a real-data example for assessing and comparing the performance of the considered methods. The district-level observations for Chanthaburi and Narathiwat Provinces are presented in Table 3-16.

TABLE 3-16 Road traffic fatality data by district in Chanthaburi and Narathiwat Provinces for January 2025

Province	Observation						
Chanthaburi	1	1	0	0	0	0	2
	2	1	1				
Narathiwat	0	1	0	0	0	1	0
	2	2	0	1	1	0	

Both datasets included a significant quantity of zero observations, reflecting districts with no recorded incidents throughout the research period. The positive numbers varied across districts, reflecting variations in accident severity within each province. The analysis began by examining the distribution of the positive observations. The investigation started by scrutinizing the distribution of the positive observations, considering these factors. The appropriateness of candidate models for the non-zero component was evaluated using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). The findings Table 3-17 and 3-18 demonstrate that the two-parameter Rayleigh distribution well fitted the positive data for both provinces. Furthermore, Figures 3-5 and 3-6 present the Q-Q plots and comparisons

between the empirical and fitted CDFs, providing additional graphical evidence of the adequacy of the two-parameter Rayleigh distribution for the positive observations.

TABLE 3-17 AIC and BIC values for road traffic fatality data in Chanthaburi Province, January 2025

Distribution	Rayleigh	Normal	Weibull	Lognormal	Exponential	Gamma
AIC	**10.37229**	12.0028	11.71601	10.37723	16.45218	10.84393
BIC	**9.955806**	11.58632	11.29953	9.960751	17.24394	10.42745

TABLE 3-18 AIC and BIC values for road traffic fatality data in Narathiwat Province, January 2025

Distribution	Rayleigh	Normal	Weibull	Lognormal	Exponential	Gamma
AIC	**10.37229**	12.0028	11.71601	10.37723	16.45218	10.84393
BIC	**9.955806**	11.58632	11.29953	9.960751	17.24394	10.42745

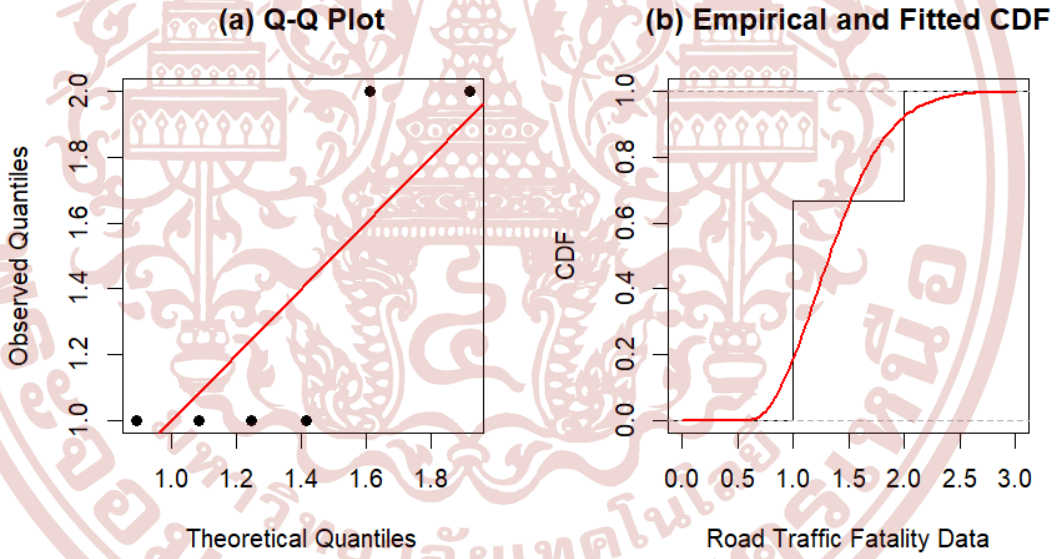


FIGURE 3-5 (a) Q-Q plot of Road Traffic Fatality Data and (b) CDF of Road Traffic Fatality Data by district in Chanthaburi Province for January 2025

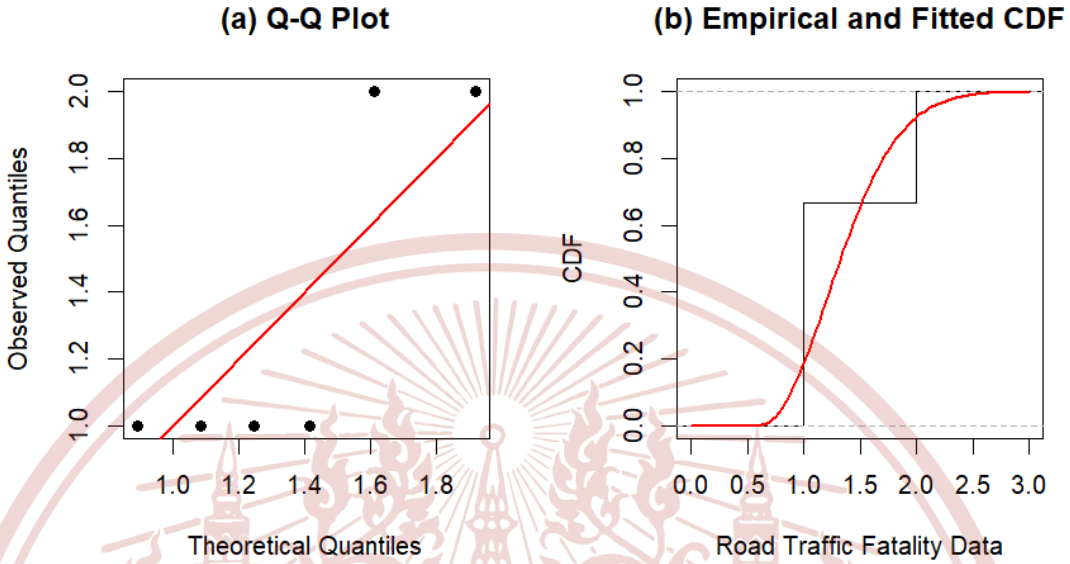


FIGURE 3-6 (a) Q-Q plot of Road Traffic Fatality Data and (b) CDF of Road Traffic Fatality Data by district in Narathiwat Province for January 2025

The existence of surplus zeros with right-skewed positive values justifies the use of the zero-inflated two-parameter Rayleigh (ZITR) distribution for data modeling. This modeling framework enables the separate analysis of districts with zero reported traffic fatalities and those with recorded fatalities, thereby accounting for structural disparities that may result from differences in traffic volume, road conditions, law enforcement practices, or other local variables. The aforementioned datasets were used to implement the recommended confidence interval methodologies for the difference and ratio of variance parameters on the empirical data. The confidence intervals were established using the methodologies outlined in this work, enabling a direct comparison of variability between the two provinces inside the ZITR model. The confidence intervals and corresponding statistics are shown in Tables 3-19 and 3-20, summarizing the findings for both the variance difference and the variance ratio. The results illustrate the practical usability and efficacy of the suggested methodologies when used on actual road traffic fatality data.

TABLE 3-19 Confidence Interval Estimates for the Difference in Variances of ZITR: January 2025 Road Traffic Fatalities in Chanthaburi and Narathiwat Provinces

Method	Lower	Upper	Length
MCMC	-0.203039	0.0875902	0.290629
HPD	-0.20342	0.0864157	**0.289836**
GCI	-0.810728	0.9226709	1.733399
MOVER	-0.802604	0.9636477	1.766252
AN	-0.800628	0.8185817	1.619209
PB	-0.383617	0.3557924	0.739409
BS	-0.561543	0.5189722	1.080516

TABLE 3-20 Confidence Interval Estimates for the Ratio of ZITR Variances Based on January 2025 Road Traffic Fatalities in Chanthaburi and Narathiwat Provinces

Method	Lower	Upper	Length
MCMC	0.7045763	1.150502	0.445926
HPD	0.7165365	1.159801	**0.443265**
GCI	0.2871594	3.637387	3.350228
MOVER	0.3048449	3.994555	3.68971
AN	-2.29433	4.327352	6.621682
PB	0.454967	1.905308	1.450341
BS	-0.071835	2.088031	2.159866

The examination of road traffic fatality data from January 2025 for Chanthaburi and Narathiwat demonstrated significant disparities in the efficacy of the interval estimate techniques under the ZITR model. The MCMC and HPD approaches yielded the smallest intervals for the variance difference, characterized by stable and interpretable limits. Conversely, the GCI, MOVER, and AN techniques produced much broader intervals, indicating increased estimate uncertainty. The bootstrap techniques (PB and BS) produced intervals of modest breadth, which were nonetheless broader than those derived from the Bayesian approaches. The same trend was seen for the variance ratio. The HPD interval was somewhat shorter than the MCMC interval, but the GCI and MOVER approaches yielded much broader ranges. The AN and BS approaches produced intervals that included negative values, which are technically invalid for variance ratios. The sample sizes in this application were somewhat modest. The simulation findings demonstrate that, in small-sample scenarios, the PB, BS, and AN approaches often do not attain coverage probabilities at the nominal level. This underrepresentation likely led to their diminished performance in the empirical study. Conversely, the HPD technique maintained coverage probabilities at the nominal level while generating very concise intervals. While the bootstrap approaches sometimes yielded shorter intervals in the simulations, their coverage was not reliably sufficient. The findings from both the simulation research and the empirical data analysis support the use of the HPD approach. It offers a more dependable balance between coverage precision and interval length for both the variance difference and the variance ratio within the zero-inflated two-parameter Rayleigh distribution.

3.6 Conclusion

This chapter presents several methods for constructing confidence intervals for the variance, the difference of variances, and the ratio of variances under the Zero-Inflated Two-Parameter Rayleigh (ZITR) distribution. Due to the complexity of direct theoretical comparisons among the proposed methods, this study employs Monte Carlo simulation together with real-data analysis to evaluate the performance of each method. The evaluation is based on coverage probabilities (CPs) and expected lengths (ELs) of the confidence intervals obtained from the simulation results.

The results show that, for estimating the variance under the ZITR distribution, the GCI, MCMC, and HPD methods all achieve coverage probabilities higher than the

nominal significance level. In terms of expected length, it is found that the most efficient methods are those that provide relatively shorter confidence intervals while still maintaining appropriate coverage probabilities. Among all competing approaches, the HPD method yields the best overall performance. However, it should be noted that the HPD method may not always perform optimally in every situation. In particular, for certain parameter settings, such as $\mu = 2$ combined with $\lambda = 0.2$ or 0.5, and under relatively high values of δ, the coverage probability of the HPD interval may occasionally fall below the nominal level. Despite this limitation, the method generally produces coverage probabilities close to the target level and consistently yields the shortest confidence intervals compared with the other methods considered.

For the variance difference and variance ratio, only selected subsets of parameter configurations are presented in order to provide an overall picture and facilitate clearer comparisons of the confidence interval methods under different scenarios. This study develops interval estimation procedures for the difference and ratio of variances under the Zero-Inflated Two-Parameter Rayleigh distribution. The methods considered include Bayesian MCMC, Bayesian HPD, the asymptotic approximate normal (AN) method, the generalized confidence interval (GCI), MOVER, the percentile bootstrap, and the bootstrap with standard error. Their performance is evaluated through a Monte Carlo simulation study and illustrated using real data on road traffic fatalities. The comparison is based on two criteria: coverage probability (CP) and expected length (EL). For the variance difference, the Bayesian HPD method generally demonstrates the best overall performance across most sample sizes, achieving coverage probabilities close to the nominal level while producing shorter confidence intervals than the competing methods.

However, an exception is observed when $\lambda_1 = 0.2$, $\lambda_2 = 1$, and $\delta_1 = \delta_2 = 0.3$, where the CPs of both the Bayesian MCMC and HPD methods fall slightly below the nominal level, whereas the GCI and MOVER methods maintain adequate coverage. Nevertheless, the Bayesian approaches still yield comparatively shorter intervals. A similar pattern is observed for the variance ratio. The HPD method performs well in most settings, particularly for small to moderate sample sizes. However, when there is substantial heterogeneity in the scale parameters, the Bayesian methods do not fully attain the nominal coverage level. In such cases, the GCI method provides more reliable coverage and, in some settings, even produces shorter intervals than MOVER. These results indicate that although the Bayesian HPD method is generally the most efficient, scenarios involving pronounced parameter heterogeneity may favor generalized approaches.

As also observed in the previous chapter, the Bayesian methods require relatively longer computational times than the non-Bayesian methods, particularly because MCMC sampling is required for posterior inference and the construction of HPD intervals. Therefore, although the Bayesian HPD method generally provides favorable statistical performance, its computational cost should also be considered when selecting an interval estimation method. In addition, the performance of the GCI method depends on the choice of the generalized pivotal quantity (GPQ). The GPQ considered in this study provided satisfactory results; however, alternative GPQ constructions may further improve the coverage probabilities and expected lengths. Future research could therefore investigate alternative GPQs for the variance, variance difference, and variance ratio under the ZITR distribution.

Chapter 4
CONFIDENCE INTERVALS FOR COEFFICIENTS OF VARIATION FOR THE ZERO-INFLATED TWO-PARAMETER RAYLEIGH DISTRIBUTIONS

This chapter focuses on the construction of confidence intervals for the coefficient of variation, the difference between coefficients of variation, and the ratio of coefficients of variation under the Zero-Inflated Two-Parameter Rayleigh (ZITR) distribution. For the estimation of a single coefficient of variation, the proposed methods include Bayesian Markov Chain Monte Carlo (MCMC), Highest Posterior Density (HPD), Approximate Normal (AN), Generalized Confidence Interval (GCI), Percentile Bootstrap (PB), and Bootstrap with Standard Error (BS) methods. For constructing confidence intervals for the difference and ratio of coefficients of variation under the ZITR distribution, the proposed methods consist of MCMC, HPD, GCI, Method of Variance Estimates Recovery (MOVER), AN, PB, and BS methods.

4.1 Introduction

In many practical applications, datasets contain an excessive number of zero observations. Ignoring these excess zeros may lead to inaccurate statistical inference and inadequate model fitting. To solve this problem, zero-inflated models have been widely used as a flexible framework for modeling data with a large proportion of zeros. The basic idea was proposed by Aitchison (1955), who suggested a mixture model with a degenerate distribution that gives positive probability to zero and a continuous distribution that describes the positive observations. This framework has then been generalized to many probability distributions, including the zero-inflated Rayleigh distribution, which combines a point mass at zero with the standard one-parameter Rayleigh distribution for modeling the positive part of the data.

Many real world data sets have a large proportion of zero observations and positively skewed continuous positive values. Such data are common in road traffic safety, where many districts report no fatalities during a certain time period while the rest of the districts report different levels of fatality. Such features are also seen in environmental monitoring, reliability engineering, insurance claims, and public health studies. In these applications, researchers are often not only interested in modeling the distribution of the data but also interested in comparing the relative variability of different populations or regions. There is therefore a need for statistical procedures that work well in the presence of excess zeros and positively skewed observations to guarantee reliable statistical inference.

The coefficient of variation (CV) is defined as the ratio of the standard deviation to the mean and is commonly used to measure the dispersion of data. Since the CV is a unit-free parameter, its value is typically expressed as a percentage. A higher CV indicates greater relative variability, whereas a lower CV suggests less relative variability. Therefore, the CV can be applied in a wide range of fields. For example, Billings et al. (1993) estimated the CV to examine the impact of socioeconomic status on hospitals. Romano et al. (2005) analyzed the shear and tensile bond strength of adhesion to tooth structures, while Saelee et al. (2013) investigated the variability of agricultural production using approximate confidence intervals for the CV.

In addition, comparisons of coefficients of variation have been widely used because the CV provides an effective way to compare variability between two or more datasets. The most common approaches for comparing two CVs are based on either the difference or the ratio of the CVs from two independent samples. For example, Verrill and Johnson (2007) introduced confidence bounds for the ratio of two CVs under normal distributions using simulation-based procedures. Sangnawakij and Niwitpong (2017) proposed confidence intervals for the single coefficient of variation and the difference between coefficients of variation in two-parameter exponential distributions.

Furthermore, many researchers have focused on studying the difference and ratio of CVs for datasets containing excess zeros. For instance, Yosboonruang et al. (2020) proposed confidence intervals for the difference between CVs of delta-lognormal distributions. Later, Yosboonruang et al. (2022) demonstrated that the ratio of CVs can be used to compare rainfall variability between two regions. More recently, Janthasuwan et al. (2025) presented confidence intervals for the ratio of CVs under the Delta-Birnbaum–Saunders distribution and applied the proposed methods to compare wind speed data in Thailand.

This study aims to investigate the estimation of confidence intervals for the difference and ratio of CVs under the ZITR distribution. Several inferential approaches are considered, including Bayesian MCMC, Bayesian HPD credible intervals, AN, GCI, MOVER, PB, and BS. The performances of these methods are evaluated through simulation studies under various scenarios using coverage probability (CP) and expected length (EL) as the evaluation criteria. Furthermore, the proposed confidence interval methods are illustrated through an empirical analysis of road traffic fatality data from Lampang and Mae Hong Son provinces, Thailand, recorded in March 2026. The data contain a substantial proportion of zero observations, corresponding to districts with no reported fatalities, making them well suited for assessing the performance and practical applicability of the proposed methods under zero-inflated settings.

4.2 The confidence intervals for the coefficient of variation for ZITR distribution

Let $X=(X_1,X_2,\ldots,X_n)$ denote a random sample drawn from the Zero-Inflated Two-Parameter Rayleigh (ZITR) distribution, denoted by $X \sim ZITR(\mu,\lambda,\delta)$, where $P(X_i=0)=\delta$ and $n_0 \sim \text{Binomial}(n,\delta)$. Suppose that a random sample of size n is observed, consisting of n_0 zero observations and $n_1=n-n_0$ positive observations. In this section, the focus is on the variance parameter, where the population variance of X is given by

$$CV(X)=\kappa=\frac{\sqrt{\dfrac{4(1-\delta)-(1-\delta)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda}+\dfrac{\mu}{\sqrt{\lambda}}\Gamma\left(\frac{1}{2}\right)\delta(1-\delta)+\mu^2\delta(1-\delta)}}{(1-\delta)\left(\mu+\dfrac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right)} \qquad (4\text{-}1)$$

The following section presents the method for constructing confidence intervals for κ

4.2.1 Bayesian MCMC

The CV parameter κ is estimated for data assumed to follow the ZITR. Here we use weakly informative priors to incorporate some prior information while maintaining posterior propriety (Dey et al., 2014). The prior distributions are selected according to the characteristics and the support of the corresponding parameters. Specifically, we give the scale parameter λ, the location parameter μ, and the zero-inflation parameter δ be Gamma, Uniform, and Beta prior, respectively.

All Bayesian analyses are conducted in the R environment using the R2 OpenBUGS package. Posterior samples are generated using a Gibbs sampling algorithm implemented in OpenBUGS. The following describes the procedure for constructing two-sided Bayesian credible intervals for κ based on these MCMC posterior samples.

$$CI_{\kappa(\mathrm{MCMC})} = \left(L_{\kappa(\mathrm{MCMC})}, U_{\kappa(\mathrm{MCMC})}\right) = \left(\kappa_{\mathrm{MCMC}}\left(\alpha/2\right), \kappa_{\mathrm{MCMC}}\left(1-\alpha/2\right)\right). \tag{4-2}$$

where $\kappa_{\mathrm{MCMC}}(\alpha/2)$ represent the $100(\alpha/2)\%$ percentile of κ_{MCMC}

4.2.2 Bayesian HPD

The Bayesian HPD method establishes an interval based on the posterior distribution that contains the most probable parameter values while minimizing the interval length (Box and Tiao, 1973) In this study, HPD are used to estimate CV parameter, denoted by κ. The computation of these intervals is carried out using the R package HDInterval. Accordingly, the HPD criterion yields a two-sided Bayesian credible interval for κ, which is denoted as

$$CI_{\kappa(\mathrm{HPD})} = \left(L_{\kappa(\mathrm{HPD})}, U_{\kappa(\mathrm{HPD})}\right). \tag{4-3}$$

where $L_{\kappa(\mathrm{HPD})}$ and $U_{\kappa(\mathrm{HPD})}$ represent the lower and upper bounds of κ_{HPD} credible interval.

4.2.3 Approximate normal (AN)

The approximate normal method considered in this study is based on the Delta method, which provides a large-sample approximation for the distribution of a smooth function of asymptotically normal estimators. Let $\hat{\delta}$, $\hat{\mu}$, and $\hat{\lambda}$ denote the MLEs of the parameters δ, μ, and λ, respectively. For the ZITR distribution, the CV is defined as

$$g(\delta,\mu,\lambda) = \frac{\sqrt{\xi}}{\psi} = \frac{\sqrt{\dfrac{4(1-\delta)-(1-\delta)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda} + \dfrac{\mu}{\sqrt{\lambda}}\Gamma\left(\frac{1}{2}\right)\delta(1-\delta) + \mu^2\delta(1-\delta)}}{(1-\delta)\left(\mu + \dfrac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right)}$$

where $\psi = (1-\delta)\left(\mu + \frac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right)$

and $\xi = \frac{4(1-\delta)-\pi(1-\delta)^2}{4\lambda} + \frac{\mu\sqrt{\pi}\delta(1-\delta)}{\sqrt{\lambda}} + \mu^2\delta(1-\delta)$ denote the mean and variance of the ZITR distribution, respectively.

The delta method approximation is derived by doing a first-order Taylor expansion of $g(\hat{\delta},\hat{\mu},\hat{\lambda})$ around the actual parameter values (δ,μ,λ). These values are articulated as follows:

$$\hat{\kappa} \approx g(\delta,\mu,\lambda) + \frac{\partial g}{\partial \delta}(\hat{\delta}-\delta) + \frac{\partial g}{\partial \mu}(\hat{\mu}-\mu) + \frac{\partial g}{\partial \lambda}(\hat{\lambda}-\lambda).$$

Consequently, the variance of $\hat{\kappa}$ can be estimated as follows:

$$V(\hat{\kappa}) \approx \left(\frac{\partial}{\partial \delta} g(\delta,\lambda,\mu)\right)^2 V(\hat{\delta}-\delta) + \left(\frac{\partial}{\partial \mu} g(\delta,\lambda,\mu)\right)^2 V(\hat{\mu}-\mu) + \left(\frac{\partial}{\partial \lambda} g(\delta,\lambda,\mu)\right)^2 V(\hat{\lambda}-\lambda)$$

The following is the derivation of the partial derivatives of $g(\delta,\mu,\lambda)$

$$\frac{\partial}{\partial \delta} g(\delta,\lambda,\mu) = \frac{\psi\left(\frac{1}{2\sqrt{\xi}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta)-4}{4\lambda} + \frac{\mu}{\sqrt{\lambda}}\Gamma\left(\frac{1}{2}\right)[1-2\delta] + \mu^2[1-2\delta]\right) - \left(\sqrt{\xi}\right)\left(-1\left(\left(\mu + \frac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\psi^2}$$

$$\frac{\partial}{\partial \mu} g(\delta,\lambda,\mu) = \frac{\left(\frac{1}{2\sqrt{\xi}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta(1-\delta)}{\sqrt{\lambda}} + 2\mu\delta(1-\delta)\right)\psi - \left(\sqrt{\xi}\right)(1-\delta)}{\psi^2}$$

$$\frac{\partial}{\partial \lambda} g(\delta,\lambda,\mu) = \frac{\psi\left(\frac{1}{2\sqrt{\xi}}\right)\left(\frac{(1-\delta)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2 - 4(1-\delta)}{4\lambda^2} - \frac{\mu\Gamma\left(\frac{1}{2}\right)\delta(1-\delta)}{2\lambda^{\frac{3}{2}}}\right) - \sqrt{\xi}\left((1-\delta)\left(-\frac{1}{2}\lambda^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)}{\psi^2}$$

Therefore

$$\hat{\kappa} \approx g(\delta,\mu,\lambda) + \frac{\partial}{\partial \delta} g(\delta,\lambda,\mu)(\hat{\delta}-\delta) + \frac{\partial}{\partial \mu} g(\delta,\lambda,\mu)(\hat{\mu}-\mu) + \frac{\partial}{\partial \lambda} g(\delta,\lambda,\mu)(\hat{\lambda}-\lambda)$$

$$V(\hat{\kappa}) \approx \left(\frac{\partial}{\partial\delta} g(\delta,\lambda,\mu)\right)^2 V\left(\hat{\delta}-\delta\right) + \left(\frac{\partial}{\partial\mu} g(\delta,\lambda,\mu)\right)^2 V(\hat{\mu}-\mu) + \left(\frac{\partial}{\partial\lambda} g(\delta,\lambda,\mu)\right)^2 V\left(\hat{\lambda}-\lambda\right)$$

$$V(\hat{\kappa}) = V\left(\frac{\partial}{\partial\delta} g(\delta,\lambda,\mu)\left(\hat{\delta}-\delta\right) + \frac{\partial}{\partial\mu} g(\delta,\lambda,\mu)(\hat{\mu}-\mu) + \frac{\partial}{\partial\lambda} g(\delta,\lambda,\mu)\left(\hat{\lambda}-\lambda\right)\right)$$

$$= \left(\left(\frac{\psi\left(\frac{1}{2\sqrt{\xi}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta)-4}{4\lambda} + \frac{\mu}{\sqrt{\lambda}}\Gamma\left(\frac{1}{2}\right)[1-2\delta] + \mu^2[1-2\delta] \right) - \left(\sqrt{\xi}\right)\left(-1\left(\left(\mu+\frac{1}{\sqrt{\lambda}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\psi^2} \right)^2 V\left(\hat{\delta}-\delta\right) \right.$$

$$+ \left(\frac{\psi\left(\frac{1}{2\sqrt{\xi}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta(1-\delta)}{\sqrt{\lambda}} + 2\mu\delta(1-\delta) \right) - \left(\sqrt{\xi}\right)(1-\delta)}{\psi^2} \right)^2 V(\hat{\mu}-\mu)$$

$$\left. + \left(\frac{\psi\left(\frac{1}{2\sqrt{\xi}}\right)\left(\frac{(1-\delta)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2 - 4(1-\delta)}{4\lambda^2} - \frac{\mu\Gamma\left(\frac{1}{2}\right)\delta(1-\delta)}{2\lambda^{\frac{3}{2}}} \right) - \sqrt{\xi}\left((1-\delta)\left(-\frac{1}{2}\lambda^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)}{\psi^2} \right)^2 V\left(\hat{\lambda}-\lambda\right) \right)$$

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$$V(\hat{\kappa})=\left(\begin{array}{l}\left(\dfrac{\psi\left(\dfrac{1}{2\sqrt{\xi}}\right)\left(\dfrac{2\left(\Gamma\left(\dfrac{1}{2}\right)\right)^2(1-\delta)-4}{4\lambda}+\dfrac{\mu}{\sqrt{\lambda}}\Gamma\left(\dfrac{1}{2}\right)[1-2\delta]+\mu^2[1-2\delta]\right)-\left(\sqrt{\xi}\right)\left(-1\left(\left(\mu+\dfrac{1}{\sqrt{\lambda}}\Gamma\left(\dfrac{3}{2}\right)\right)\right)\right)}{\psi^2}\right)^2\left(\dfrac{\delta(1-\delta)}{n}\right)\\ +\left(\dfrac{\psi\left(\dfrac{1}{2\sqrt{\xi}}\right)\left(\dfrac{\Gamma\left(\dfrac{1}{2}\right)\delta(1-\delta)}{\sqrt{\lambda}}+2\mu\delta(1-\delta)\right)-\left(\sqrt{\xi}\right)(1-\delta)}{\psi^2}\right)^2\left(\dfrac{1}{2\mu+\dfrac{1}{n_{(1)}}\sum_{i=1}^{n_{(1)}}\left(x_{i(1)}-\mu\right)^{-2}}\right)\\ +\left(\dfrac{\psi\left(\dfrac{1}{2\sqrt{\xi}}\right)\left(\dfrac{(1-\delta)^2\left(\Gamma\left(\dfrac{1}{2}\right)\right)^2-4(1-\delta)}{4\lambda^2}-\dfrac{\mu\Gamma\left(\dfrac{1}{2}\right)\delta(1-\delta)}{2\lambda^{\frac{3}{2}}}\right)-\sqrt{\xi}\left((1-\delta)\left(-\dfrac{1}{2}\lambda^{-\frac{3}{2}}\Gamma\left(\dfrac{3}{2}\right)\right)\right)}{\psi^2}\right)^2\dfrac{\lambda^2}{n_{(1)}}\end{array}\right)$$

So estimate of $V(\hat{\kappa})$ defined $\hat{V}(\hat{\kappa})$ expressed as

$$\hat{V}(\hat{\kappa})=\left(\begin{array}{l}\left(\dfrac{\hat{\psi}\left(\dfrac{1}{2\sqrt{\hat{\xi}}}\right)\left(\dfrac{2\left(\Gamma\left(\dfrac{1}{2}\right)\right)^2(1-\hat{\delta})-4}{4\hat{\lambda}}+\dfrac{\hat{\mu}}{\sqrt{\hat{\lambda}}}\Gamma\left(\dfrac{1}{2}\right)[1-2\hat{\delta}]+\hat{\mu}^2[1-2\hat{\delta}]\right)-\left(\sqrt{\hat{\xi}}\right)\left(-1\left(\left(\hat{\mu}+\dfrac{1}{\sqrt{\hat{\lambda}}}\Gamma\left(\dfrac{3}{2}\right)\right)\right)\right)}{\psi^2}\right)^2\left(\dfrac{\hat{\delta}(1-\hat{\delta})}{n}\right)\\ +\left(\dfrac{\hat{\psi}\left(\dfrac{1}{2\sqrt{\hat{\xi}}}\right)\left(\dfrac{\Gamma\left(\dfrac{1}{2}\right)\hat{\delta}(1-\hat{\delta})}{\sqrt{\hat{\lambda}}}+2\hat{\mu}\hat{\delta}(1-\hat{\delta})\right)-\left(\sqrt{\hat{\xi}}\right)(1-\hat{\delta})}{\hat{\psi}^2}\right)^2\left(\dfrac{1}{2\hat{\mu}+\dfrac{1}{n_{(1)}}\sum_{i=1}^{n_{(1)}}\left(x_{i(1)}-\hat{\mu}\right)^{-2}}\right)\\ +\left(\dfrac{\hat{\psi}\left(\dfrac{1}{2\sqrt{\hat{\xi}}}\right)\left(\dfrac{(1-\hat{\delta})^2\left(\Gamma\left(\dfrac{1}{2}\right)\right)^2-4(1-\hat{\delta})}{4\hat{\lambda}^2}-\dfrac{\hat{\mu}\Gamma\left(\dfrac{1}{2}\right)\hat{\delta}(1-\hat{\delta})}{2\hat{\lambda}^{\frac{3}{2}}}\right)-\sqrt{\hat{\xi}}\left((1-\hat{\delta})\left(-\dfrac{1}{2}\hat{\lambda}^{-\frac{3}{2}}\Gamma\left(\dfrac{3}{2}\right)\right)\right)}{\hat{\psi}^2}\right)^2\dfrac{\hat{\lambda}^2}{n_{(1)}}\end{array}\right)\qquad(4\text{-}4)$$

Consequently, the estimated normal confidence interval for ξ is formulated as

$$CI_{\kappa(AN)} = \left(\hat{\kappa} - z_{\alpha/2}\sqrt{\hat{V}(\hat{\kappa})}, \hat{\kappa} + z_{\alpha/2}\sqrt{\hat{V}(\hat{\kappa})}\right), \tag{4-5}$$

Here, $z_{\alpha/2}$ represents the higher $\alpha/2$ quantile of the standard normal distribution, whereas $\hat{V}(\hat{\kappa})$ signifies the variance estimator derived using the delta technique.

4.2.4 Generalized confidence interval (GCI)

Section 2.2 presents the generalized pivotal quantities (GPQs) for the parameters λ and μ, as suggested by Krishnamoorthy et al., which may be articulated as follows:

$$Q_a = Q_\mu,$$

and

$$Q_\lambda = \frac{1}{2Q_b^{\,2}}$$

where $Q_a = \hat{a} - \hat{b}\frac{\hat{a}^*}{\hat{b}^*}$ and $Q_b = \frac{\hat{b}}{\hat{b}^*}$. Furthermore, Wu and Hsieh () construct the pivotal quantities based on VST to construct intervals, which is

$$Q_\delta = \sin^2\left(\arcsin\sqrt{\hat{\delta}} - \frac{T}{2\sqrt{n}}\right),$$

where $T = 2\sqrt{n}\left(\arcsin\sqrt{\hat{\delta}} - \arcsin\sqrt{\delta}\right) \sim N(0,1)$. Hence, considering the three pivotal Q_a, Q_λ, and Q_δ, which are independent of the parameter values, GPQ for the variance of *ZITR* distribution is given by

$$Q_\kappa = \frac{\sqrt{\dfrac{4(1-Q_\delta) - (1-Q_\delta)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4Q_\lambda} + \dfrac{Q_\mu}{\sqrt{Q_\lambda}}\Gamma\left(\frac{1}{2}\right)Q_\delta(1-Q_\delta) + Q_\mu^2 Q_\delta(1-Q_\delta)}}{(1-Q_\delta)\left(Q_\mu + \dfrac{1}{\sqrt{Q_\lambda}}\Gamma\left(\frac{3}{2}\right)\right)}. \tag{4-6}$$

The associated GCI for κ is

$$CI_{\kappa(GCI)} = \left(L_{\kappa(GCI)}, U_{\kappa(GCI)}\right) = \left(Q_\kappa(\alpha/2), Q_\kappa(1-\alpha/2)\right). \tag{4-7}$$

where $Q_\kappa(\alpha/2)$ represent the $100(\alpha/2)\%$ percentile of Q_κ

4.2.5 Percentile Bootstrap (PB)

The bootstrap is a simulation-based methodology developed by Efron (1993) that estimates the sampling distribution of a dataset via repeated resampling. Bootstrap samples are produced by repeatedly sampling with replacement from the original dataset. This method, when iterated, generates a sequence of bootstrap estimates of κ , from which an empirical distribution is constructed. The PB for κ is determined according to the empirical quantiles of the ordered bootstrap estimations as

$$CI_{\kappa(PB)} = \left(L_{\kappa(PB)}, U_{\kappa(PB)}\right) = \left((\hat{\kappa}^*_{(\alpha/2)}, \hat{\kappa}^*_{(1-\alpha/2)})\right). \quad (4\text{-}8)$$

In this expression, $\hat{\kappa}^*_{(\alpha/2)}$ and $\hat{\kappa}^*_{(1-\alpha/2)}$ are the empirical quantiles of order $\alpha/2$ and $1-\alpha/2$ of the bootstrap samples.

4.2.6 Bootstrap with standard error (BS)

The standard error does not require an explicit analytical representation in the case of Bootstrap with standard error, which provides an alternative to the conventional interval estimation procedures. The variability of the estimator is directly obtained from the resampling technique. In this context, several bootstrap samples are created, and the empirical distribution obtained from this process is used to estimate the standard error, which will be used to construct confidence intervals. This strategy is particularly advantageous in situations with a limited sample size or where the theoretical variance cannot be expressed in a closed form.

Let $\hat{\kappa}^{*b}$, where $b = 1, 2, \ldots, B$, be the bootstrap estimates of the parameter of interest obtained from B resamples. The associated standard error is calculated as the sample standard deviation of $\hat{\kappa}^{*b}$. Consequently, the two-sided $100(1-\alpha)\%$ confidence interval for κ, obtained from the bootstrap standard error, may be expressed as

$$CI_{\kappa(BS)} = \left(L_{\kappa(BS)}, U_{\kappa(BS)}\right) = \left(\bar{\kappa} - Z_{\alpha/2}SE, \bar{\kappa} + Z_{\alpha/2}SE\right), \quad (4\text{-}9)$$

where SE is standard error estimate from resampling and $z_{1-\alpha/2}$ is the upper $(1-\alpha/2)$ -quantile of the standard normal distribution.

4.3 The confidence intervals for difference and ratio for the coefficients of variation for ZITR distribution

This section examines confidence intervals for the difference and ratio of two coefficients of variation, using the methodologies of MCMC, HPD, GCI, MOVER, AN, PB, and BS interval approaches. Let $X_i = (X_{i1}, \ldots, X_{in_i})$, where $i = 1, 2$, signify two independent random samples drawn from the zero-inflated two-parameter Rayleigh distribution, referred to as $ZITR(\mu_i, \lambda_i, \delta_i)$. In this context, μ_i, λ_i, and δ_i represent the location parameter, the scale parameter, and the probability of zero, respectively. The variance of $X_i = (X_{i1}, \ldots, X_{in_i})$ is expressed as

$$\theta_{CV} = \kappa_1 - \kappa_2$$

$$= \left(\frac{\sqrt{\frac{4(1-\delta_1)-(1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda_1} + \frac{\mu_1}{\sqrt{\lambda_1}}\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1) + \mu_1^2\delta_1(1-\delta_1)}}{(1-\delta_1)\left(\mu_1 + \frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)} - \frac{\sqrt{\frac{4(1-\delta_2)-(1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda_2} + \frac{\mu_2}{\sqrt{\lambda_2}}\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2) + \mu_2^2\delta_2(1-\delta_2)}}{(1-\delta_2)\left(\mu_2 + \frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)} \right), \quad (4\text{-}10)$$

and

$$\tau_{CV} = \kappa_1 / \kappa_2$$

$$= \left(\frac{\sqrt{\frac{4(1-\delta_1)-(1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda_1} + \frac{\mu_1}{\sqrt{\lambda_1}}\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1) + \mu_1^2\delta_1(1-\delta_1)}}{(1-\delta_1)\left(\mu_1 + \frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)} \Big/ \frac{\sqrt{\frac{4(1-\delta_2)-(1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda_2} + \frac{\mu_2}{\sqrt{\lambda_2}}\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2) + \mu_2^2\delta_2(1-\delta_2)}}{(1-\delta_2)\left(\mu_2 + \frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)} \right). \quad (4\text{-}11)$$

Respectively. The following section outlines the method for constructing confidence intervals for θ_{CV} and τ_{CV}.

4.3.1 Bayesian MCMC

Bayesian inference implemented via MCMC combines prior information with the observed data to obtain the posterior distributions of the model parameters. Let $X_{ij} = (X_{i1}, \ldots, X_{in_i})$, for $i = 1, 2$ and $j = 1, 2, \ldots, n_i$, denote two independent random samples from the ZITR distribution. In this study, Bayesian inference is conducted for the difference and ratio of the coefficients of variation. Since the joint posterior distribution of the model parameters does not admit a closed-form expression, posterior inference is performed using MCMC sampling. The resulting posterior samples are subsequently used to estimate the posterior distributions of the quantities of interest and to construct Bayesian credible intervals. The prior distributions for the model parameters are as follows: a uniform prior for the location parameter μ, a gamma prior for the scale parameter λ, and a beta prior for δ. These prior choices ensure that the parameter constraints are satisfied and improve numerical stability during computation. All analyses are implemented in R using the R2 OpenBUGS package. MCMC for the

difference in CVs, denoted by θ, and the ratio of CVs, denoted by τ, are constructed based on the corresponding posterior distributions as follows

$$CI_{\theta_{CV}(\text{MCMC})} = \left(L_{\theta_{CV}(\text{MCMC})}, U_{\theta_{CV}(\text{MCMC})}\right) = \left(\theta_{CV(\text{MCMC})}(\alpha/2), \theta_{CV(\text{MCMC})}(1-\alpha/2)\right), \quad (4\text{-}12)$$

and

$$CI_{\tau_{CV}(\text{MCMC})} = \left(L_{\tau_{CV}(\text{MCMC})}, U_{\tau_{CV}(\text{MCMC})}\right) = \left(\tau_{\text{CV(MCMC)}}(\alpha/2), \tau_{\text{CV(MCMC)}}(1-\alpha/2)\right). \quad (4\text{-}13)$$

4.3.2 Bayesian HPD

The studies use HPD credible intervals to evaluate the uncertainty associated with the difference and ratio for coefficients of variation, represented by θ_{CV} and τ_{CV}, respectively. These intervals are generated using posterior samples acquired by the HDInterval package inside the R environment. According to the simulated posterior samples, the two-sided HPD credible intervals for θ_{CV} and τ_{CV} are provided by

$$CI_{\theta_{CV}(HPD)} = \left(L_{\theta_{CV}(HPD)}, U_{\theta_{CV}(HPD)}\right), \quad (4\text{-}14)$$

and

$$CI_{\tau_{CV}(HPD)} = \left(L_{\tau_{CV}(HPD)}, U_{\tau_{CV}(HPD)}\right). \quad (4\text{-}15)$$

4.3.3 Approximate normal (AN)

This section derives variance approximations for the estimators of the difference and ratio of CVs, denoted by $\hat{\theta}$ and $\hat{\tau}$, respectively, using the delta method. Let $\hat{\kappa}_i = g(\hat{\delta}_i, \hat{\mu}_i, \hat{\lambda}_i)$, be the estimator of CV for the ZITR distribution, where

$$g(\delta_i, \mu_i, \lambda_i) = \frac{\sqrt{\frac{4(1-\delta_i)-(1-\delta_i)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\lambda_i} + \frac{\mu_i}{\sqrt{\lambda_i}}\Gamma\left(\frac{1}{2}\right)\delta_i(1-\delta_i) + \mu_i^2\delta_i(1-\delta_i)}}{(1-\delta_i)\left(\mu_i + \frac{1}{\sqrt{\lambda_i}}\Gamma\left(\frac{3}{2}\right)\right)}.$$

To obtain approximate variances for $\hat{\theta}$ and $\hat{\tau}$, the delta method is employed. This approach provides a convenient way to approximate the sampling distribution of a smooth function of estimators through linearization. Let $G = g(\hat{U}_1, \hat{U}_2, \hat{U}_3, \hat{U}_4, \hat{U}_5, \hat{U}_6)$ denote an estimator of the continuously differentiable function $g(U_1, U_2, U_3, U_4, U_5, U_6)$. A first-order Taylor series expansion of $g(\cdot)$ about the true parameter vector $(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6)$, corresponding to $(U_1, U_2, U_3, U_4, U_5, U_6)$, yields an asymptotic approximation to the distribution of G. Under regularity conditions and for sufficiently large sample sizes, G is approximately normally distributed. Consequently, approximate expressions for the variance of G can be obtained and subsequently used to construct confidence intervals for the parameters θ and τ.

$$g(U_1,U_2,U_3,U_4,U_5,U_6) \approx g\left((\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6)\right) + \sum_{i=1}^{6} \frac{\partial g\left((\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6)\right)}{\partial u_i}(U_i-\omega_i).$$

In this setting, define the vector of basic statistics as

$$U_1=\hat{\delta}_1,\quad U_2=\hat{\mu}_1,\quad U_3=\hat{\lambda}_1,\quad U_4=\hat{\delta}_2,\quad U_5=\hat{\mu}_2,\quad U_6=\hat{\lambda}_2,$$

where $\hat{\delta}_1,\hat{\mu}_1,\hat{\lambda}_1,\hat{\delta}_2,\hat{\mu}_2,\hat{\lambda}_2$ are the maximum likelihood estimators (MLEs) of $\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2$, respectively. Substituting these estimators into the function $g(\cdot)$ yields the estimator of θ as

$$\hat{\theta}=g\left(\hat{\delta}_1,\hat{\mu}_1,\hat{\lambda}_1,\hat{\delta}_2,\hat{\mu}_2,\hat{\lambda}_2\right)=\frac{\sqrt{\frac{4\left(1-\hat{\delta}_1\right)-\left(1-\hat{\delta}_1\right)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\hat{\lambda}_1}+\frac{\hat{\mu}_1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{1}{2}\right)\hat{\delta}_1\left(1-\hat{\delta}_1\right)+\hat{\mu}_1^2\hat{\delta}_1\left(1-\hat{\delta}_1\right)}}{\left(1-\hat{\delta}_1\right)\left(\hat{\mu}_1+\frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)}-\frac{\sqrt{\frac{4\left(1-\hat{\delta}_2\right)-\left(1-\hat{\delta}_2\right)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\hat{\lambda}_2}+\frac{\hat{\mu}_2}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{1}{2}\right)\hat{\delta}_2\left(1-\hat{\delta}_2\right)+\hat{\mu}_2^2\hat{\delta}_2\left(1-\hat{\delta}_2\right)}}{\left(1-\hat{\delta}_2\right)\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)}$$

and the estimator of τ as

$$\hat{\tau}=g\left(\hat{\delta}_1,\hat{\mu}_1,\hat{\lambda}_1,\hat{\delta}_2,\hat{\mu}_2,\hat{\lambda}_2\right)=\frac{\sqrt{\frac{4\left(1-\hat{\delta}_1\right)-\left(1-\hat{\delta}_1\right)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\hat{\lambda}_1}+\frac{\hat{\mu}_1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{1}{2}\right)\hat{\delta}_1\left(1-\hat{\delta}_1\right)+\hat{\mu}_1^2\hat{\delta}_1\left(1-\hat{\delta}_1\right)}}{\left(1-\hat{\delta}_1\right)\left(\hat{\mu}_1+\frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)}\Bigg/\frac{\sqrt{\frac{4\left(1-\hat{\delta}_2\right)-\left(1-\hat{\delta}_2\right)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4\hat{\lambda}_2}+\frac{\hat{\mu}_2}{\sqrt{\hat{\lambda}_2}}\sqrt{\pi}\hat{\delta}_2\left(1-\hat{\delta}_2\right)+\hat{\mu}_2^2\hat{\delta}_2\left(1-\hat{\delta}_2\right)}}{\left(1-\hat{\delta}_2\right)\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)}$$

The partial derivatives of $\hat{\theta}$ up to the first order with regard to the components of the parameter vector are derived as follows:

$$\frac{\partial}{\partial\delta_1}g\left(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2\right)=\frac{\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_1)-4}{4\lambda_1}+\frac{\mu_1}{\sqrt{\lambda_1}}\Gamma\left(\frac{1}{2}\right)[1-2\delta_1]+\mu_1^2[1-2\delta_1]\right)-\left(\sqrt{\xi_1}\right)\left(-1\left(\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\psi_1^2}$$

$$\frac{\partial}{\partial\mu_1}g\left(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2\right)=\frac{\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{\sqrt{\lambda_1}}+2\mu_1\delta_1(1-\delta_1)\right)-\left(\sqrt{\xi_1}\right)(1-\delta_1)}{\psi_1^2}$$

$$\frac{\partial}{\partial\lambda_1}g\left(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2\right)=\frac{\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{(1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta_1)}{4\lambda_1^2}-\frac{\mu_1\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{2\lambda_1^{\frac{3}{2}}}\right)-\sqrt{\xi_1}\left((1-\delta_1)\left(-\frac{1}{2}\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)}{\psi_1^2}$$

$$\frac{\partial}{\partial\delta_2}g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)=-\frac{\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_2)-4}{4\lambda_2}+\frac{\mu_2}{\sqrt{\lambda_2}}\Gamma\left(\frac{1}{2}\right)[1-2\delta_2]+\mu_2{}^2[1-2\delta_2]\right)-\left(\sqrt{\xi_2}\right)\left(-1\left(\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\psi_2{}^2}$$

$$\frac{\partial}{\partial\mu_2}g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)=-\frac{\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{\sqrt{\lambda_2}}+2\mu_2\delta_2(1-\delta_2)\right)-\left(\sqrt{\xi_2}\right)(1-\delta_2)}{\psi_2{}^2}$$

$$\frac{\partial}{\partial\lambda_2}g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)=-\frac{\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{(1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta_2)}{4\lambda_2{}^2}-\frac{\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{2\lambda_2{}^{\frac{3}{2}}}\right)-\sqrt{\xi_2}\left((1-\delta_2)\left(-\frac{1}{2}\lambda_2{}^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)}{\psi_2{}^2}$$

Therefore,

$$\hat{\theta}\approx g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\delta_1}\left(\hat{\delta}_1-\delta_1\right)+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\mu_1}\left(\hat{\mu}_1-\mu_1\right)$$
$$+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\lambda_1}\left(\hat{\lambda}_1-\lambda_1\right)+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\delta_2}\left(\hat{\delta}_2-\delta_2\right)+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\mu_2}\left(\hat{\mu}_2-\mu_2\right)$$
$$+\frac{\partial g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2)}{\partial\lambda_2}\left(\hat{\lambda}_2-\lambda_2\right)$$

$$V(\hat{\theta})=\left(\frac{\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_1)-4}{4\lambda_1}+\frac{\mu_1}{\sqrt{\lambda_1}}\Gamma\left(\frac{1}{2}\right)[1-2\delta_1]+\mu_1^2[1-2\delta_1]\right)-\left(\sqrt{\xi_1}\right)\left(-1\left(\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\psi_1^2}\right)^2 V(\hat{\delta}_1-\delta_1)$$

$$+\left(\frac{\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{\sqrt{\lambda_1}}+2\mu_1\delta_1(1-\delta_1)\right)-\left(\sqrt{\xi_1}\right)(1-\delta_1)}{\psi_1^2}\right)^2 V(\hat{\mu}_1-\mu_1)$$

$$+\left(\frac{\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{(1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta_1)}{4\lambda_1^2}-\frac{\mu_1\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{2\lambda_1^{\frac{3}{2}}}\right)-\sqrt{\xi_1}\left((1-\delta_1)\left(-\frac{1}{2}\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)}{\psi_1^2}\right)^2 V(\hat{\lambda}_1-\lambda_1)$$

$$+\left(\frac{\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_2)-4}{4\lambda_2}+\frac{\mu_2}{\sqrt{\lambda_2}}\Gamma\left(\frac{1}{2}\right)[1-2\delta_2]+\mu_2^2[1-2\delta_2]\right)-\left(\sqrt{\xi_2}\right)\left(-1\left(\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\psi_2^2}\right)^2 V(\hat{\delta}_2-\delta_2)$$

$$+\left(\frac{\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{\sqrt{\lambda_2}}+2\mu_2\delta_2(1-\delta_2)\right)-\left(\sqrt{\xi_2}\right)(1-\delta_2)}{\psi_2^2}\right)^2 V(\hat{\mu}_2-\mu_2)$$

$$+\left(\frac{\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{(1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta_2)}{4\lambda_2^2}-\frac{\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{2\lambda_2^{\frac{3}{2}}}\right)-\sqrt{\xi_2}\left((1-\delta_2)\left(-\frac{1}{2}\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)}{\psi_2^2}\right)^2 V(\hat{\lambda}_2-\lambda_2)$$

$$V\left(\hat{\theta}\right)=\left(\frac{\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2\left(1-\delta_1\right)-4}{4\lambda_1}+\frac{\mu_1}{\sqrt{\lambda_1}}\Gamma\left(\frac{1}{2}\right)\left[1-2\delta_1\right]+\mu_1^{\,2}\left[1-2\delta_1\right]\right)-\left(\sqrt{\xi_1}\right)\left(-1\left(\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\psi_1^{\,2}}\right)^2\left(\frac{\delta_1\left(1-\delta_1\right)}{n_1}\right)$$

$$+\left(\frac{\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_1\left(1-\delta_1\right)}{\sqrt{\lambda_1}}+2\mu_1\delta_1\left(1-\delta_1\right)\right)-\left(\sqrt{\xi_1}\right)\left(1-\delta_1\right)}{\psi_1^{\,2}}\right)^2\left(\frac{1}{2\mu_1+\frac{1}{n_{1(1)}}\sum_{j=1}^{n_{1(1)}}\left(x_{1j(1)}-\mu_1\right)^{-2}}\right)$$

$$+\left(\frac{\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{\left(1-\delta_1\right)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4\left(1-\delta_1\right)}{4\lambda_1^{\,2}}-\frac{\mu_1\Gamma\left(\frac{1}{2}\right)\delta_1\left(1-\delta_1\right)}{2\lambda_1^{\frac{3}{2}}}\right)-\sqrt{\xi_1}\left(\left(1-\delta_1\right)\left(-\frac{1}{2}\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)}{\psi_1^{\,2}}\right)^2\frac{\lambda_1^{\,2}}{n_{1(1)}}$$

$$+\left(\frac{\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2\left(1-\delta_2\right)-4}{4\lambda_2}+\frac{\mu_2}{\sqrt{\lambda_2}}\Gamma\left(\frac{1}{2}\right)\left[1-2\delta_2\right]+\mu_2^{\,2}\left[1-2\delta_2\right]\right)-\left(\sqrt{\xi_2}\right)\left(-1\left(\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\psi_2^{\,2}}\right)^2\left(\frac{\delta_2\left(1-\delta_2\right)}{n_2}\right)$$

$$+\left(\frac{\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_2\left(1-\delta_2\right)}{\sqrt{\lambda_2}}+2\mu_2\delta_2\left(1-\delta_2\right)\right)-\left(\sqrt{\xi_2}\right)\left(1-\delta_2\right)}{\psi_2^{\,2}}\right)^2\left(\frac{1}{2\mu_2+\frac{1}{n_{2(1)}}\sum_{j=1}^{n_{2(1)}}\left(x_{2j(1)}-\mu_2\right)^{-2}}\right)$$

$$+\left(\frac{\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{\left(1-\delta_2\right)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4\left(1-\delta_2\right)}{4\lambda_2^{\,2}}-\frac{\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2\left(1-\delta_2\right)}{2\lambda_2^{\frac{3}{2}}}\right)-\sqrt{\xi_2}\left(\left(1-\delta_2\right)\left(-\frac{1}{2}\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)}{\psi_2^{\,2}}\right)^2\frac{\lambda_2^{\,2}}{n_{2(1)}}$$

So estimate of $V\left(\hat{\theta}\right)$ defined $\hat{V}\left(\hat{\theta}\right)$ expressed as

$$
\begin{aligned}
V\left(\hat{\theta}\right)= & \left(\frac{\hat{\psi}_1\left(\frac{1}{2\sqrt{\hat{\xi}_1}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2\left(1-\hat{\delta}_1\right)-4}{4\hat{\lambda}_1}+\frac{\hat{\mu}_1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{1}{2}\right)\left[1-2\hat{\delta}_1\right]+\hat{\mu}_1^{\,2}\left[1-2\hat{\delta}_1\right]\right)-\left(\sqrt{\hat{\xi}_1}\right)\left(-1\left(\left(\hat{\mu}_1+\frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\hat{\psi}_1^{\,2}}\right)^2\left(\frac{\hat{\delta}_1\left(1-\hat{\delta}_1\right)}{n_1}\right) \\
& +\left(\frac{\hat{\psi}_1\left(\frac{1}{2\sqrt{\hat{\xi}_1}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\hat{\delta}_1\left(1-\hat{\delta}_1\right)}{\sqrt{\hat{\lambda}_1}}+2\hat{\mu}_1\hat{\delta}_1\left(1-\hat{\delta}_1\right)\right)-\left(\sqrt{\hat{\xi}_1}\right)\left(1-\hat{\delta}_1\right)}{\hat{\psi}_1^{\,2}}\right)^2\left(\frac{1}{2\hat{\mu}_1+\frac{1}{n_{1(1)}}\sum_{j=1}^{n_{1(1)}}\left(x_{1j(1)}-\hat{\mu}_1\right)^{-2}}\right) \\
& +\left(\frac{\hat{\psi}_1\left(\frac{1}{2\sqrt{\hat{\xi}_1}}\right)\left(\frac{\left(1-\hat{\delta}_1\right)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4\left(1-\hat{\delta}_1\right)}{4\hat{\lambda}_1^{\,2}}-\frac{\hat{\mu}_1\Gamma\left(\frac{1}{2}\right)\hat{\delta}_1\left(1-\hat{\delta}_1\right)}{2\hat{\lambda}_1^{\frac{3}{2}}}\right)-\sqrt{\hat{\xi}_1}\left(\left(1-\hat{\delta}_1\right)\left(-\frac{1}{2}\hat{\lambda}_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)}{\hat{\psi}_1^{\,2}}\right)^2\frac{\hat{\lambda}_1^{\,2}}{n_{1(1)}} \\
& +\left(\frac{\hat{\psi}_2\left(\frac{1}{2\sqrt{\hat{\xi}_2}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2\left(1-\hat{\delta}_2\right)-4}{4\hat{\lambda}_2}+\frac{\hat{\mu}_2}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{1}{2}\right)\left[1-2\hat{\delta}_2\right]+\hat{\mu}_2^{\,2}\left[1-2\hat{\delta}_2\right]\right)-\left(\sqrt{\hat{\xi}_2}\right)\left(-1\left(\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\hat{\psi}_2^{\,2}}\right)^2\left(\frac{\hat{\delta}_2\left(1-\hat{\delta}_2\right)}{n_2}\right) \\
& +\left(\frac{\hat{\psi}_2\left(\frac{1}{2\sqrt{\hat{\xi}_2}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\hat{\delta}_2\left(1-\hat{\delta}_2\right)}{\sqrt{\hat{\lambda}_2}}+2\hat{\mu}_2\hat{\delta}_2\left(1-\hat{\delta}_2\right)\right)-\left(\sqrt{\hat{\xi}_2}\right)\left(1-\hat{\delta}_2\right)}{\hat{\psi}_2^{\,2}}\right)^2\left(\frac{1}{2\hat{\mu}_2+\frac{1}{n_{2(1)}}\sum_{j=1}^{n_{2(1)}}\left(x_{2j(1)}-\hat{\mu}_2\right)^{-2}}\right) \\
& +\left(\frac{\hat{\psi}_2\left(\frac{1}{2\sqrt{\hat{\xi}_2}}\right)\left(\frac{\left(1-\hat{\delta}_2\right)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4\left(1-\hat{\delta}_2\right)}{4\hat{\lambda}_2^{\,2}}-\frac{\hat{\mu}_2\Gamma\left(\frac{1}{2}\right)\hat{\delta}_2\left(1-\hat{\delta}_2\right)}{2\hat{\lambda}_2^{\frac{3}{2}}}\right)-\sqrt{\hat{\xi}_2}\left(\left(1-\hat{\delta}_2\right)\left(-\frac{1}{2}\hat{\lambda}_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)}{\hat{\psi}_2^{\,2}}\right)^2\frac{\hat{\lambda}_2^{\,2}}{n_{2(1)}}
\end{aligned}
\tag{4-16}
$$

For ratio of CVs. The first-order partial derivatives with regard to the parts of the parameter vector are derived as follows:

$$\frac{\partial}{\partial \delta_1} g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2) = \frac{\kappa_2 \dfrac{\psi_1\left(\dfrac{1}{2\sqrt{\xi_1}}\right)\left(\dfrac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_1)-4}{4\lambda_1}+\dfrac{\mu_1}{\sqrt{\lambda_1}}\Gamma\left(\frac{1}{2}\right)[1-2\delta_1]+\mu_1^{\,2}[1-2\delta_1]\right)-\left(\sqrt{\xi_1}\right)\left(-1\left(\left(\mu_1+\dfrac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\psi_1^{\,2}}}{(\kappa_2)^2}$$

$$\frac{\partial}{\partial \mu_1} g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2) = \frac{\kappa_2\left(\psi_1\left(\dfrac{1}{2\sqrt{\xi_1}}\right)\left(\dfrac{\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{\sqrt{\lambda_1}}+2\mu_1\delta_1(1-\delta_1)\right)-\left(\sqrt{\xi_1}\right)(1-\delta_1)\right)}{\dfrac{\psi_1^{\,2}}{\kappa^2{}_2}}$$

$$\frac{\partial}{\partial \lambda_1} g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2) = \frac{\kappa_2\left(\psi_1\left(\dfrac{1}{2\sqrt{\xi_1}}\right)\left(\dfrac{(1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta_1)}{4\lambda_1^{\,2}}-\dfrac{\mu_1\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{2\lambda_1^{\frac{3}{2}}}\right)-\sqrt{\xi_1}\left((1-\delta_1)\left(-\dfrac{1}{2}\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\dfrac{\psi_1^{\,2}}{\kappa^2{}_2}}$$

$$\frac{\partial}{\partial \delta_2} g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2) = -\frac{\kappa_1\left(\psi_2\left(\dfrac{1}{2\sqrt{\xi_2}}\right)\left(\dfrac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_2)-4}{4\lambda_2}+\dfrac{\mu_2}{\sqrt{\lambda_2}}\Gamma\left(\frac{1}{2}\right)[1-2\delta_2]+\mu_2^{\,2}[1-2\delta_2]\right)-\left(\sqrt{\xi_2}\right)\left(-1\left(\left(\mu_2+\dfrac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)\right)}{\dfrac{\psi_2^{\,2}}{\kappa_2^2}}$$

$$\frac{\partial}{\partial \mu_2} g(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2) = -\left(\frac{\kappa_1\left(\psi_2\left(\dfrac{1}{2\sqrt{\xi_2}}\right)\left(\dfrac{\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{\sqrt{\lambda_2}}+2\mu_2\delta_2(1-\delta_2)\right)-\left(\sqrt{\xi_2}\right)(1-\delta_2)\right)}{\dfrac{\psi_2^{\,2}}{\kappa_2^2}}\right)$$

$$\frac{\partial}{\partial\lambda_2}g\left(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2\right)=-\left(\frac{\kappa_1\left(\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{\left(1-\delta_2\right)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4\left(1-\delta_2\right)}{4{\lambda_2}^2}-\frac{\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2\left(1-\delta_2\right)}{2{\lambda_2}^{\frac{3}{2}}}\right)-\sqrt{\xi_2}\left(\left(1-\delta_2\right)\left(-\frac{1}{2}{\lambda_2}^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\frac{{\psi_2}^2}{\kappa_2^2}}\right)$$

Therefore

$$\hat{\tau}\approx g\left(\delta_1,\mu_1,\lambda_1,\delta_2,\mu_2,\lambda_2\right)+\frac{\partial g\left(\hat{\delta}_1,\hat{\mu}_1,\hat{\lambda}_1,\hat{\delta}_2,\hat{\mu}_2,\hat{\lambda}_2\right)}{\partial\hat{\delta}_1}\left(\hat{\delta}_1-\delta_1\right)+\frac{\partial g\left(\hat{\delta}_1,\hat{\mu}_1,\hat{\lambda}_1,\hat{\delta}_2,\hat{\mu}_2,\hat{\lambda}_2\right)}{\partial\hat{\mu}_1}\left(\hat{\mu}_1-\mu_1\right)$$
$$+\frac{\partial g\left(\hat{\delta}_1,\hat{\mu}_1,\hat{\lambda}_1,\hat{\delta}_2,\hat{\mu}_2,\hat{\lambda}_2\right)}{\partial\hat{\lambda}_1}\left(\hat{\lambda}_1-\lambda_1\right)+\frac{\partial g\left(\hat{\delta}_1,\hat{\mu}_1,\hat{\lambda}_1,\hat{\delta}_2,\hat{\mu}_2,\hat{\lambda}_2\right)}{\partial\hat{\delta}_2}\left(\hat{\delta}_2-\delta_2\right)+\frac{\partial g\left(\hat{\delta}_1,\hat{\mu}_1,\hat{\lambda}_1,\hat{\delta}_2,\hat{\mu}_2,\hat{\lambda}_2\right)}{\partial\hat{\mu}_2}\left(\hat{\mu}_2-\mu_2\right)$$
$$+\frac{\partial g\left(\hat{\delta}_1,\hat{\mu}_1,\hat{\lambda}_1,\hat{\delta}_2,\hat{\mu}_2,\hat{\lambda}_2\right)}{\partial\hat{\lambda}_2}\left(\hat{\lambda}_2-\lambda_2\right)$$

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$$V(\hat{\tau})=\left(-\frac{\kappa_2\left(\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_1)-4}{4\lambda_1}+\frac{\mu_1}{\sqrt{\lambda_1}}\Gamma\left(\frac{1}{2}\right)[1-2\delta_1]+\mu_1^2[1-2\delta_1]\right)-\left(\sqrt{\xi_1}\right)\left(-1\left(\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)\right)}{\frac{\psi_1^2}{\kappa_2^2}}\right)^2 V(\hat{\delta}_1-\delta_1)$$

$$+\left(\frac{\kappa_2\left(\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{\sqrt{\lambda_1}}+2\mu_1\delta_1(1-\delta_1)\right)-\left(\sqrt{\xi_1}\right)(1-\delta_1)\right)}{\frac{\psi_1^2}{\kappa_2^2}}\right)^2 V(\hat{\mu}_1-\mu_1)$$

$$+\left(\frac{\kappa_2\left(\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{(1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta_1)}{4\lambda_1^2}-\frac{\mu_1\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{2\lambda_1^{\frac{3}{2}}}\right)-\sqrt{\xi_1}\left((1-\delta_1)\left(-\frac{1}{2}\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\frac{\psi_1^2}{\kappa_2^2}}\right)^2 V(\hat{\lambda}_1-\lambda_1)$$

$$+\left(-\frac{\kappa_1\left(\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_2)-4}{4\lambda_2}+\frac{\mu_2}{\sqrt{\lambda_2}}\Gamma\left(\frac{1}{2}\right)[1-2\delta_2]+\mu_2^2[1-2\delta_2]\right)-\left(\sqrt{\xi_2}\right)\left(-1\left(\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)\right)}{\frac{\psi_2^2}{\kappa_2^2}}\right)^2 V(\hat{\delta}_2-\delta_2)$$

$$+\left(-\frac{\kappa_1\left(\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{\sqrt{\lambda_2}}+2\mu_2\delta_2(1-\delta_2)\right)-\left(\sqrt{\xi_2}\right)(1-\delta_2)\right)}{\frac{\psi_2^2}{\kappa_2^2}}\right)^2 V(\hat{\mu}_2-\mu_2)$$

$$+\left(-\frac{\kappa_1\left(\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{(1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta_2)}{4\lambda_2^2}-\frac{\mu_2\sqrt{\pi}\delta_2(1-\delta_2)}{2\lambda_2^{\frac{3}{2}}}\right)-\sqrt{\xi_2}\left((1-\delta_2)\left(-\frac{1}{2}\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\frac{\psi_2^2}{\kappa_2^2}}\right)^2 V(\hat{\lambda}_2-\lambda_2)$$

$$V(\hat{\tau})=\left(-\left(\frac{\kappa_1\left(\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_1)-4}{4\lambda_1}+\frac{\mu_1}{\sqrt{\lambda_1}}\Gamma\left(\frac{1}{2}\right)[1-2\delta_1]+\mu_1^2[1-2\delta_1]\right)-\left(\sqrt{\xi_1}\right)\left(-1\left(\left(\mu_1+\frac{1}{\sqrt{\lambda_1}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)\right)}{\frac{\psi_1^2}{\kappa_2^2}}\right)\right)^2\left(\frac{\delta_1(1-\delta_1)}{n_1}\right)$$

$$+\left(\frac{\kappa_2\left(\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{\sqrt{\lambda_1}}+2\mu_1\delta_1(1-\delta_1)\right)-\left(\sqrt{\xi_1}\right)(1-\delta_1)\right)}{\frac{\psi_1^2}{\kappa_2^2}}\left(\frac{1}{2\hat{\mu}_1+\frac{1}{n_{1(1)}}\sum_{i=1}^{n_{1(1)}}\left(x_{1j(1)}-\hat{\mu}_1\right)^{-2}}\right)\right)^2$$

$$+\left(\frac{\kappa_2\left(\psi_1\left(\frac{1}{2\sqrt{\xi_1}}\right)\left(\frac{(1-\delta_1)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta_1)}{4\lambda_1^2}-\frac{\mu_1\Gamma\left(\frac{1}{2}\right)\delta_1(1-\delta_1)}{2\lambda_1^{\frac{3}{2}}}\right)-\sqrt{\xi_1}\left((1-\delta_1)\left(-\frac{1}{2}\lambda_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\frac{\psi_1^2}{\kappa_2^2}}\right)^2\frac{\lambda_1^2}{n_{1(1)}}$$

$$+\left(-\left(\frac{\kappa_1\left(\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2(1-\delta_2)-4}{4\lambda_2}+\frac{\mu_2}{\sqrt{\lambda_2}}\Gamma\left(\frac{1}{2}\right)[1-2\delta_2]+\mu_2^2[1-2\delta_2]\right)-\left(\sqrt{\xi_2}\right)\left(-1\left(\left(\mu_2+\frac{1}{\sqrt{\lambda_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)\right)}{\frac{\psi_2^2}{\kappa_2^2}}\right)\right)^2\left(\frac{\delta_2(1-\delta_2)}{n_2}\right)$$

$$+\left(-\left(\frac{\kappa_1\left(\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{\sqrt{\lambda_2}}+2\mu_2\delta_2(1-\delta_2)\right)-\left(\sqrt{\xi_2}\right)(1-\delta_2)\right)}{\frac{\psi_2^2}{\kappa_2^2}}\right)\right)^2\left(\frac{1}{2\mu_2+\frac{1}{n_{2(1)}}\sum_{j=1}^{n_{2(1)}}\left(x_{2j(1)}-\mu_2\right)^{-2}}\right)$$

$$+\left(-\left(\frac{\kappa_1\left(\psi_2\left(\frac{1}{2\sqrt{\xi_2}}\right)\left(\frac{(1-\delta_2)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4(1-\delta_2)}{4\lambda_2^2}-\frac{\mu_2\Gamma\left(\frac{1}{2}\right)\delta_2(1-\delta_2)}{2\lambda_2^{\frac{3}{2}}}\right)-\sqrt{\xi_2}\left((1-\delta_2)\left(-\frac{1}{2}\lambda_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\frac{\psi_2^2}{\kappa_2^2}}\right)\right)^2\frac{\lambda_2^2}{n_{2(1)}}$$

So estimate of $V(\hat{\tau})$ defined $\hat{V}(\hat{\tau})$ expressed as

$$\hat{V}(\hat{\tau})=\left(\left(\frac{\hat{\kappa}_1\left(\hat{\psi}_2\left(\frac{1}{2\sqrt{\hat{\xi}_2}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2\left(1-\hat{\delta}_1\right)-4}{4\hat{\lambda}_1}+\frac{\hat{\mu}_1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{1}{2}\right)\left[1-2\hat{\delta}_1\right]+\hat{\mu}_2{}^2\left[1-2\hat{\delta}_1\right]\right)-\left(\sqrt{\hat{\xi}_2}\right)\left(-1\left(\left(\hat{\mu}_1+\frac{1}{\sqrt{\hat{\lambda}_1}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)\right)}{\frac{\hat{\psi}_2{}^2}{\hat{\kappa}_2^2}}\right)^2\left(\frac{\hat{\delta}_1\left(1-\hat{\delta}_1\right)}{n_1}\right)\right.$$

$$+\left(\frac{\hat{\kappa}_2\left(\hat{\psi}_1\left(\frac{1}{2\sqrt{\hat{\xi}_1}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\hat{\delta}_1\left(1-\hat{\delta}_1\right)}{\sqrt{\hat{\lambda}_1}}+2\hat{\mu}_1\hat{\delta}_1\left(1-\hat{\delta}_1\right)\right)-\left(\sqrt{\hat{\xi}_1}\right)\left(1-\delta_1\right)\right)}{\frac{\hat{\psi}_1{}^2}{\hat{\kappa}_2^2}}\right)^2\left(\frac{1}{2\hat{\mu}_1+\frac{1}{n_{1(1)}}\sum_{j=1}^{n_{1(1)}}\left(x_{1j(1)}-\hat{\mu}_1\right)^{-2}}\right)$$

$$+\left(\frac{\hat{\kappa}_2\left(\hat{\psi}_1\left(\frac{1}{2\sqrt{\hat{\xi}_1}}\right)\left(\frac{\left(1-\hat{\delta}_1\right)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4\left(1-\hat{\delta}_1\right)}{4\hat{\lambda}_1{}^2}-\frac{\hat{\mu}_1\Gamma\left(\frac{1}{2}\right)\hat{\delta}_1\left(1-\hat{\delta}_1\right)}{2\hat{\lambda}_1^{\frac{3}{2}}}\right)-\sqrt{\hat{\xi}_1}\left(\left(1-\hat{\delta}_1\right)\left(-\frac{1}{2}\hat{\lambda}_1^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\frac{\hat{\psi}_1{}^2}{\hat{\kappa}_2^2}}\right)^2\frac{\hat{\lambda}_1{}^2}{n_{1(1)}}$$

$$+\left(-\left(\frac{\hat{\kappa}_1\left(\hat{\psi}_2\left(\frac{1}{2\sqrt{\hat{\xi}_2}}\right)\left(\frac{2\left(\Gamma\left(\frac{1}{2}\right)\right)^2\left(1-\hat{\delta}_2\right)-4}{4\hat{\lambda}_2}+\frac{\hat{\mu}_2}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{1}{2}\right)\left[1-2\hat{\delta}_2\right]+\hat{\mu}_2{}^2\left[1-2\hat{\delta}_2\right]\right)-\left(\sqrt{\hat{\xi}_2}\right)\left(-1\left(\left(\hat{\mu}_2+\frac{1}{\sqrt{\hat{\lambda}_2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)\right)}{\frac{\hat{\psi}_2{}^2}{\hat{\kappa}_2^2}}\right)\right)^2\left(\frac{\hat{\delta}_2\left(1-\hat{\delta}_2\right)}{n_2}\right)$$

$$+\left(-\left(\frac{\hat{\kappa}_1\left(\hat{\psi}_2\left(\frac{1}{2\sqrt{\hat{\xi}_2}}\right)\left(\frac{\Gamma\left(\frac{1}{2}\right)\hat{\delta}_2\left(1-\hat{\delta}_2\right)}{\sqrt{\hat{\lambda}_2}}+2\hat{\mu}_2\hat{\delta}_2\left(1-\hat{\delta}_2\right)\right)-\left(\sqrt{\hat{\xi}_2}\right)\left(1-\delta_2\right)\right)}{\frac{\hat{\psi}_2{}^2}{\hat{\kappa}_2^2}}\right)\right)^2\left(\frac{1}{2\hat{\mu}_2+\frac{1}{n_{2(1)}}\sum_{j=1}^{n_{2(1)}}\left(x_{2j(1)}-\hat{\mu}_2\right)^{-2}}\right)$$

$$+\left(-\left(\frac{\hat{\kappa}_1\left(\hat{\psi}_2\left(\frac{1}{2\sqrt{\hat{\xi}_2}}\right)\left(\frac{\left(1-\hat{\delta}_2\right)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2-4\left(1-\hat{\delta}_2\right)}{4\hat{\lambda}_2{}^2}-\frac{\hat{\mu}_2\Gamma\left(\frac{1}{2}\right)\hat{\delta}_2\left(1-\hat{\delta}_2\right)}{2\hat{\lambda}_2^{\frac{3}{2}}}\right)-\sqrt{\hat{\xi}_2}\left(\left(1-\hat{\delta}_2\right)\left(-\frac{1}{2}\hat{\lambda}_2^{-\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)\right)\right)\right)}{\frac{\hat{\psi}_2{}^2}{\hat{\kappa}_2^2}}\right)\right)^2\frac{\hat{\lambda}_2{}^2}{n_{2(1)}} \qquad (4\text{-}17)$$

Consequently, the estimated normal confidence interval for θ_{CV} and τ_{CV} is formulated as

$$CI_{\theta_{CV}(AN)} = \left(L_{\theta_{CV}(AN)}, U_{\theta_{CV}(AN)}\right) = \left(\hat{\theta}_{CV} - z_{1-\alpha/2}\sqrt{\hat{V}\left(\hat{\theta}_{CV}\right)}, \hat{\theta}_{CV} + z_{1-\alpha/2}\sqrt{\hat{V}\left(\hat{\theta}_{CV}\right)}\right) \quad (4\text{-}18)$$

and

$$CI_{\tau_{CV}(AN)} = \left(L_{\tau_{CV}(AN)}, U_{\tau_{CV}(AN)}\right) = \left(\hat{\tau}_{CV} - z_{1-\alpha/2}\sqrt{\hat{V}\left(\hat{\tau}_{CV}\right)}, \hat{\tau}_{CV} + z_{1-\alpha/2}\sqrt{\hat{V}\left(\hat{\tau}_{CV}\right)}\right), \quad (4\text{-}19)$$

respectively, where denotes the upper $\alpha/2$ quantile of the standard normal distribution. The explicit formulations of $\hat{V}(\hat{\theta})$ and $\hat{V}(\hat{\tau})$ are derived by the delta technique

4.3.4 Generalized confidence interval (GCI)

The GPQ for κ_i can be defined using for random variable x_i, where $i = 1, 2$ as follow

$$Q_{\kappa_i} = \frac{\sqrt{\frac{4\left(1-Q_{\hat{\delta}_i}\right)-\left(1-Q_{\hat{\delta}_i}\right)^2\left(\Gamma\left(\frac{1}{2}\right)\right)^2}{4Q_{\hat{\lambda}_i}} + \frac{Q_{\hat{\mu}_i}}{\sqrt{Q_{\hat{\lambda}_i}}}\Gamma\left(\frac{1}{2}\right)Q_{\hat{\delta}_i}\left(1-Q_{\hat{\delta}_i}\right) + Q_{\hat{\mu}_i}^2 Q_{\hat{\delta}_i}\left(1-Q_{\hat{\delta}_i}\right)}}{\left(1-Q_{\hat{\delta}_i}\right)\left(Q_{\hat{\mu}_i} + \frac{1}{\sqrt{Q_{\hat{\lambda}_i}}}\Gamma\left(\frac{3}{2}\right)\right)} \quad (4\text{-}20)$$

where Q_{δ_i}, Q_{μ_i} and Q_{λ_i} be pivotal quantities of δ_i, μ_i and λ_i respectively. The estimators of $\hat{\theta}_{mean}$ and $\hat{\tau}_{mean}$ obtained using the GCI method are denoted by Q_θ and Q_τ respectively, defined as

$$Q_{\theta_{CV}} = Q_{\kappa_1} - Q_{\kappa_2}, \quad (4\text{-}21)$$

and

$$Q_{\tau_{CV}} = \frac{Q_{\kappa_1}}{Q_{\kappa_2}}, \quad (4\text{-}22)$$

The empirical distributions of $Q_{\theta_{CV}}$ and $Q_{\tau_{CV}}$, which do not include unknown parameters, may be derived by Monte Carlo simulation. The $(1-\alpha)$ generalized confidence intervals are generated using the empirical quantiles of $Q_{\theta_{CV}}$ and $Q_{\tau_{CV}}$. In particular,

$$CI_{\theta_{CV}(\mathrm{GCI})} = \left(Q_{\theta_{CV}}(\alpha/2), Q_{\theta_{CV}}(1-\alpha/2)\right), \quad (4\text{-}23)$$

and

$$CI_{\tau_{CV}(\mathrm{GCI})} = \left(Q_{\tau_{CV}}(\alpha/2), Q_{\tau_{CV}}(1-\alpha/2)\right). \quad (4\text{-}24)$$

where $Q_{\theta_{CV}}(\alpha/2)$ and $Q_{\tau_{CV}}(\alpha/2)$ represent the $100(\alpha/2)$ th percentiles of $Q_{\theta_{CV}}$ and $Q_{\tau_{CV}}$, respectively.

4.3.5 Method of variance estimates recovery (MOVER)

Donner and Zou (2012) introduced a methodology for calculating confidence intervals for both the difference and the ratio of two distribution parameters. This method is especially advantageous when obtaining analytical formulas for the coefficients of variation of the estimators proves challenging, hence providing a pragmatic option for interval estimation of both metrics. This research presents the confidence ranges for the differences in coefficients of variation and the coefficients of variation ratio, represented by θ and τ, as

$$CI_{\theta_{CV}(\text{MOVER})} = \left(L_{\theta_{CV}(\text{MOVER})}, U_{\theta_{CV}(\text{MOVER})}\right), \tag{4-25}$$

and

$$CI_{\tau_{CV}(\text{MOVER})} = \left(L_{\tau_{CV}(\text{MOVER})}, U_{\tau_{CV}(\text{MOVER})}\right). \tag{4-26}$$

The boundaries for θ_{CV} are specified as follows

$$L_{\theta_{CV}(\text{MOVER})} = (\hat{\kappa}_1 - \hat{\kappa}_2) - \sqrt{(\hat{\kappa}_1 - l_1)^2 - (u_2 - \hat{\kappa}_2)^2},$$

and

$$U_{\theta_{CV}(\text{MOVER})} = (\hat{\kappa}_1 - \hat{\kappa}_2) + \sqrt{(u_1 - \hat{\kappa}_1)^2 - (\hat{\kappa}_2 - l_2)^2}.$$

The associated limits for the coefficient of variation for ratio τ_{CV} are specified by

$$L_{\tau_{CV}(\text{MOVER})} = \frac{\hat{\kappa}_1\hat{\kappa}_2 - \sqrt{(\hat{\kappa}_1\hat{\kappa}_2)^2 - l_1u_2(2\hat{\kappa}_1 - l_1)(2\hat{\kappa}_2 - u_2)}}{u_2(2\hat{\kappa}_2 - u_2)},$$

and

$$U_{\tau_{CV}(\text{MOVER})} = \frac{\hat{\kappa}_1\hat{\kappa}_2 + \sqrt{(\hat{\kappa}_1\hat{\kappa}_2)^2 - u_1l_2(2\hat{\kappa}_1 - u_1)(2\hat{\kappa}_2 - l_2)}}{l_2(2\hat{\kappa}_2 - l_2)}.$$

In this context, (l_1, u_1) and (l_2, u_2) represent the confidence intervals for the respective coefficients of variation parameters, derived by the GCI methodology.

4.3.6 Percentile Bootstrap (PB)

The Percentile Bootstrap procedure is a nonparametric resampling technique that was initially introduced by Efron and Tibshirani (1993). It does not necessitate stringent distributional assumptions. Consider a random sample $X_i = (X_{i1}, X_{i2}, \ldots, X_{in_i})$ that is selected from the ZITR distribution. The process of obtaining bootstrap replications involves drawing observations with replacement from the observed data, resulting in $X_i^{*b} = (X_{i1}^{*b}, X_{i2}^{*b}, \ldots, X_{in_i}^{*b})$. This process is repeated B times. Let θ and τ represent the difference and the ratio of the coefficients of variation for the two populations, respectively, in this framework. Each resample is used to calculate the corresponding bootstrap statistics, $\hat{\theta}^{*b}$ and $\hat{\tau}^{*b}$, for $b = 1, 2, \ldots, B$. The confidence intervals are subsequently constructed by utilizing the empirical distributions of these bootstrap replicates. Consequently, the two-sided Percentile Bootstrap confidence interval for the difference in CVs is defined as follows:

$$CI_{\theta_{CV}(PB)} = \left(L_{\theta_{CV}(PB)}, U_{\theta_{CV}(PB)}\right) = \left(\hat{\theta}^*_{CV}\left(\frac{\alpha}{2}\right), \hat{\theta}^*_{CV}\left(1-\frac{\alpha}{2}\right)\right), \quad (4\text{-}27)$$

Here, $\hat{\theta}^*\left(\frac{\alpha}{2}\right)$ and $\hat{\theta}^*\left(1-\frac{\alpha}{2}\right)$ denote the empirical $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$ quantiles of the ordered bootstrap estimates $\hat{\theta}^*_{CV}$.

The two-sided Percentile Bootstrap confidence interval for the ratio in CVs is

$$CI_{\tau_{CV}(PB)} = \left(L_{\tau_{CV}(PB)}, U_{\tau_{CV}(PB)}\right) = \left(\hat{\tau}^*_{CV}\left(\frac{\alpha}{2}\right), \hat{\tau}^*_{CV}\left(1-\frac{\alpha}{2}\right)\right), \quad (4\text{-}28)$$

where $\hat{\tau}^*\left(\frac{\alpha}{2}\right)$ and $\hat{\tau}^*\left(1-\frac{\alpha}{2}\right)$ represent the empirical $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$ quantiles of the ordered bootstrap estimates $\hat{\tau}^*\left(\frac{\alpha}{2}\right)$.

4.3.7 The bootstrap with standard error (BS)

The bootstrap-based standard error approach prevents the necessity for explicit analytical derivation of variance formulas by approximating the variability of an estimator through repeated resampling from the observed data. This renders the method especially advantageous in scenarios where the underlying distribution is intricate or intractable, as the dispersion of the resampled statistics can be directly employed to quantify uncertainty.

In this framework, the difference and the ratio of CVs under the ZITR model are denoted by θ and τ, respectively. Let $\hat{\theta}^{*b}$ and $\hat{\tau}^{*b}$ be the corresponding bootstrap estimates obtained from the resamples for $b = 1, 2, \ldots, B$. The bootstrap sample means are subsequently defined as

$$\bar{\theta}^*_{CV} = \frac{1}{B}\sum_{b=1}^{B}\hat{\theta}^{*b}_{CV}, \qquad \bar{\tau}^* = \frac{1}{B}\sum_{b=1}^{B}\hat{\tau}^{*b}_{CV}.$$

The standard errors are obtained from the variability of the bootstrap replicates and are calculated using the sample standard deviations of the corresponding bootstrap distributions:

$$SE(\hat{\theta}_{CV}) = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\theta}^{*b}_{CV} - \bar{\theta}^*_{CV}\right)^2}, \text{ and } SE(\hat{\tau}_{CV}) = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\tau}^{*b}_{CV} - \bar{\tau}^*_{CV}\right)^2}.$$

Thus, the $100(1-\alpha)\%$ confidence range on both sides for the difference between CVs is given by

$$CI_{\theta_{CV}(BS)} = \left(L_{\theta_{CV}(BS)}, U_{\theta_{CV}(BS)}\right) = \left(\bar{\theta}^*_{CV} - Z_{\alpha/2}SE(\hat{\theta}_{CV}), \bar{\theta}^*_{CV} + Z_{\alpha/2}SE(\hat{\theta}_{CV})\right), \quad (4\text{-}29)$$

and the confidence interval for the ratio of CVs is

$$CI_{\tau_{CV}(BS)} = \left(L_{\tau_{CV}(BS)}, U_{\tau_{CV}(BS)}\right) = \left(\bar{\tau}^* - Z_{\alpha/2}SE(\hat{\tau}_{CV}), \bar{\tau}^* + Z_{\alpha/2}SE(\hat{\tau}_{CV})\right), \quad (4\text{-}30)$$

where $Z_{\alpha/2}$ represents the higher $\alpha/2$ quantile of the ordinary normal distribution.

4.4 Simulation study and results

This study evaluates the performance of the proposed methods for constructing confidence intervals for a single CV, as well as the difference and ratio of coefficients of variation between two ZITR populations. For estimating the confidence interval of a single coefficient of variation, the MCMC, HPD, AN, GCI, PB, and BS methods are employed. For estimating confidence intervals for the difference and ratio of CVs, the MCMC, HPD, AN, GCI, MOVER, PB, and BS methods are employed.

The performance evaluation was conducted through data simulation using the R software and the Monte Carlo simulation technique. The performance of the competing methods was assessed based on the CP and EL. A method was considered appropriate if its CP was close to or greater than the nominal confidence level of 0.95. Among the methods satisfying this criterion, the method with the shortest EL was regarded as the most efficient.

For a single coefficient of variation, data were generated from the ZITR, denoted by $X \sim ZITR(\mu, \lambda, \delta)$. The sample sizes were set to $n = 20, 50, 70, 100,$ and 200, with the zero-inflation probability specified as $\delta = 0.1, 0.2,$ and 0.3. The scale parameter was set to $\lambda = 0.2, 0.5,$ and 1, while the location parameter was set to $\mu = 1$ and 2. For each simulation scenario, $M = 1,000$ Monte Carlo replications were performed. For the bootstrap methods, $B = 1,000$ resamples were generated in each replication. The GCI method was implemented using $q = 2,500$ pivotal quantities. For the Bayesian methods, $T = 20,000$ Gibbs sampling iterations were generated, with the first $C = 5,000$ iterations discarded as burn-in

For difference and ratio for coefficient of variation, the data were generated for two independent ZITR distributions: $X_i \sim ZITR(\mu_i, \lambda_i, \delta_i)$, $i = 1, 2$. Sample sizes were considered $(n_1, n_2) = (20, 20), (20, 50), (20, 70), (20, 100), (50, 50), (50, 70), (50, 100), (70, 70),$ $(70, 100)$, $(100, 100)$, and $(200, 200)$. The probability of zeros were set at $(\delta_1, \delta_2) =$ $(0.1, 0.1)$, $(0.1, 0.3)$ and $(0.3, 0.3)$. The scale parameter $(\lambda_1, \lambda_2) = (0.2, 0.2), (0.2, 1),$ and $(1, 1)$. The location parameter fixed $(\mu_1, \mu_2) = (0.2, 0.2)$ and $(1, 0.2)$. The number of generated M = 1000 replications for the Monte Carlo simulation, 1000 resamples for the bootstrap method, 2500 pivotal quantities were utilized for the GCI, and we generated T = 20,000 realizations of Bayesian using Gibbs sampling with a burn-in of C = 5000.

The simulation study evaluating the performance of the confidence interval estimation methods for a single coefficient of variation is presented in Tables 4-1 to 4-6 and Figure 4-1. The simulation results indicate that the Bayesian MCMC, Bayesian HPD, and GCI methods perform well across the considered scenarios, including cases with small sample sizes. However, the AN and BS methods exhibit undercoverage in several scenarios, particularly when the sample sizes are small. The undercoverage of the AN method is more pronounced when the location parameter is relatively large compared with the scale parameter; however, its coverage performance improves as the sample size increases. Similarly, the PB method shows a gradual improvement in coverage performance with increasing sample size, with its CP values becoming closer to the nominal confidence level of 0.95. In contrast, the BS method tends to produce shorter ELs but lower CP values, particularly for small sample sizes. For all methods,

increasing the sample size generally results in shorter ELs, indicating improved precision of the interval estimates. Overall, considering both CP and EL, the Bayesian HPD method demonstrates the most desirable performance, as it maintains acceptable coverage probabilities while providing relatively shorter confidence intervals.

TABLE 4-1 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the CV under ZITR with $\mu = 0.2$ and $\lambda = 0.2$

λ	δ	n	Coverage probability					
			PB	BS	GCI	AN	MCMC	HPD
0.2	0.1	20	0.837	0.742	0.942	0.943	0.987	0.989
			(0.37664)	(0.37094)	(0.42359)	(0.63123)	(0.33194)	**(0.32263)**
		50	0.912	0.815	0.957	0.995	0.970	0.978
			(0.23750)	(0.24013)	(0.24808)	(0.57866)	(0.22199)	**(0.21807)**
		70	0.900	0.794	0.949	1.000	0.974	0.972
			(0.19862)	(0.20047)	(0.20466)	(0.56246)	(0.18811)	**(0.18530)**
		100	0.926	0.803	0.952	0.999	0.962	0.961
			(0.16610)	(0.16502)	(0.16964)	(0.55060)	(0.15938)	**(0.15752)**
		200	0.940	0.832	0.946	1.000	0.950	0.952
			(0.11700)	(0.11745)	(0.11819)	(0.53521)	(0.11503)	**(0.11420)**
	0.2	20	0.905	0.796	0.957	0.922	0.977	0.959
			(0.51105)	(0.50894)	(0.55751)	(0.68983)	(0.40199)	**(0.39203)**
		50	0.926	0.807	0.946	0.978	0.958	0.949
			(0.31602)	(0.31110)	(0.32551)	(0.56748)	**(0.28122)**	(0.27703)
		70	0.937	0.817	0.956	0.994	0.969	0.961
			(0.26495)	(0.26399)	(0.27102)	(0.53992)	(0.24236)	**(0.23970)**
		100	0.928	0.815	0.943	0.999	0.953	0.942
			(0.22123)	(0.22105)	(0.22451)	(0.52009)	**(0.20700)**	(0.20532)
		200	0.942	0.834	0.955	1.000	0.957	0.953
			(0.15464)	(0.15270)	(0.15614)	(0.48930)	(0.14999)	**(0.14928)**
	0.3	20	0.917	0.817	0.945	0.901	0.938	0.907
			(0.64589)	(0.63287)	(0.70992)	(0.78552)	(0.47175)	(0.46097)
		50	0.943	0.818	0.955	0.966	0.950	0.936
			(0.39300)	(0.38483)	(0.40473)	(0.58804)	**(0.34054)**	(0.33603)
		70	0.950	0.830	0.953	0.988	0.962	0.951
			(0.33096)	(0.33121)	(0.33791)	(0.54385)	(0.29779)	**(0.29516)**
		100	0.940	0.819	0.953	0.987	0.951	0.944
			(0.27286)	(0.27276)	(0.27713)	(0.50069)	**(0.25186)**	(0.25018)
		200	0.942	0.824	0.947	1.000	0.947	0.944
			(0.19185)	(0.19385)	(0.19318)	**(0.46370)**	(0.18363)	(0.18290)

TABLE 4-2 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the CV under ZITR with $\mu = 0.2$ and $\lambda = 0.5$

λ	δ	n	Coverage probability					
			PB	BS	GCI	AN	MCMC	HPD
0.5	0.1	20	0.860	0.781	0.941	0.949	0.981	0.990
			(0.37942)	(0.38937)	(0.41742)	(0.63081)	(0.33948)	**(0.33066)**
		50	0.905	0.795	0.951	0.999	0.965	0.969
			(0.23594)	(0.23393)	(0.24345)	(0.56379)	(0.22504)	**(0.22113)**
		70	0.912	0.798	0.951	0.998	0.959	0.960
			(0.20055)	(0.19675)	(0.20510)	(0.54140)	(0.19388)	**(0.19109)**
		100	0.940	0.821	0.957	0.999	0.961	0.961
			(0.16740)	(0.16688)	(0.17003)	(0.52268)	(0.16398)	**(0.16209)**
		200	0.938	0.817	0.955	1.000	0.958	0.962
			(0.11729)	(0.11568)	(0.11819)	(0.50294)	(0.11680)	**(0.11594)**
	0.2	20	0.888	0.774	0.955	0.918	0.979	0.973
			(0.50841)	(0.50358)	(0.54500)	(0.69116)	(0.40576)	**(0.39618)**
		50	0.930	0.808	0.957	0.984	0.973	0.964
			(0.31559)	(0.32038)	(0.32310)	(0.55426)	(0.28405)	**(0.28005)**
		70	0.924	0.805	0.941	0.989	0.956	0.947
			(0.26342)	(0.26366)	(0.26782)	(0.51903)	**(0.24522)**	(0.24271)
		100	0.949	0.826	0.962	0.998	0.969	0.964
			(0.22172)	(0.22230)	(0.22445)	(0.49442)	(0.21072)	**(0.20909)**
		200	0.948	0.831	0.959	1.000	0.960	0.959
			(0.15451)	(0.15593)	(0.15559)	(0.45681)	(0.15103)	**(0.15031)**
	0.3	20	0.921	0.829	0.943	0.872	0.955	0.929
			(0.63935)	(0.64533)	(0.68737)	(0.76105)	**(0.47213)**	(0.46160)
		50	0.927	0.808	0.935	0.948	0.950	0.933
			(0.39123)	(0.38725)	(0.39971)	(0.55657)	**(0.34260)**	(0.33834)
		70	0.958	0.840	0.962	0.968	0.968	0.962
			(0.32796)	(0.32579)	(0.33356)	(0.51323)	(0.29811)	**(0.29559)**
		100	0.932	0.832	0.944	0.967	0.943	0.938
			(0.27290)	(0.27668)	(0.27611)	**(0.47083)**	(0.25465)	(0.25304)
		200	0.944	0.839	0.950	0.997	0.942	0.941
			(0.19074)	(0.19095)	**(0.19223)**	(0.42447)	(0.18428)	(0.18356)

TABLE 4-3 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the CV under ZITR with $\mu = 0.2$ and $\lambda = 1$

λ	δ	n	Coverage probability					
			PB	BS	GCI	AN	MCMC	HPD
1	0.1	20	0.863	0.763	0.955	0.951	0.971	0.987
			(0.37592)	(0.37544)	(0.40789)	(0.59929)	(0.34315)	**(0.33464)**
		50	0.901	0.789	0.948	0.996	0.958	0.966
			(0.23660)	(0.23496)	(0.24267)	(0.51809)	(0.23042)	**(0.22665)**
		70	0.913	0.811	0.952	0.997	0.955	0.966
			(0.19860)	(0.19993)	(0.20232)	(0.50088)	(0.19603)	**(0.19323)**
		100	0.925	0.802	0.948	0.999	0.950	0.954
			(0.16599)	(0.16450)	(0.16822)	(0.48404)	(0.16520)	**(0.16326)**
		200	0.948	0.818	0.963	1.000	0.959	0.965
			(0.11722)	(0.11700)	(0.11804)	(0.45619)	(0.11681)	**(0.11593)**
	0.2	20	0.923	0.810	0.956	0.922	0.984	0.981
			(0.51250)	(0.50917)	(0.53979)	(0.66351)	(0.41035)	**(0.40105)**
		50	0.932	0.825	0.939	0.968	0.963	0.956
			(0.31753)	(0.31927)	(0.32319)	(0.51180)	(0.28953)	**(0.28561)**
		70	0.935	0.821	0.954	0.988	0.960	0.964
			(0.26453)	(0.25860)	(0.26749)	(0.47659)	(0.24805)	**(0.24550)**
		100	0.945	0.830	0.954	0.996	0.964	0.962
			(0.21970)	(0.22174)	(0.22136)	(0.45242)	(0.21032)	**(0.20870)**
		200	0.951	0.820	0.961	0.999	0.966	0.963
			(0.15509)	(0.15284)	(0.15589)	(0.40971)	(0.15190)	**(0.15119)**
	0.3	20	0.926	0.832	0.943	0.842	0.974	0.971
			(0.64631)	(0.65221)	(0.65678)	(0.68454)	(0.47593)	**(0.46529)**
		50	0.931	0.813	0.939	0.930	0.948	0.939
			(0.38751)	(0.38752)	(0.39432)	(0.52317)	(0.34271)	(0.33865)
		70	0.925	0.846	0.926	0.938	0.937	0.933
			(0.32772)	(0.33203)	(0.33176)	(0.47465)	(0.29999)	(0.29747)
		100	0.941	0.836	0.948	0.968	0.957	0.951
			(0.27360)	(0.27357)	(0.27608)	(0.42631)	(0.25660)	**(0.25497)**
		200	0.928	0.823	0.943	0.986	0.941	0.935
			(0.19099)	(0.18997)	(0.19188)	**(0.37077)**	(0.18441)	(0.18371)

TABLE 4-4 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the CV under ZITR with $\mu = 1$ and $\lambda = 0.2$

λ	δ	n	Coverage probability					
			PB	BS	GCI	AN	MCMC	HPD
0.2	0.1	20	0.875	0.761	0.929	0.862	0.929	0.952
			(0.37610)	(0.37780)	(0.38915)	(0.45088)	(0.34136)	**(0.33108)**
		50	0.917	0.808	0.940	0.942	0.928	0.948
			(0.24266)	(0.24517)	(0.24292)	(0.34772)	(0.22808)	(0.22353)
		70	0.932	0.793	0.944	0.966	0.929	0.949
			(0.20606)	(0.20161)	(0.20573)	**(0.32173)**	(0.19377)	(0.19044)
		100	0.925	0.807	0.949	0.981	0.932	0.948
			(0.17089)	(0.16957)	(0.17088)	**(0.29995)**	(0.16233)	(0.16012)
		200	0.944	0.838	0.948	0.994	0.937	0.948
			(0.12163)	(0.12112)	(0.12178)	**(0.26985)**	(0.11706)	(0.11608)
	0.2	20	0.909	0.793	0.930	0.827	0.978	0.977
			(0.51580)	(0.51247)	(0.52074)	(0.56189)	(0.40867)	**(0.39809)**
		50	0.935	0.844	0.948	0.895	0.962	0.965
			(0.32126)	(0.32239)	(0.32175)	(0.38848)	(0.28720)	**(0.28293)**
		70	0.947	0.852	0.953	0.928	0.965	0.965
			(0.26824)	(0.27196)	(0.26810)	(0.34139)	(0.24590)	**(0.24318)**
		100	0.937	0.814	0.957	0.940	0.960	0.964
			(0.22324)	(0.22236)	(0.22375)	(0.30320)	(0.20913)	**(0.20746)**
		200	0.956	0.835	0.961	0.982	0.960	0.961
			(0.15725)	(0.15500)	(0.15694)	(0.24848)	(0.15056)	**(0.14998)**
	0.3	20	0.942	0.809	0.949	0.853	0.975	0.972
			(0.65285)	(0.66301)	(0.66654)	(0.69342)	(0.48033)	**(0.46968)**
		50	0.941	0.819	0.954	0.891	0.969	0.957
			(0.39052)	(0.39504)	(0.39129)	(0.44449)	(0.34196)	**(0.33757)**
		70	0.953	0.840	0.957	0.900	0.972	0.965
			(0.32924)	(0.32122)	(0.32990)	(0.38178)	(0.29786)	**(0.29533)**
		100	0.949	0.849	0.949	0.900	0.956	0.954
			(0.27306)	(0.27795)	(0.27347)	(0.32452)	(0.25264)	**(0.25109)**
		200	0.945	0.826	0.950	0.928	0.952	0.953
			(0.19182)	(0.19437)	(0.19191)	(0.23913)	(0.18261)	**(0.18204)**

TABLE 4-5 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the CV under ZITR with $\mu = 1$ and $\lambda = 0.5$

λ	δ	n	Coverage probability					
			PB	BS	GCI	AN	MCMC	HPD
0.5	0.1	20	0.883	0.760	0.904	0.778	0.911	0.957
			(0.39090)	(0.38772)	(0.39499)	(0.41271)	(0.35413)	**(0.34415)**
		50	0.910	0.791	0.945	0.882	0.943	0.959
			(0.25141)	(0.25251)	(0.24815)	(0.30001)	(0.23427)	**(0.22969)**
		70	0.919	0.814	0.935	0.927	0.925	0.941
			(0.21483)	(0.21242)	(0.21259)	(0.27367)	(0.20136)	(0.19805)
		100	0.918	0.791	0.934	0.943	0.942	0.942
			(0.17689)	(0.17990)	(0.17561)	(0.25118)	(0.16822)	(0.16613)
		200	0.937	0.817	0.950	0.981	0.934	0.937
			(0.12619)	(0.12601)	**(0.12561)**	(0.21408)	(0.12196)	(0.12102)
	0.2	20	0.916	0.802	0.924	0.816	0.980	0.984
			(0.53102)	(0.54187)	(0.52431)	(0.55153)	(0.41872)	**(0.40867)**

TABLE 4-5 (CON)

λ	δ	n	Coverage probability					
			PB	BS	GCI	AN	MCMC	HPD
		50	0.937	0.819	0.950	0.875	0.964	0.967
			(0.32369)	(0.31685)	(0.32074)	(0.36495)	(0.28869)	**(0.28455)**
		70	0.937	0.823	0.945	0.884	0.954	0.957
			(0.27413)	(0.27076)	(0.27262)	(0.31567)	(0.25127)	**(0.24864)**
		100	0.945	0.836	0.954	0.909	0.960	0.957
			(0.22745)	(0.22850)	(0.22647)	(0.27251)	(0.21323)	**(0.21169)**
		200	0.947	0.816	0.957	0.929	0.962	0.958
			(0.16002)	(0.15588)	(0.15942)	(0.20843)	(0.15384)	**(0.15330)**
	0.3	20	0.919	0.837	0.941	0.845	0.969	0.964
			(0.64719)	(0.64873)	(0.64669)	(0.69071)	(0.48009)	**(0.46997)**
		50	0.937	0.835	0.947	0.866	0.959	0.949
			(0.39298)	(0.39328)	(0.39218)	(0.44498)	**(0.34417)**	(0.33991)
		70	0.940	0.834	0.947	0.887	0.953	0.949
			(0.33057)	(0.32679)	(0.32986)	(0.37767)	**(0.29952)**	(0.29713)
		100	0.952	0.830	0.954	0.901	0.963	0.956
			(0.27325)	(0.27694)	(0.27235)	(0.31217)	(0.25305)	**(0.25156)**
		200	0.943	0.828	0.947	0.912	0.953	0.949
			(0.19250)	(0.19382)	(0.19233)	(0.22291)	**(0.18360)**	(0.18307)

TABLE 4-6 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the CV under ZITR with $\mu = 1$ and $\lambda = 1$

λ	δ	n	Coverage probability					
			PB	BS	GCI	AN	MCMC	HPD
1	0.1	20	0.878	0.756	0.880	0.741	0.948	0.971
			(0.39843)	(0.39935)	(0.39407)	(0.37834)	(0.35838)	**(0.34877)**
		50	0.912	0.816	0.945	0.830	0.945	0.959
			(0.26122)	(0.26147)	(0.25605)	(0.27157)	(0.24181)	**(0.23740)**
		70	0.921	0.806	0.952	0.842	0.946	0.953
			(0.22190)	(0.21618)	(0.21834)	(0.24193)	(0.20749)	**(0.20426)**
		100	0.936	0.834	0.942	0.878	0.947	0.949
			(0.18606)	(0.18670)	(0.18365)	(0.21436)	(0.17589)	(0.17384)
		200	0.942	0.819	0.951	0.923	0.939	0.944
			(0.13078)	(0.12960)	**(0.13027)**	(0.17573)	(0.12664)	(0.12578)
	0.2	20	0.897	0.791	0.910	0.807	0.964	0.970
			(0.53584)	(0.53642)	(0.52414)	(0.54117)	(0.42166)	**(0.41205)**
		50	0.932	0.848	0.939	0.840	0.957	0.962
			(0.33338)	(0.33758)	(0.32920)	(0.35913)	(0.29618)	**(0.29221)**
		70	0.947	0.831	0.947	0.863	0.956	0.964
			(0.28071)	(0.27980)	(0.27813)	(0.30766)	(0.25723)	**(0.25469)**
		100	0.936	0.812	0.940	0.857	0.949	0.955
			(0.23221)	(0.23149)	(0.23075)	(0.25797)	(0.21764)	**(0.21610)**
		200	0.954	0.855	0.958	0.895	0.965	0.961
			(0.16305)	(0.16300)	(0.16246)	(0.18873)	(0.15717)	**(0.15665)**
	0.3	20	0.915	0.813	0.938	0.824	0.960	0.956
			(0.64688)	(0.64870)	(0.64087)	(0.68264)	(0.48019)	**(0.47018)**
		50	0.953	0.848	0.955	0.870	0.963	0.959
			(0.39829)	(0.39889)	(0.39553)	(0.44928)	(0.34811)	**(0.34398)**
		70	0.936	0.819	0.942	0.895	0.961	0.946
			(0.33025)	(0.33071)	(0.32908)	(0.37740)	**(0.29964)**	(0.29735)
		100	0.953	0.834	0.962	0.910	0.968	0.961
			(0.27675)	(0.27385)	(0.27588)	(0.31852)	(0.25652)	**(0.25509)**
		200	0.935	0.837	0.938	0.893	0.942	0.940
			(0.19391)	(0.19234)	(0.19360)	(0.22554)	(0.18531)	(0.18481)

TABLE 4-7 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference of Two CVs under the ZITR Distribution with $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 0.2)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.2:0.2	0.1:0.1	20:20	0.93	0.850	0.957	0.957	0.983	0.989	0.983
			(0.54611)	(0.56021)	(0.62506)	(0.61977)	(0.95328)	(0.41789)	**(0.41630)**
		20:50	0.904	0.805	0.958	0.958	0.989	0.986	0.989
			(0.44910)	(0.44855)	(0.50021)	(0.49716)	(0.88093)	(0.40481)	**(0.40191)**
		20:70	0.901	0.817	0.963	0.960	0.994	0.990	0.991
			(0.43359)	(0.43220)	(0.48440)	(0.48172)	(0.87665)	(0.38863)	**(0.38463)**
		20:100	0.881	0.787	0.944	0.945	0.991	0.983	0.988
			(0.41279)	(0.40753)	(0.46165)	(0.45960)	(0.85112)	(0.37187)	**(0.36691)**
		50:50	0.936	0.852	0.945	0.942	0.999	0.962	0.962
			(0.33836)	(0.34912)	(0.35583)	(0.35447)	(0.83334)	(0.28961)	**(0.28871)**
		50:70	0.948	0.825	0.955	0.955	0.999	0.972	0.973
			(0.31292)	(0.31857)	(0.32708)	(0.32597)	(0.81744)	(0.29519)	**(0.29419)**
		50:100	0.926	0.810	0.940	0.942	0.999	0.965	0.967
			(0.29165)	(0.28371)	(0.30347)	(0.30259)	(0.80576)	(0.27509)	**(0.27378)**
		70:70	0.947	0.824	0.955	0.955	0.998	0.955	0.954
			(0.28265)	(0.28214)	(0.29314)	(0.29230)	(0.80921)	(0.24691)	**(0.24615)**
		70:100	0.940	0.819	0.951	0.952	1.000	0.969	0.969
			(0.26122)	(0.26383)	(0.26970)	(0.26908)	(0.80112)	(0.24925)	**(0.24845)**
		100:100	0.948	0.842	0.950	0.948	0.999	0.954	0.955
			(0.23646)	(0.23534)	(0.24269)	(0.24209)	(0.78546)	(0.21409)	**(0.21349)**
		200:200	0.933	0.807	0.947	0.946	1.000	0.951	0.945
			(0.16532)	(0.16327)	(0.16799)	(0.16788)	(0.75730)	**(0.15703)**	(0.15662)
	0.1:0.3	20:20	0.951	0.843	0.960	0.959	0.949	0.924	0.921
			(0.75722)	(0.76189)	(0.84406)	(0.83974)	(1.04946)	(0.56725)	(0.56432)
		20:50	0.936	0.834	0.946	0.948	0.973	0.948	0.955
			(0.54886)	(0.55395)	(0.59922)	(0.59556)	(0.89237)	(0.48056)	**(0.47879)**
		20:70	0.917	0.829	0.958	0.960	0.976	0.971	0.977
			(0.50690)	(0.50292)	(0.55190)	(0.54954)	(0.85351)	(0.45200)	**(0.44990)**
		20:100	0.904	0.813	0.949	0.948	0.970	0.965	0.978
			(0.47379)	(0.46728)	(0.51901)	(0.51680)	(0.83224)	(0.42355)	**(0.42053)**
		50:50	0.945	0.833	0.953	0.951	0.984	0.950	0.947
			(0.46108)	(0.45153)	(0.47911)	(0.47815)	(0.83221)	**(0.40711)**	(0.40533)
		50:70	0.937	0.823	0.940	0.940	0.995	0.941	0.942
			(0.40939)	(0.41348)	(0.42170)	(0.42109)	**(0.80018)**	(0.37252)	(0.37137)
		50:100	0.936	0.827	0.940	0.940	0.992	0.944	0.950
			(0.36364)	(0.36420)	(0.37519)	(0.37400)	(0.78039)	(0.33756)	**(0.33659)**
		70:70	0.952	0.820	0.954	0.955	0.997	0.947	0.946
			(0.38605)	(0.39015)	(0.39664)	**(0.39580)**	(0.79547)	(0.35190)	(0.35062)
		70:100	0.939	0.849	0.941	0.944	0.998	0.949	0.951
			(0.34093)	(0.34726)	(0.34914)	(0.34858)	(0.77323)	(0.31720)	**(0.31628)**
		100:100	0.948	0.847	0.952	0.952	0.999	0.949	0.950
			(0.32188)	(0.32223)	(0.32765)	(0.32727)	(0.75423)	(0.30026)	**(0.29924)**
		200:200	0.940	0.832	0.939	0.941	1.000	0.936	0.933
			(0.22498)	(0.22511)	(0.22707)	(0.22678)	**(0.70655)**	(0.21708)	(0.21643)
	0.3:0.3	20:20	0.934	0.870	0.944	0.940	0.926	0.970	0.970
			(0.93437)	(0.93813)	(1.03460)	(1.02971)	(1.13382)	(0.60492)	**(0.60269)**
		20:50	0.940	0.850	0.947	0.950	0.931	0.960	0.963
			(0.76232)	(0.75888)	(0.82534)	(0.82232)	(0.99256)	(0.58632)	**(0.58301)**
		20:70	0.935	0.849	0.950	0.952	0.944	0.962	0.959
			(0.72833)	(0.71723)	(0.78941)	(0.78781)	(0.97195)	(0.55925)	**(0.55485)**
		20:100	0.931	0.835	0.953	0.952	0.943	0.955	0.949
			(0.70921)	(0.70550)	(0.76962)	(0.76767)	(0.94830)	**(0.54073)**	(0.53525)
		50:50	0.941	0.827	0.948	0.945	0.951	0.954	0.952
			(0.55988)	(0.56168)	(0.57729)	(0.57630)	(0.85220)	(0.44920)	**(0.44788)**
		50:70	0.932	0.827	0.936	0.931	0.974	0.948	0.951
			(0.51791)	(0.51138)	(0.53267)	(0.53160)	(0.81589)	(0.45413)	**(0.45266)**

TABLE 4-7 (CON)

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability						
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		50:100	0.951	0.844	0.951	0.952	0.973	0.964	0.968
			(0.48431)	(0.48938)	(0.49690)	(0.49647)	(0.78451)	(0.42902)	**(0.42723)**
		70:70	0.936	0.832	0.936	0.940	0.973	0.950	0.948
			(0.46632)	(0.46422)	(0.47885)	(0.47774)	(0.78479)	**(0.39030)**	(0.38925)
		70:100	0.951	0.860	0.957	0.960	0.985	0.962	0.960
			(0.43140)	(0.43071)	(0.44013)	(0.43964)	(0.75285)	(0.39168)	**(0.39051)**
		100:100	0.939	0.832	0.945	0.943	0.988	0.947	0.947
			(0.38713)	(0.38834)	(0.39454)	(0.39441)	**(0.73106)**	(0.33777)	(0.33683)
		200:200	0.938	0.832	0.940	0.938	1.000	0.940	0.943
			(0.27191)	(0.27437)	(0.27447)	(0.27429)	**(0.66483)**	(0.25174)	(0.25107)

TABLE 4-8 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference of Two CVs under the ZITR Distribution with $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.2:1	0.1:0.1	20:20	0.923	0.825	0.944	0.947	0.977	0.968	0.966
			(0.53846)	(0.52584)	(0.60303)	(0.59789)	(0.90611)	(0.42196)	**(0.42041)**
		20:50	0.919	0.832	0.950	0.954	0.989	0.985	0.987
			(0.45476)	(0.45805)	(0.50195)	(0.49928)	(0.85147)	(0.41263)	**(0.40989)**
		20:70	0.897	0.783	0.945	0.945	0.984	0.989	0.991
			(0.43145)	(0.42330)	(0.47812)	(0.47510)	(0.83035)	(0.39193)	**(0.38830)**
		20:100	0.883	0.797	0.953	0.954	0.990	0.987	0.990
			(0.41342)	(0.41654)	(0.46261)	(0.46098)	(0.83046)	(0.37500)	**(0.37026)**
		50:50	0.931	0.806	0.940	0.944	1.000	0.969	0.971
			(0.33727)	(0.32428)	(0.35206)	(0.35076)	(0.80417)	(0.29395)	**(0.29298)**
		50:70	0.934	0.821	0.949	0.949	0.995	0.963	0.967
			(0.31316)	(0.31383)	(0.32508)	(0.32384)	(0.78449)	(0.30041)	**(0.29944)**
		50:100	0.921	0.813	0.936	0.937	0.998	0.966	0.963
			(0.29133)	(0.28717)	(0.30213)	(0.30132)	(0.76919)	(0.27900)	**(0.27779)**
		70:70	0.940	0.822	0.947	0.946	1.000	0.953	0.953
			(0.28341)	(0.27868)	(0.29162)	(0.29084)	(0.77505)	(0.25667)	**(0.25591)**
		70:100	0.938	0.823	0.948	0.950	1.000	0.959	0.960
			(0.26216)	(0.26441)	(0.26977)	(0.26897)	(0.75747)	(0.25418)	**(0.25339)**
		100:100	0.942	0.841	0.955	0.953	1.000	0.965	0.967
			(0.23784)	(0.23891)	(0.24214)	(0.24172)	(0.74867)	(0.21974)	**(0.21911)**
		200:200	0.948	0.835	0.959	0.960	1.000	0.964	0.963
			(0.16613)	(0.16483)	(0.16790)	(0.16781)	(0.71350)	(0.15748)	**(0.15705)**
	0.1:0.3	20:20	0.936	0.844	0.948	0.946	0.949	0.922	0.925
			(0.76652)	(0.78374)	(0.83298)	(0.82878)	(1.06451)	(0.57611)	(0.57314)
		20:50	0.932	0.824	0.952	0.951	0.969	0.974	0.976
			(0.54726)	(0.54106)	(0.59241)	(0.58934)	(0.86517)	(0.48500)	**(0.48331)**
		20:70	0.913	0.820	0.944	0.942	0.963	0.956	0.963
			(0.50937)	(0.51642)	(0.55303)	(0.54967)	(0.82445)	(0.45662)	**(0.45442)**
		20:100	0.903	0.789	0.949	0.949	0.970	0.969	0.976
			(0.47173)	(0.46674)	(0.51556)	(0.51260)	(0.79562)	(0.42628)	**(0.42340)**
		50:50	0.932	0.834	0.934	0.935	0.981	0.947	0.947
			(0.46214)	(0.45543)	(0.47496)	(0.47415)	**(0.82628)**	(0.41367)	(0.41191)
		50:70	0.950	0.821	0.953	0.955	0.990	0.957	0.958
			(0.40790)	(0.40653)	(0.41917)	(0.41837)	(0.77904)	(0.37658)	**(0.37548)**
		50:100	0.954	0.822	0.942	0.942	0.997	0.956	0.957
			(0.36249)	(0.35211)	(0.37244)	(0.37158)	(0.74770)	(0.33965)	**(0.33865)**
		70:70	0.937	0.828	0.944	0.944	0.993	0.950	0.949
			(0.38629)	(0.38769)	(0.39406)	(0.39358)	(0.76485)	**(0.35705)**	(0.35574)

TABLE 4-8 (CON)

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		70:100	0.949	0.842	0.953	0.952	0.996	0.956	0.955
			(0.33952)	(0.34380)	(0.34618)	(0.34565)	(0.73545)	(0.31974)	**(0.31881)**
		100:100	0.949	0.841	0.949	0.950	0.996	0.955	0.954
			(0.32037)	(0.31817)	(0.32465)	(0.32445)	(0.72390)	(0.30265)	**(0.30162)**
		200:200	0.941	0.819	0.938	0.942	0.999	0.935	0.932
			(0.22443)	(0.22514)	(0.22612)	(0.22602)	**(0.66100)**	(0.21810)	(0.21745)
	0.3:0.3	20:20	0.943	0.854	0.950	0.951	0.935	0.967	0.970
			(0.93295)	(0.93010)	(1.01796)	(1.01378)	(1.20351)	(0.60965)	**(0.60756)**
		20:50	0.938	0.823	0.945	0.945	0.947	0.967	0.969
			(0.76185)	(0.73234)	(0.81886)	(0.81765)	(0.98687)	(0.58838)	**(0.58504)**
		20:70	0.935	0.828	0.945	0.945	0.936	0.949	0.944
			(0.73437)	(0.72166)	(0.79300)	(0.79087)	(0.95722)	(0.56500)	(0.56056)
		20:100	0.927	0.832	0.952	0.954	0.938	0.941	0.929
			(0.70312)	(0.70936)	(0.76310)	**(0.76106)**	(0.92375)	(0.53871)	(0.53341)
		50:50	0.945	0.835	0.949	0.950	0.972	0.954	0.960
			(0.55816)	(0.55238)	(0.57311)	(0.57220)	(0.84303)	(0.45761)	**(0.45620)**
		50:70	0.955	0.837	0.959	0.961	0.970	0.961	0.964
			(0.51327)	(0.51126)	(0.52675)	(0.52643)	(0.79819)	(0.45540)	**(0.45398)**
		50:100	0.941	0.831	0.945	0.942	0.963	0.951	0.952
			(0.48080)	(0.48669)	(0.49344)	(0.49247)	(0.75607)	(0.42892)	**(0.42713)**
		70:70	0.942	0.833	0.942	0.942	0.972	0.946	0.950
			(0.46550)	(0.46656)	(0.47440)	(0.47392)	(0.76157)	(0.39795)	**(0.39682)**
		70:100	0.953	0.843	0.954	0.954	0.973	0.954	0.956
			(0.42949)	(0.42529)	(0.43790)	(0.43693)	(0.72644)	(0.39307)	**(0.39188)**
		100:100	0.944	0.834	0.950	0.950	0.983	0.950	0.955
			(0.38785)	(0.39225)	(0.39365)	(0.39302)	(0.69665)	(0.34405)	**(0.34312)**
		200:200	0.957	0.827	0.961	0.961	0.998	0.960	0.959
			(0.27046)	(0.26835)	(0.27297)	(0.27291)	(0.61600)	(0.25142)	**(0.25076)**

TABLE 4-9 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference of Two CVs under the ZITR Distribution with $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (1,1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
1:1	0.1:0.1	20:20	0.923	0.832	0.947	0.947	0.973	0.992	0.993
			(0.55167)	(0.54289)	(0.60164)	(0.59583)	(0.85541)	(0.43394)	**(0.43235)**
		20:50	0.901	0.815	0.946	0.948	0.982	0.989	0.993
			(0.45070)	(0.45808)	(0.48359)	(0.48077)	(0.79935)	(0.41971)	**(0.41703)**
		20:70	0.887	0.797	0.947	0.951	0.989	0.984	0.989
			(0.42773)	(0.42851)	(0.46146)	(0.45912)	(0.79274)	(0.39959)	**(0.39588)**
		20:100	0.884	0.798	0.958	0.958	0.987	0.987	0.997
			(0.41129)	(0.41484)	(0.44402)	(0.44211)	(0.78377)	(0.38251)	**(0.37791)**
		50:50	0.933	0.837	0.936	0.938	0.995	0.960	0.963
			(0.34023)	(0.33965)	(0.35018)	(0.34889)	(0.74496)	(0.30391)	**(0.30299)**
		50:70	0.936	0.822	0.944	0.945	0.998	0.969	0.970
			(0.31247)	(0.30797)	(0.32208)	(0.32094)	(0.73565)	(0.30745)	**(0.30647)**
		50:100	0.923	0.819	0.941	0.942	0.999	0.960	0.962
			(0.29014)	(0.29246)	(0.29781)	(0.29688)	(0.71848)	(0.28580)	**(0.28456)**
		70:70	0.956	0.855	0.963	0.960	0.995	0.978	0.981
			(0.28554)	(0.28499)	(0.29147)	(0.29096)	(0.71848)	(0.26376)	**(0.26301)**

TABLE 4-9 (CON)

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		70:100	0.942	0.819	0.947	0.949	0.998	0.967	0.971
			(0.26108)	(0.26184)	(0.26624)	(0.26544)	(0.70122)	(0.25897)	**(0.25815)**
		100:100	0.924	0.815	0.930	0.928	1.000	0.949	0.949
			(0.23698)	(0.23863)	(0.24075)	(0.24031)	**(0.69297)**	(0.22367)	(0.22305)
		200:200	0.946	0.831	0.947	0.946	1.000	0.948	0.952
			(0.16668)	(0.16669)	(0.16790)	(0.16774)	(0.65717)	(0.15923)	**(0.15882)**
	0.1:0.3	20:20	0.923	0.841	0.937	0.938	0.937	0.916	0.915
			(0.76389)	(0.77342)	(0.82502)	(0.82155)	(0.98795)	(0.58230)	(0.57937)
		20:50	0.923	0.812	0.937	0.939	0.934	0.947	0.951
			(0.54973)	(0.54720)	(0.58022)	(0.57742)	(0.80217)	(0.49197)	**(0.49031)**
		20:70	0.900	0.817	0.934	0.938	0.970	0.938	0.949
			(0.50327)	(0.50027)	(0.53367)	(0.53097)	**(0.76881)**	(0.46054)	(0.45839)
		20:100	0.887	0.787	0.933	0.935	0.958	0.943	0.952
			(0.47113)	(0.47042)	(0.50156)	(0.49857)	(0.74614)	(0.43313)	**(0.43024)**
		50:50	0.935	0.814	0.942	0.941	0.972	0.937	0.933
			(0.45770)	(0.45442)	(0.46816)	(0.46742)	**(0.74424)**	(0.41584)	(0.41417)
		50:70	0.936	0.814	0.943	0.944	0.984	0.946	0.945
			(0.40805)	(0.40026)	(0.41678)	(0.41595)	**(0.70818)**	(0.38231)	(0.38122)
		50:100	0.939	0.826	0.951	0.950	0.984	0.950	0.952
			(0.36362)	(0.36317)	(0.37046)	(0.36959)	(0.67887)	(0.34686)	**(0.34589)**
		70:70	0.956	0.825	0.954	0.950	0.988	0.958	0.952
			(0.38845)	(0.38572)	(0.39473)	(0.39381)	(0.69771)	(0.36296)	**(0.36172)**
		70:100	0.938	0.835	0.940	0.941	0.990	0.951	0.950
			(0.33866)	(0.34037)	(0.34411)	(0.34365)	(0.66760)	(0.32388)	**(0.32297)**
		100:100	0.946	0.814	0.950	0.948	0.994	0.945	0.944
			(0.32055)	(0.31104)	**(0.32473)**	(0.32431)	(0.65376)	(0.30611)	(0.30514)
		200:200	0.931	0.833	0.934	0.932	1.000	0.933	0.933
			(0.22417)	(0.22168)	(0.22572)	(0.22566)	**(0.59771)**	(0.21885)	(0.21822)
	0.3:0.3	20:20	0.942	0.864	0.947	0.945	0.907	0.977	0.979
			(0.93711)	(0.94655)	(0.99934)	(0.99492)	(1.08285)	(0.61472)	**(0.61260)**
		20:50	0.942	0.838	0.954	0.955	0.928	0.971	0.973
			(0.75309)	(0.75982)	(0.79258)	(0.79123)	(0.91182)	(0.59042)	**(0.58714)**
		20:70	0.934	0.814	0.944	0.947	0.917	0.963	0.964
			(0.72832)	(0.72581)	(0.76862)	(0.76735)	(0.89505)	(0.56746)	**(0.56301)**
		20:100	0.939	0.823	0.950	0.949	0.909	0.968	0.966
			(0.70561)	(0.70561)	(0.74308)	(0.74191)	(0.86919)	(0.54553)	**(0.54029)**
		50:50	0.945	0.827	0.946	0.943	0.950	0.959	0.962
			(0.55240)	(0.54335)	(0.56452)	(0.56322)	(0.74526)	(0.45585)	**(0.45451)**
		50:70	0.945	0.838	0.946	0.946	0.935	0.963	0.966
			(0.51136)	(0.51149)	(0.52068)	(0.51969)	(0.71264)	(0.45847)	**(0.45704)**
		50:100	0.947	0.840	0.955	0.956	0.958	0.964	0.963
			(0.48008)	(0.47703)	(0.48837)	(0.48768)	(0.68397)	(0.43277)	**(0.43099)**
		70:70	0.943	0.839	0.945	0.943	0.948	0.953	0.956
			(0.46640)	(0.46759)	(0.47282)	(0.47250)	(0.67787)	(0.40723)	**(0.40609)**
		70:100	0.941	0.844	0.942	0.944	0.962	0.957	0.956
			(0.42715)	(0.42630)	(0.43269)	(0.43258)	(0.64463)	(0.39535)	**(0.39421)**
		100:100	0.946	0.841	0.944	0.945	0.965	0.950	0.948
			(0.38686)	(0.38911)	(0.39091)	(0.39095)	(0.61166)	**(0.34544)**	(0.34453)
		200:200	0.944	0.840	0.946	0.947	0.990	0.947	0.947
			(0.27118)	(0.27354)	(0.27272)	(0.27256)	**(0.52692)**	(0.25270)	(0.25204)

TABLE 4-10 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference of Two CVs under the ZITR Distribution with $(\mu_1,\mu_2)=(1,0.2)$, $(\lambda_1,\lambda_2)=(0.2,0.2)$, and $(\delta_1,\delta_2)=(0.1,0.1),(0.1,0.3),(0.3,0.3)$

λ_1,λ_2	$\delta_1:\delta_2$	Coverage probability							
		$n_1:n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.2:0.2	0.1:0.1	20:20	0.913	0.827	0.936	0.938	0.961	0.944	0.952
			(0.55807)	(0.55677)	(0.60519)	(0.60124)	(0.84880)	(0.42414)	**(0.42227)**
		20:50	0.901	0.810	0.945	0.948	0.977	0.957	0.969
			(0.44921)	(0.44718)	(0.46845)	(0.46604)	(0.74744)	(0.41245)	**(0.40944)**
		20:70	0.896	0.788	0.931	0.930	0.986	0.946	0.959
			(0.43659)	(0.43993)	(0.45139)	(0.44958)	(0.73464)	(0.39745)	**(0.39309)**
		20:100	0.879	0.792	0.931	0.933	0.981	0.950	0.964
			(0.42181)	(0.42413)	(0.43560)	(0.43435)	(0.72656)	(0.38386)	**(0.37853)**
		50:50	0.934	0.855	0.943	0.945	0.993	0.949	0.950
			(0.34191)	(0.34493)	(0.35130)	(0.34999)	(0.68076)	(0.29345)	**(0.29245)**
		50:70	0.944	0.820	0.946	0.948	0.996	0.963	0.965
			(0.31900)	(0.31135)	(0.32425)	(0.32325)	(0.66750)	(0.30101)	**(0.29989)**
		50:100	0.932	0.834	0.934	0.934	0.997	0.941	0.951
			(0.29725)	(0.29867)	(0.30108)	(0.30042)	(0.65995)	(0.28185)	**(0.28036)**
		70:70	0.944	0.833	0.945	0.945	0.995	0.961	0.965
			(0.28897)	(0.28816)	(0.29500)	(0.29401)	(0.65243)	(0.25419)	**(0.25338)**
		70:100	0.940	0.821	0.940	0.946	0.999	0.947	0.951
			(0.26641)	(0.26502)	(0.26955)	(0.26907)	(0.64031)	(0.25408)	**(0.25316)**
		100:100	0.948	0.832	0.955	0.955	1.000	0.953	0.959
			(0.24046)	(0.24095)	(0.24381)	(0.24326)	(0.63529)	(0.21668)	**(0.21607)**
		200:200	0.943	0.826	0.940	0.941	1.000	0.944	0.943
			(0.16823)	(0.16732)	(0.16993)	(0.16984)	**(0.60724)**	(0.15629)	(0.15588)
	0.1:0.3	20:20	0.942	0.859	0.951	0.952	0.927	0.884	0.882
			(0.77029)	(0.77101)	(0.83416)	**(0.83084)**	(0.95705)	(0.57705)	(0.57428)
		20:50	0.911	0.822	0.946	0.948	0.950	0.891	0.910
			(0.55337)	(0.55097)	(0.57178)	(0.56880)	**(0.75292)**	(0.48918)	(0.48748)
		20:70	0.906	0.814	0.937	0.937	0.946	0.887	0.907
			(0.51152)	(0.51258)	(0.52624)	(0.52422)	(0.72091)	(0.45915)	(0.45686)
		20:100	0.908	0.838	0.946	0.950	0.952	0.891	0.916
			(0.47537)	(0.48521)	(0.48845)	**(0.48578)**	(0.68800)	(0.43017)	(0.42711)
		50:50	0.941	0.837	0.949	0.949	0.966	0.913	0.913
			(0.46390)	(0.46898)	(0.47561)	(0.47443)	**(0.69437)**	(0.41132)	(0.40974)
		50:70	0.942	0.845	0.950	0.951	0.970	0.925	0.922
			(0.41420)	(0.41208)	**(0.42117)**	(0.42021)	(0.65838)	(0.37838)	(0.37727)
		50:100	0.949	0.834	0.949	0.948	0.979	0.930	0.933
			(0.36843)	(0.36186)	(0.37309)	(0.37235)	**(0.63004)**	(0.34304)	(0.34206)
		70:70	0.965	0.836	0.962	0.965	0.980	0.935	0.931
			(0.38948)	(0.38320)	(0.39645)	**(0.39581)**	(0.64739)	(0.35579)	(0.35453)
		70:100	0.934	0.842	0.939	0.939	0.983	0.914	0.914
			(0.34359)	(0.34351)	(0.34719)	(0.34693)	(0.61408)	(0.31950)	(0.31861)
		100:100	0.952	0.830	0.952	0.957	0.987	0.911	0.909
			(0.32487)	(0.32343)	(0.32931)	**(0.32889)**	(0.60382)	(0.30273)	(0.30176)
		200:200	0.953	0.848	0.959	0.957	0.999	0.941	0.939
			(0.22833)	(0.23135)	(0.22944)	**(0.22923)**	(0.54072)	(0.21872)	(0.21810)
	0.3:0.3	20:20	0.937	0.859	0.946	0.944	0.911	0.945	0.954
			(0.93556)	(0.92652)	(0.99252)	(0.98762)	(1.10503)	(0.61588)	**(0.61355)**
		20:50	0.927	0.822	0.932	0.932	0.898	0.965	0.968
			(0.76727)	(0.74933)	(0.78388)	(0.78258)	(0.93061)	(0.59126)	**(0.58795)**
		20:70	0.946	0.869	0.945	0.945	0.911	0.972	0.976
			(0.73315)	(0.73973)	(0.74689)	(0.74568)	(0.89838)	(0.56523)	**(0.56076)**
		20:100	0.925	0.835	0.929	0.928	0.919	0.975	0.970
			(0.71216)	(0.70345)	(0.72422)	(0.72307)	(0.88285)	(0.54430)	**(0.53881)**
		50:50	0.949	0.836	0.952	0.953	0.949	0.953	0.955
			(0.55933)	(0.56336)	(0.56962)	(0.56909)	(0.76297)	(0.45200)	**(0.45056)**

TABLE 4-10 (CON)

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		50:70	0.949	0.826	0.952	0.951	0.945	0.957	0.959
			(0.51331)	(0.50460)	(0.51968)	(0.51954)	(0.72704)	(0.45408)	**(0.45258)**
		50:100	0.938	0.838	0.940	0.941	0.952	0.956	0.956
			(0.48066)	(0.47590)	(0.48489)	(0.48425)	(0.69076)	(0.42810)	**(0.42635)**
		70:70	0.944	0.843	0.948	0.945	0.949	0.948	0.951
			(0.46718)	(0.46834)	(0.47371)	(0.47326)	(0.67752)	(0.40121)	**(0.40006)**
		70:100	0.945	0.830	0.945	0.947	0.957	0.958	0.960
			(0.43008)	(0.43105)	(0.43332)	(0.43330)	(0.64354)	(0.39177)	**(0.39059)**
		100:100	0.940	0.827	0.942	0.943	0.967	0.947	0.948
			(0.38652)	(0.38615)	(0.39009)	(0.39006)	**(0.61877)**	(0.33898)	(0.33805)
		200:200	0.950	0.822	0.947	0.951	0.987	0.954	0.957
			(0.27170)	(0.26964)	(0.27279)	(0.27301)	(0.53027)	(0.25176)	**(0.25110)**

TABLE 4-11 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference of Two CVs under the ZITR Distribution with $(\mu_1, \mu_2) = (1, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.2:1	0.1:0.1	20:20	0.925	0.842	0.948	0.948	0.972	0.982	0.983
			(0.54269)	(0.54221)	(0.57996)	(0.57554)	(0.81006)	(0.43041)	**(0.42872)**
		20:50	0.893	0.817	0.940	0.939	0.968	0.962	0.973
			(0.45796)	(0.45731)	(0.47192)	(0.46977)	(0.72255)	(0.42086)	**(0.41784)**
		20:70	0.884	0.803	0.938	0.940	0.977	0.962	0.973
			(0.43200)	(0.42893)	(0.44499)	(0.44276)	(0.69603)	(0.39883)	**(0.39492)**
		20:100	0.881	0.794	0.940	0.938	0.974	0.956	0.973
			(0.42152)	(0.42471)	(0.43385)	(0.43203)	(0.68526)	(0.38556)	**(0.38048)**
		50:50	0.955	0.830	0.959	0.960	0.996	0.974	0.973
			(0.34035)	(0.33913)	(0.34708)	(0.34590)	(0.65569)	(0.29947)	**(0.29856)**
		50:70	0.935	0.823	0.942	0.944	0.991	0.966	0.969
			(0.31879)	(0.31815)	(0.32255)	(0.32114)	(0.63647)	(0.30606)	**(0.30499)**
		50:100	0.935	0.835	0.945	0.947	0.998	0.966	0.969
			(0.29519)	(0.29224)	(0.29833)	(0.29722)	(0.61653)	(0.28376)	**(0.28236)**
		70:70	0.947	0.838	0.951	0.950	0.997	0.964	0.967
			(0.28801)	(0.28442)	(0.29181)	(0.29079)	(0.62096)	(0.26023)	**(0.25946)**
		70:100	0.934	0.818	0.935	0.939	0.996	0.953	0.958
			(0.26710)	(0.26440)	(0.26907)	(0.26835)	(0.60437)	(0.25799)	**(0.25715)**
		100:100	0.940	0.824	0.942	0.941	0.997	0.947	0.949
			(0.24060)	(0.24342)	(0.24285)	(0.24216)	**(0.59380)**	(0.22120)	(0.22056)
		200:200	0.946	0.819	0.949	0.951	1.000	0.945	0.946
			(0.16895)	(0.16749)	(0.16974)	**(0.16945)**	(0.54967)	(0.15755)	(0.15713)
	0.1:0.3	20:20	0.931	0.831	0.939	0.938	0.942	0.904	0.903
			(0.76387)	(0.75431)	(0.80536)	(0.80171)	(0.99053)	(0.58043)	(0.57770)
		20:50	0.920	0.837	0.947	0.945	0.947	0.924	0.931
			(0.55309)	(0.55113)	(0.56885)	(0.56643)	(0.74413)	(0.49407)	(0.49238)
		20:70	0.896	0.814	0.935	0.935	0.929	0.923	0.932
			(0.50667)	(0.50090)	(0.52125)	(0.51850)	(0.69955)	(0.46135)	(0.45909)
		20:100	0.884	0.786	0.933	0.934	0.945	0.914	0.933
			(0.47475)	(0.48006)	(0.48602)	(0.48391)	(0.65560)	(0.43310)	(0.43003)
		50:50	0.945	0.836	0.950	0.948	0.966	0.923	0.919
			(0.46136)	(0.45588)	**(0.46862)**	(0.46774)	(0.69040)	(0.41473)	(0.41314)
		50:70	0.936	0.818	0.940	0.942	0.974	0.925	0.924
			(0.40833)	(0.40965)	(0.41293)	(0.41209)	**(0.63471)**	(0.37852)	(0.37741)
		50:100	0.934	0.834	0.944	0.944	0.963	0.921	0.925
			(0.36656)	(0.36296)	(0.37026)	(0.36931)	**(0.59526)**	(0.34585)	(0.34487)

TABLE 4-11 (CON)

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		70:70	0.951	0.834	0.955	0.957	0.968	0.935	0.931
			(0.38813)	(0.38700)	**(0.39194)**	(0.39174)	(0.62287)	(0.35885)	(0.35762)
		70:100	0.937	0.839	0.938	0.937	0.981	0.939	0.938
			(0.34383)	(0.34214)	(0.34676)	(0.34621)	**(0.57513)**	(0.32373)	(0.32285)
		100:100	0.945	0.820	0.945	0.945	0.984	0.945	0.943
			(0.32278)	(0.31448)	(0.32539)	(0.32491)	**(0.56551)**	(0.30457)	(0.30362)
		200:200	0.955	0.834	0.958	0.961	0.995	0.941	0.936
			(0.22613)	(0.22306)	(0.22746)	**(0.22721)**	(0.48619)	(0.21881)	(0.21816)
	0.3:0.3	20:20	0.948	0.837	0.950	0.950	0.943	0.980	0.984
			(0.93754)	(0.92908)	(0.97775)	(0.97428)	(1.22114)	(0.61732)	**(0.61508)**
		20:50	0.941	0.859	0.952	0.951	0.920	0.974	0.980
			(0.77190)	(0.77723)	(0.78465)	(0.78303)	(0.95566)	(0.59632)	**(0.59279)**
		20:70	0.932	0.837	0.938	0.939	0.919	0.974	0.976
			(0.73129)	(0.71714)	(0.74349)	(0.74233)	(0.90435)	(0.56758)	**(0.56324)**
		20:100	0.938	0.830	0.943	0.941	0.903	0.971	0.971
			(0.70811)	(0.70697)	(0.71919)	(0.71842)	(0.85603)	(0.54456)	**(0.53919)**
		50:50	0.942	0.826	0.950	0.951	0.961	0.954	0.957
			(0.55642)	(0.54336)	(0.56418)	(0.56322)	(0.79343)	(0.45341)	**(0.45207)**
		50:70	0.953	0.823	0.953	0.950	0.943	0.968	0.970
			(0.51291)	(0.50378)	(0.51765)	(0.51705)	(0.72659)	(0.45712)	**(0.45569)**
		50:100	0.945	0.811	0.949	0.945	0.954	0.960	0.967
			(0.47994)	(0.47200)	(0.48306)	(0.48270)	(0.67792)	(0.43028)	**(0.42850)**
		70:70	0.952	0.842	0.949	0.950	0.963	0.965	0.967
			(0.46727)	(0.46959)	(0.47186)	(0.47122)	(0.69361)	(0.40274)	**(0.40162)**
		70:100	0.938	0.823	0.941	0.937	0.958	0.944	0.944
			(0.42689)	(0.42173)	(0.43008)	(0.42981)	**(0.63335)**	(0.39270)	(0.39152)
		100:100	0.956	0.861	0.957	0.957	0.981	0.960	0.962
			(0.38636)	(0.39499)	(0.38916)	(0.38850)	(0.60571)	(0.34031)	**(0.33942)**
		200:200	0.956	0.861	0.957	0.957	0.981	0.960	0.962
			(0.38636)	(0.39499)	(0.38916)	(0.38850)	(0.60571)	(0.34031)	**(0.33942)**

TABLE 4-12 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for the Difference of Two CVs under the ZITR Distribution with $(\mu_1, \mu_2) = (1, 0.2)$, $(\lambda_1, \lambda_2) = (1,1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
1:1	0.1:0.1	20:20	0.908	0.848	0.928	0.928	0.936	0.981	0.986
			(0.56405)	(0.56385)	(0.58646)	(0.58204)	(0.72991)	(0.43331)	**(0.43173)**
		20:50	0.879	0.798	0.924	0.930	0.948	0.975	0.981
			(0.47324)	(0.47190)	(0.47196)	(0.46962)	(0.66313)	(0.43303)	**(0.43004)**
		20:70	0.874	0.772	0.910	0.910	0.955	0.970	0.979
			(0.45532)	(0.46305)	(0.45181)	(0.44948)	(0.65181)	(0.41549)	**(0.41133)**
		20:100	0.885	0.792	0.910	0.910	0.946	0.954	0.970
			(0.44578)	(0.44870)	(0.43899)	(0.43740)	(0.63882)	(0.40099)	**(0.39576)**
		50:50	0.939	0.819	0.944	0.940	0.976	0.965	0.965
			(0.35446)	(0.35655)	(0.35704)	(0.35563)	(0.59195)	(0.31002)	**(0.30905)**
		50:70	0.946	0.833	0.954	0.952	0.984	0.974	0.978
			(0.33074)	(0.33699)	(0.33110)	(0.32977)	(0.57951)	(0.31559)	**(0.31444)**
		50:100	0.930	0.832	0.943	0.943	0.987	0.960	0.966
			(0.31340)	(0.32075)	(0.31101)	(0.31020)	(0.55952)	(0.29661)	**(0.29515)**
		70:70	0.946	0.842	0.946	0.949	0.988	0.964	0.969
			(0.30013)	(0.30309)	(0.30159)	(0.30081)	(0.56790)	(0.26782)	**(0.26698)**
		70:100	0.932	0.838	0.940	0.938	0.994	0.966	0.969
			(0.27829)	(0.28055)	(0.27820)	(0.27737)	(0.55202)	(0.26759)	**(0.26666)**
		100:100	0.950	0.843	0.950	0.951	0.997	0.961	0.959
			(0.25062)	(0.24892)	(0.25105)	(0.25061)	(0.53271)	(0.22836)	**(0.22769)**

TABLE 4-12 (CON)

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		200:200	0.949	0.846	0.954	0.955	0.999	0.954	0.956
			(0.17566)	(0.17388)	(0.17569)	(0.17553)	(0.49384)	(0.16321)	**(0.16274)**
	0.1:0.3	20:20	0.935	0.845	0.948	0.952	0.904	0.902	0.903
			(0.77686)	(0.78084)	(0.80988)	**(0.80675)**	(0.87352)	(0.58949)	(0.58691)
		20:50	0.903	0.815	0.925	0.929	0.901	0.914	0.921
			(0.57201)	(0.56693)	(0.57326)	(0.57061)	(0.66997)	(0.50625)	(0.50446)
		20:70	0.892	0.807	0.929	0.927	0.903	0.922	0.935
			(0.52721)	(0.53608)	(0.52544)	(0.52284)	(0.62188)	(0.47424)	(0.47185)
		20:100	0.900	0.801	0.932	0.933	0.898	0.919	0.937
			(0.49996)	(0.49210)	(0.49416)	(0.49230)	(0.59233)	(0.44796)	(0.44480)
		50:50	0.939	0.829	0.942	0.943	0.925	0.939	0.938
			(0.47398)	(0.46427)	(0.47836)	(0.47785)	(0.60325)	(0.42364)	(0.42210)
		50:70	0.940	0.835	0.940	0.945	0.942	0.938	0.939
			(0.42142)	(0.42289)	(0.42269)	(0.42215)	(0.55316)	(0.38875)	(0.38768)
		50:100	0.943	0.816	0.943	0.939	0.936	0.927	0.931
			(0.38081)	(0.37295)	(0.38011)	(0.37911)	(0.51365)	(0.35554)	(0.35447)
		70:70	0.942	0.839	0.948	0.947	0.945	0.939	0.937
			(0.39779)	(0.39740)	(0.40120)	(0.40046)	(0.53697)	(0.36792)	(0.36674)
		70:100	0.949	0.828	0.952	0.950	0.938	0.941	0.940
			(0.35336)	(0.35119)	(0.35383)	**(0.35366)**	(0.49777)	(0.33203)	(0.33115)
		100:100	0.943	0.832	0.950	0.950	0.958	0.935	0.931
			(0.32971)	(0.32639)	(0.33108)	**(0.33096)**	(0.48401)	(0.31136)	(0.31040)
		200:200	0.944	0.839	0.943	0.944	0.982	0.943	0.939
			(0.23244)	(0.23252)	(0.23318)	(0.23294)	**(0.41202)**	(0.22503)	(0.22439)
	0.3:0.3	20:20	0.933	0.861	0.946	0.947	0.907	0.975	0.976
			(0.93767)	(0.94050)	(0.96455)	(0.96148)	(1.07664)	(0.62586)	**(0.62353)**
		20:50	0.954	0.831	0.952	0.953	0.899	0.976	0.979
			(0.77000)	(0.74421)	(0.76851)	(0.76736)	(0.90044)	(0.59931)	**(0.59602)**
		20:70	0.930	0.834	0.937	0.938	0.912	0.968	0.970
			(0.74489)	(0.73419)	(0.73707)	(0.73671)	(0.86387)	(0.57463)	**(0.57035)**
		20:100	0.953	0.853	0.952	0.951	0.891	0.970	0.970
			(0.72321)	(0.72412)	(0.71641)	(0.71549)	(0.85214)	(0.55412)	**(0.54902)**
		50:50	0.956	0.855	0.959	0.957	0.938	0.968	0.972
			(0.55880)	(0.56189)	(0.56171)	(0.56195)	(0.71425)	(0.45198)	**(0.45067)**
		50:70	0.952	0.860	0.958	0.954	0.925	0.962	0.964
			(0.51897)	(0.51560)	(0.52132)	(0.52053)	(0.66759)	(0.46195)	**(0.46055)**
		50:100	0.954	0.849	0.958	0.958	0.928	0.963	0.962
			(0.48418)	(0.48928)	(0.48387)	(0.48344)	(0.63228)	(0.43348)	**(0.43178)**
		70:70	0.938	0.829	0.934	0.934	0.922	0.944	0.944
			(0.46860)	(0.46153)	(0.47136)	(0.47091)	(0.61902)	(0.40138)	(0.40019)
		70:100	0.938	0.826	0.941	0.944	0.937	0.947	0.951
			(0.43257)	(0.42980)	(0.43363)	(0.43360)	(0.58952)	(0.39709)	**(0.39594)**
		100:100	0.944	0.834	0.943	0.942	0.945	0.948	0.951
			(0.39091)	(0.39118)	(0.39169)	(0.39149)	(0.54732)	(0.34802)	**(0.34708)**
		200:200	0.945	0.850	0.947	0.949	0.972	0.947	0.945
			(0.27270)	(0.27460)	(0.27330)	(0.27353)	**(0.43888)**	(0.24921)	(0.24853)

TABLE 4-13 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for ratio of Two CVs under the ZITR Distribution with $(\mu_1,\mu_2)=(0.2,0.2)$, $(\lambda_1,\lambda_2)=(0.2,0.2)$, and $(\delta_1,\delta_2)=(0.1,0.1),(0.1,0.3),(0.3,0.3)$

λ_1,λ_2	$\delta_1:\delta_2$	$n_1:n_2$	Coverage probability						
			PB	BS	GCI	MOVER	AN	BCI	HPD
0.2:0.2	0.1:0.1	20:20	0.930	0.855	0.957	0.957	0.999	0.989	0.981
			(1.04826)	(1.08818)	(0.99795)	(0.99152)	(7.44508)	(0.67670)	**(0.66355)**
		20:50	0.904	0.798	0.958	0.958	1.000	0.986	0.986
			(0.77829)	(0.78033)	(0.82669)	(0.82519)	(6.80031)	(0.66126)	**(0.64678)**
		20:70	0.901	0.807	0.963	0.960	1.000	0.990	0.992
			(0.73981)	(0.74031)	(0.80175)	(0.80069)	(6.50132)	(0.63673)	**(0.62203)**
		20:100	0.881	0.777	0.944	0.945	1.000	0.983	0.987
			(0.69735)	(0.68897)	(0.76516)	(0.76463)	(6.05151)	(0.61059)	**(0.59571)**
		50:50	0.936	0.856	0.945	0.942	1.000	0.962	0.959
			(0.59699)	(0.61499)	(0.58408)	(0.58294)	(7.33827)	(0.47320)	**(0.46826)**
		50:70	0.948	0.822	0.955	0.955	1.000	0.972	0.971
			(0.53852)	(0.55186)	(0.53769)	(0.53667)	(6.19673)	(0.48107)	**(0.47543)**
		50:100	0.926	0.807	0.940	0.942	1.000	0.965	0.964
			(0.49740)	(0.48707)	(0.50353)	(0.50304)	(6.59437)	(0.45223)	**(0.44651)**
		70:70	0.947	0.830	0.955	0.955	1.000	0.955	0.953
			(0.48883)	(0.48823)	(0.48100)	(0.48006)	(6.20446)	(0.40283)	**(0.39943)**
		70:100	0.940	0.825	0.951	0.952	1.000	0.969	0.964
			(0.44341)	(0.44751)	(0.44355)	(0.44311)	(6.03022)	(0.40665)	**(0.40283)**
		100:100	0.948	0.840	0.950	0.948	1.000	0.954	0.952
			(0.40384)	(0.40259)	(0.39925)	(0.39868)	(5.85993)	(0.34955)	**(0.34715)**
		200:200	0.933	0.807	0.947	0.946	1.000	0.951	0.951
			(0.27867)	(0.27591)	(0.27786)	(0.27782)	(5.55541)	(0.25793)	**(0.25667)**
	0.1:0.3	20:20	0.940	0.839	0.959	0.955	0.994	0.923	0.937
			(0.74183)	(0.75364)	(0.74951)	**(0.74903)**	(3.73535)	(0.60827)	(0.59471)
		20:50	0.909	0.807	0.949	0.947	1.000	0.961	0.977
			(0.55448)	(0.56230)	(0.60287)	(0.60320)	(3.67821)	(0.50905)	**(0.49697)**
		20:70	0.883	0.779	0.959	0.961	1.000	0.974	0.979
			(0.51614)	(0.51134)	(0.56432)	(0.56562)	(3.27652)	(0.47571)	**(0.46445)**
		20:100	0.892	0.783	0.951	0.948	1.000	0.972	0.981
			(0.49956)	(0.49441)	(0.55035)	(0.55143)	(3.27634)	(0.45412)	**(0.44299)**
		50:50	0.938	0.827	0.948	0.946	1.000	0.950	0.958
			(0.43321)	(0.42390)	(0.43546)	(0.43513)	(3.26381)	(0.40207)	**(0.39704)**
		50:70	0.928	0.817	0.944	0.941	1.000	0.942	0.962
			(0.39326)	(0.39697)	(0.39765)	(0.39814)	(3.15251)	(0.37108)	**(0.36628)**
		50:100	0.924	0.813	0.940	0.940	1.000	0.949	0.959
			(0.35235)	(0.35302)	(0.36100)	(0.36061)	(3.11204)	(0.33601)	**(0.33145)**
		70:70	0.951	0.818	0.956	0.954	1.000	0.945	0.958
			(0.36267)	(0.36638)	(0.36303)	(0.36294)	(3.19366)	(0.34275)	**(0.33914)**
		70:100	0.938	0.833	0.934	0.935	1.000	0.946	0.944
			(0.32340)	(0.33200)	(0.32687)	(0.32689)	**(3.06683)**	(0.30907)	(0.30588)
		100:100	0.944	0.840	0.950	0.949	1.000	0.950	0.959
			(0.29800)	(0.29916)	(0.29825)	(0.29825)	(2.95615)	(0.28638)	**(0.28399)**
		200:200	0.939	0.816	0.941	0.940	1.000	0.939	0.944
			(0.20796)	(0.20861)	(0.20809)	(0.20798)	**(2.79270)**	(0.20353)	(0.20235)
	0.3:0.3	20:20	0.934	0.875	0.944	0.940	0.983	0.970	0.972
			(1.20345)	(1.22728)	(1.18231)	(1.18272)	(3.82819)	(0.77453)	**(0.75674)**
		20:50	0.940	0.843	0.947	0.950	1.000	0.960	0.947
			(0.89712)	(0.89787)	(0.94534)	(0.94642)	(3.19903)	**(0.68824)**	(0.67316)
		20:70	0.935	0.834	0.950	0.952	1.000	0.962	0.949
			(0.85635)	(0.84529)	(0.91108)	(0.91431)	(3.14375)	**(0.65360)**	(0.63902)
		20:100	0.931	0.834	0.953	0.952	0.998	0.955	0.943
			(0.82493)	(0.82419)	(0.88476)	(0.88596)	(3.14070)	**(0.62395)**	(0.61001)
		50:50	0.941	0.822	0.948	0.945	0.996	0.954	0.947
			(0.66504)	(0.67228)	(0.65747)	(0.65777)	(3.00758)	**(0.53700)**	(0.53002)
		50:70	0.932	0.825	0.936	0.931	1.000	0.948	0.947
			(0.61408)	(0.61150)	(0.61512)	(0.61486)	**(2.92412)**	(0.53763)	(0.53068)
		50:100	0.951	0.848	0.951	0.952	1.000	0.964	0.961
			(0.56711)	(0.57550)	(0.57243)	(0.57276)	(2.69026)	(0.50074)	**(0.49449)**

TABLE 4-13 (CON)

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability						
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		70:70	0.936	0.829	0.936	0.940	1.000	0.950	0.947
			(0.55050)	(0.55149)	(0.54885)	(0.54822)	(2.90048)	**(0.46278)**	(0.45811)
		70:100	0.951	0.861	0.957	0.960	1.000	0.962	0.960
			(0.50846)	(0.50923)	(0.50915)	(0.50935)	(2.78277)	(0.46108)	**(0.45660)**
		100:100	0.939	0.831	0.945	0.943	1.000	0.947	0.946
			(0.45420)	(0.45676)	(0.45265)	(0.45284)	**(2.70876)**	(0.39691)	(0.39369)
		200:200	0.938	0.839	0.940	0.938	1.000	0.940	0.937
			(0.31596)	(0.31945)	(0.31517)	(0.31514)	**(2.44075)**	(0.29222)	(0.29060)

TABLE 4-14 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for ratio of Two CVs under the ZITR Distribution with $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.2:1	0.1:0.1	20:20	0.917	0.826	0.942	0.945	0.999	0.968	0.949
			(1.16750)	(1.14676)	(1.09358)	(1.08646)	(11.51422)	**(0.71775)**	(0.70280)
		20:50	0.922	0.826	0.951	0.954	1.000	0.984	0.986
			(0.87160)	(0.88326)	(0.91837)	(0.91718)	(10.01841)	(0.72341)	**(0.70752)**
		20:70	0.904	0.779	0.947	0.946	1.000	0.989	0.987
			(0.80337)	(0.79184)	(0.86515)	(0.86369)	(9.93983)	(0.68852)	**(0.67266)**
		20:100	0.885	0.792	0.952	0.953	1.000	0.987	0.990
			(0.76435)	(0.77190)	(0.83838)	(0.83840)	(10.26779)	(0.66534)	**(0.64948)**
		50:50	0.930	0.809	0.939	0.944	1.000	0.967	0.954
			(0.67045)	(0.64900)	(0.65241)	(0.65077)	(10.65238)	(0.52090)	**(0.51531)**
		50:70	0.934	0.811	0.950	0.948	1.000	0.962	0.955
			(0.59344)	(0.59740)	(0.59038)	(0.58905)	(9.82943)	(0.52733)	**(0.52106)**
		50:100	0.923	0.808	0.937	0.936	1.000	0.964	0.960
			(0.54603)	(0.53740)	(0.55080)	(0.55038)	(9.74916)	(0.49615)	**(0.49008)**
		70:70	0.939	0.821	0.946	0.947	1.000	0.953	0.940
			(0.54666)	(0.53973)	(0.53493)	(0.53439)	(10.10730)	**(0.45469)**	(0.45076)
		70:100	0.936	0.824	0.946	0.946	1.000	0.960	0.956
			(0.49329)	(0.49869)	(0.49218)	(0.49161)	(9.57202)	(0.45209)	**(0.44785)**
		100:100	0.944	0.848	0.954	0.951	1.000	0.966	0.959
			(0.45410)	(0.45584)	(0.44607)	(0.44579)	(9.66435)	(0.39395)	**(0.39127)**
		200:200	0.952	0.836	0.961	0.963	1.000	0.964	0.958
			(0.31262)	(0.31053)	(0.31028)	(0.31027)	(9.31447)	(0.28682)	**(0.28541)**
	0.1:0.3	20:20	0.930	0.820	0.950	0.945	0.999	0.925	0.938
			(0.80958)	(0.83053)	(0.81063)	(0.81059)	(6.23656)	(0.63441)	(0.61945)
		20:50	0.913	0.798	0.951	0.950	1.000	0.981	0.985
			(0.58020)	(0.57394)	(0.62974)	(0.63059)	(5.12277)	(0.53011)	**(0.51750)**
		20:70	0.899	0.803	0.950	0.949	1.000	0.962	0.977
			(0.55284)	(0.56195)	(0.60377)	(0.60438)	(5.16143)	(0.50317)	**(0.49106)**
		20:100	0.872	0.780	0.941	0.939	1.000	0.973	0.979
			(0.52087)	(0.51712)	(0.57506)	(0.57494)	(4.96249)	(0.47569)	**(0.46409)**
		50:50	0.938	0.834	0.946	0.944	1.000	0.953	0.966
			(0.46443)	(0.45783)	(0.46421)	(0.46434)	(5.11965)	(0.42666)	**(0.42090)**
		50:70	0.940	0.813	0.950	0.948	1.000	0.961	0.970
			(0.41518)	(0.41568)	(0.42158)	(0.42173)	(4.95559)	(0.39112)	**(0.38589)**
		50:100	0.951	0.815	0.944	0.942	1.000	0.957	0.960
			(0.37976)	(0.36972)	(0.38861)	(0.38881)	(4.98458)	(0.36036)	**(0.35546)**
		70:70	0.929	0.824	0.944	0.941	1.000	0.950	0.957
			(0.38315)	(0.38403)	(0.38349)	(0.38360)	(4.85726)	(0.36123)	**(0.35727)**
		70:100	0.948	0.844	0.955	0.953	1.000	0.955	0.960
			(0.34612)	(0.35175)	(0.35010)	(0.35015)	(4.82487)	(0.33061)	**(0.32712)**
		100:100	0.946	0.845	0.949	0.947	1.000	0.954	0.955
			(0.31725)	(0.31694)	(0.31725)	(0.31744)	(4.81504)	(0.30440)	**(0.30173)**

TABLE 4-14 (CON)

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		200:200	0.942	0.818	0.942	0.943	1.000	0.930	0.940
			(0.22109)	(0.22203)	(0.22122)	(0.22135)	**(4.70490)**	(0.21699)	(0.21573)
	0.3:0.3	20:20	0.945	0.849	0.950	0.948	0.994	0.965	0.961
			(1.25539)	(1.26866)	(1.22903)	(1.23106)	(5.22800)	(0.78770)	**(0.76858)**
		20:50	0.936	0.816	0.944	0.945	0.998	0.968	0.945
			(0.97217)	(0.94227)	(1.01863)	(1.02316)	(4.61521)	**(0.73283)**	(0.71640)
		20:70	0.937	0.814	0.946	0.946	0.998	0.949	0.928
			(0.90711)	(0.89864)	(0.96554)	(0.96713)	**(4.43234)**	(0.68551)	(0.67004)
		20:100	0.928	0.830	0.953	0.955	0.998	0.940	0.918
			(0.86028)	(0.87063)	**(0.92426)**	(0.92642)	(4.25453)	(0.64950)	(0.63531)
		50:50	0.944	0.831	0.951	0.949	1.000	0.953	0.954
			(0.70855)	(0.70594)	(0.70024)	(0.70077)	(4.37985)	(0.56999)	**(0.56227)**
		50:70	0.953	0.838	0.960	0.961	0.999	0.961	0.960
			(0.64388)	(0.64238)	(0.64506)	(0.64639)	(4.14893)	(0.56208)	**(0.55467)**
		50:100	0.938	0.818	0.944	0.942	1.000	0.951	0.945
			(0.59712)	(0.60851)	(0.60432)	(0.60468)	(4.03200)	**(0.52659)**	(0.51988)
		70:70	0.944	0.830	0.943	0.946	1.000	0.948	0.948
			(0.58498)	(0.58823)	(0.58025)	(0.58031)	**(4.05447)**	(0.49315)	(0.48793)
		70:100	0.954	0.840	0.955	0.954	1.000	0.956	0.951
			(0.54054)	(0.53770)	(0.54199)	(0.54152)	(4.01196)	(0.48903)	**(0.48404)**
		100:100	0.948	0.839	0.950	0.950	1.000	0.952	0.952
			(0.48562)	(0.49263)	(0.48358)	(0.48345)	(3.84935)	(0.42742)	**(0.42390)**
		200:200	0.956	0.828	0.962	0.961	1.000	0.961	0.958
			(0.33756)	(0.33534)	(0.33695)	(0.33708)	(3.77093)	(0.31234)	**(0.31052)**

TABLE 4-15 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for ratio of Two CVs under the ZITR Distribution with $(\mu_1, \mu_2) = (0.2, 0.2)$, $(\lambda_1, \lambda_2) = (1,1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
1:1	0.1:0.1	20:20	0.923	0.828	0.947	0.947	1.000	0.992	0.987
			(1.13431)	(1.12781)	(1.03694)	(1.02902)	(14.45970)	(0.72146)	**(0.70622)**
		20:50	0.901	0.806	0.946	0.948	1.000	0.989	0.991
			(0.83982)	(0.85520)	(0.86160)	(0.85955)	(12.39183)	(0.72583)	**(0.70995)**
		20:70	0.887	0.780	0.947	0.951	1.000	0.984	0.992
			(0.78237)	(0.78587)	(0.82160)	(0.82039)	(12.23276)	(0.69670)	**(0.68100)**
		20:100	0.884	0.792	0.958	0.958	1.000	0.987	0.997
			(0.74554)	(0.75371)	(0.79095)	(0.79041)	(11.99445)	(0.67169)	**(0.65639)**
		50:50	0.933	0.842	0.936	0.938	1.000	0.960	0.960
			(0.63405)	(0.63584)	(0.61136)	(0.61001)	(12.16727)	(0.51494)	**(0.50883)**
		50:70	0.937	0.825	0.944	0.945	1.000	0.969	0.968
			(0.57900)	(0.57240)	(0.57204)	(0.57081)	(12.21059)	(0.53264)	**(0.52592)**
		50:100	0.923	0.811	0.941	0.942	1.000	0.960	0.961
			(0.53277)	(0.53726)	(0.53221)	(0.53148)	(11.92583)	(0.50164)	**(0.49532)**
		70:70	0.956	0.847	0.963	0.960	1.000	0.978	0.974
			(0.52985)	(0.53064)	(0.51469)	(0.51419)	(12.18341)	(0.45357)	**(0.44917)**
		70:100	0.942	0.809	0.947	0.949	1.000	0.967	0.970
			(0.47802)	(0.48083)	(0.47291)	(0.47204)	(11.73854)	(0.45100)	**(0.44634)**
		100:100	0.924	0.817	0.930	0.928	1.000	0.949	0.945
			(0.43561)	(0.43910)	(0.42741)	(0.42681)	**(11.96882)**	(0.38838)	(0.38540)
		200:200	0.946	0.830	0.947	0.946	1.000	0.948	0.948
			(0.30278)	(0.30314)	(0.29932)	(0.29908)	**(11.68953)**	(0.28013)	(0.27868)
	0.1:0.3	20:20	0.921	0.818	0.940	0.940	1.000	0.916	0.930
			(0.75931)	(0.77329)	(0.74858)	(0.74895)	**(7.16678)**	(0.62911)	(0.61424)

TABLE 4-15 (CON)

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		20:50	0.897	0.791	0.939	0.941	1.000	0.953	0.970
			(0.57525)	(0.57461)	(0.60536)	(0.60638)	(6.11925)	(0.54021)	**(0.52768)**
		20:70	0.877	0.774	0.934	0.934	1.000	0.947	0.966
			(0.53556)	(0.53362)	(0.56920)	(0.56979)	(5.99314)	(0.50719)	**(0.49510)**
		20:100	0.871	0.766	0.934	0.935	1.000	0.946	0.969
			(0.51318)	(0.51200)	(0.54962)	(0.54940)	(5.91088)	(0.48289)	**(0.47163)**
		50:50	0.931	0.807	0.942	0.941	1.000	0.940	0.959
			(0.44531)	(0.44363)	(0.44255)	(0.44248)	(5.95666)	(0.42338)	**(0.41741)**
		50:70	0.933	0.807	0.944	0.945	1.000	0.946	0.958
			(0.39854)	(0.39475)	(0.40072)	(0.40065)	(5.78744)	(0.38815)	**(0.38257)**
		50:100	0.927	0.820	0.946	0.946	1.000	0.948	0.960
			(0.36824)	(0.36864)	(0.37178)	(0.37190)	(5.64887)	(0.36129)	**(0.35626)**
		70:70	0.952	0.818	0.952	0.952	1.000	0.956	0.968
			(0.36905)	(0.36427)	(0.36713)	(0.36668)	(5.69126)	(0.35750)	**(0.35330)**
		70:100	0.936	0.821	0.945	0.947	1.000	0.952	0.961
			(0.33178)	(0.33385)	(0.33338)	(0.33323)	(5.65410)	(0.32655)	**(0.32284)**
		100:100	0.942	0.812	0.945	0.947	1.000	0.946	0.953
			(0.30414)	(0.29709)	(0.30374)	(0.30356)	(5.71246)	(0.29876)	**(0.29592)**
		200:200	0.935	0.831	0.937	0.939	1.000	0.933	0.939
			(0.21297)	(0.21085)	(0.21295)	(0.21287)	**(5.59169)**	(0.21078)	(0.20947)
	0.3:0.3	20:20	0.942	0.857	0.947	0.945	0.999	0.977	0.973
			(1.25869)	(1.28483)	(1.20479)	(1.20566)	(6.82400)	(0.80076)	**(0.78137)**
		20:50	0.942	0.842	0.954	0.955	0.999	0.971	0.962
			(0.94320)	(0.96404)	(0.96785)	(0.97096)	(5.56953)	(0.72774)	**(0.71167)**
		20:70	0.934	0.819	0.944	0.947	1.000	0.963	0.960
			(0.89823)	(0.90164)	(0.93315)	(0.93577)	(5.38876)	(0.68998)	**(0.67471)**
		20:100	0.939	0.822	0.950	0.949	1.000	0.968	0.963
			(0.86371)	(0.86613)	(0.89906)	(0.90120)	(5.15010)	(0.65991)	**(0.64592)**
		50:50	0.945	0.826	0.946	0.943	1.000	0.959	0.959
			(0.69606)	(0.68986)	(0.68469)	(0.68398)	(5.24756)	(0.56715)	**(0.55950)**
		50:70	0.945	0.826	0.946	0.946	1.000	0.963	0.956
			(0.62957)	(0.63072)	(0.62511)	(0.62511)	(5.06683)	(0.55874)	**(0.55119)**
		50:100	0.947	0.840	0.955	0.956	1.000	0.964	0.958
			(0.58925)	(0.58798)	(0.59069)	(0.59082)	(4.95057)	(0.52708)	**(0.52038)**
		70:70	0.943	0.843	0.945	0.943	1.000	0.953	0.951
			(0.58409)	(0.58722)	(0.57567)	(0.57602)	(5.09755)	(0.50519)	**(0.49969)**
		70:100	0.941	0.835	0.942	0.944	1.000	0.957	0.951
			(0.52252)	(0.52128)	(0.52052)	(0.52112)	(4.92762)	(0.48086)	**(0.47587)**
		100:100	0.946	0.836	0.944	0.945	1.000	0.950	0.954
			(0.47965)	(0.48525)	(0.47512)	(0.47541)	(5.00236)	(0.42530)	**(0.42162)**
		200:200	0.944	0.838	0.946	0.947	1.000	0.947	0.941
			(0.32981)	(0.33200)	(0.32843)	(0.32833)	**(4.73595)**	(0.30644)	(0.30462)

TABLE 4-16 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for ratio of Two CVs under the ZITR Distribution with $(\mu_1, \mu_2) = (1, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 0.2)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
0.2:0.2	0.1:0.1	20:20	0.917	0.822	0.939	0.939	1.000	0.931	0.958
			(0.98523)	(0.99497)	(0.88363)	(0.87655)	(5.54751)	(0.66488)	**(0.64980)**
		20:50	0.887	0.783	0.943	0.946	1.000	0.959	0.977
			(0.74005)	(0.73777)	(0.73377)	(0.73255)	(5.32177)	(0.65779)	**(0.64150)**
		20:70	0.886	0.780	0.928	0.930	1.000	0.943	0.970
			(0.72327)	(0.73122)	(0.72308)	(0.72251)	(5.09508)	(0.64300)	**(0.62633)**
		20:100	0.872	0.773	0.927	0.927	1.000	0.947	0.970
			(0.69356)	(0.69868)	(0.70123)	(0.70167)	(4.97333)	(0.62154)	**(0.60497)**
		50:50	0.933	0.838	0.944	0.943	1.000	0.947	0.951
			(0.54968)	(0.55549)	(0.52514)	(0.52369)	(4.84343)	(0.44888)	**(0.44283)**
		50:70	0.942	0.820	0.948	0.948	1.000	0.961	0.970
			(0.51081)	(0.50228)	(0.49580)	(0.49461)	(4.61162)	(0.46578)	**(0.45876)**
		50:100	0.924	0.831	0.933	0.937	1.000	0.936	0.951
			(0.47866)	(0.48336)	(0.46985)	(0.46960)	(4.56794)	(0.44274)	**(0.43581)**
		70:70	0.940	0.824	0.943	0.946	1.000	0.956	0.966
			(0.45496)	(0.45386)	(0.44050)	(0.43919)	(4.65332)	(0.38580)	**(0.38147)**
		70:100	0.933	0.818	0.941	0.943	1.000	0.941	0.958
			(0.42427)	(0.42254)	(0.41458)	(0.41402)	(4.44729)	(0.39302)	**(0.38817)**
		100:100	0.948	0.825	0.955	0.955	1.000	0.951	0.958
			(0.37942)	(0.38174)	(0.36981)	(0.36943)	(4.40467)	(0.33071)	**(0.32781)**
		200:200	0.936	0.823	0.945	0.946	1.000	0.944	0.949
			(0.26144)	(0.25946)	(0.25862)	(0.25848)	**(4.25558)**	(0.23795)	(0.23658)
	0.1:0.3	20:20	0.926	0.836	0.952	0.953	0.997	0.873	0.919
			(0.67535)	(0.69057)	(0.64979)	**(0.64819)**	(3.37618)	(0.59190)	(0.57685)
		20:50	0.876	0.761	0.936	0.938	1.000	0.882	0.932
			(0.52066)	(0.51943)	(0.52625)	(0.52581)	**(3.17833)**	(0.50116)	(0.48813)
		20:70	0.865	0.761	0.928	0.927	1.000	0.891	0.934
			(0.50075)	(0.50542)	(0.50871)	(0.50952)	**(3.07141)**	(0.47612)	(0.46346)
		20:100	0.886	0.798	0.936	0.939	1.000	0.902	0.941
			(0.48338)	(0.49488)	(0.49307)	(0.49304)	**(2.99740)**	(0.45359)	(0.44137)
		50:50	0.935	0.820	0.948	0.946	1.000	0.904	0.934
			(0.39848)	(0.40425)	(0.39043)	(0.38991)	**(2.88818)**	(0.38198)	(0.37594)
		50:70	0.935	0.820	0.947	0.949	1.000	0.918	0.934
			(0.36120)	(0.36338)	(0.35689)	(0.35670)	**(2.85675)**	(0.35076)	(0.34513)
		50:100	0.931	0.812	0.948	0.949	1.000	0.925	0.946
			(0.34007)	(0.33573)	(0.33791)	(0.33803)	**(2.78032)**	(0.33002)	(0.32467)
		70:70	0.953	0.841	0.953	0.952	1.000	0.934	0.945
			(0.32976)	(0.32908)	(0.32457)	**(0.32420)**	(2.82368)	(0.31904)	(0.31500)
		70:100	0.929	0.834	0.935	0.935	1.000	0.911	0.926
			(0.30450)	(0.30548)	(0.30143)	(0.30126)	**(2.69638)**	(0.29474)	(0.29101)
		100:100	0.945	0.830	0.953	0.953	1.000	0.910	0.930
			(0.27336)	(0.27361)	(0.27053)	**(0.27047)**	(2.63929)	(0.26574)	(0.26302)
		200:200	0.951	0.834	0.953	0.956	1.000	0.933	0.944
			(0.19139)	(0.19478)	(0.18992)	(0.18981)	**(2.43269)**	(0.18656)	(0.18535)
	0.3:0.3	20:20	0.939	0.857	0.941	0.940	0.992	0.953	0.969
			(1.13728)	(1.15022)	(1.06045)	(1.05817)	(3.26471)	(0.76649)	**(0.74775)**
		20:50	0.926	0.820	0.935	0.935	1.000	0.968	0.968
			(0.89488)	(0.88341)	(0.88214)	(0.88447)	(3.14324)	(0.69200)	**(0.67634)**
		20:70	0.940	0.862	0.942	0.943	1.000	0.972	0.978
			(0.84968)	(0.85863)	(0.84492)	(0.84688)	(2.97728)	(0.65486)	**(0.64002)**
		20:100	0.927	0.833	0.927	0.925	1.000	0.976	0.967
			(0.82978)	(0.82412)	(0.82889)	(0.83051)	(2.85394)	(0.63064)	**(0.61661)**
		50:50	0.949	0.833	0.952	0.952	1.000	0.955	0.965
			(0.64246)	(0.65069)	(0.62413)	(0.62407)	(2.47024)	(0.52865)	**(0.52151)**

TABLE 4-16 (CON)

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability						
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		50:70	0.950	0.830	0.951	0.949	1.000	0.955	0.958
			(0.58735)	(0.58082)	(0.57684)	(0.57740)	(2.48267)	(0.52398)	**(0.51681)**
		50:100	0.937	0.840	0.942	0.945	1.000	0.954	0.953
			(0.54602)	(0.54342)	(0.53956)	(0.53993)	(2.33437)	(0.48774)	**(0.48139)**
		70:70	0.945	0.845	0.946	0.946	1.000	0.949	0.955
			(0.53300)	(0.53438)	(0.52164)	(0.52165)	(2.19088)	(0.46186)	**(0.45697)**
		70:100	0.940	0.832	0.945	0.947	1.000	0.956	0.957
			(0.48471)	(0.48783)	(0.47809)	(0.47861)	(2.15924)	(0.44421)	**(0.43954)**
		100:100	0.940	0.823	0.943	0.943	1.000	0.951	0.947
			(0.43465)	(0.43566)	(0.42787)	(0.42815)	(2.15295)	**(0.38473)**	(0.38139)
		200:200	0.951	0.820	0.948	0.948	1.000	0.956	0.955
			(0.30034)	(0.29848)	(0.29761)	(0.29784)	(1.94487)	(0.27905)	**(0.27747)**

TABLE 4-17 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for ratio of Two CVs under the ZITR Distribution with $(\mu_1, \mu_2) = (1, 0.2)$, $(\lambda_1, \lambda_2) = (0.2, 1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVE	AN	MCMC	HPD
0.2:1	0.1:0.1	20:20	0.917	0.827	0.941	0.944	1.000	0.981	0.989
			(1.06715)	(1.08167)	(0.95781)	(0.95009)	(10.18142)	(0.70394)	**(0.68734)**
		20:50	0.882	0.799	0.935	0.936	1.000	0.963	0.977
			(0.83128)	(0.83190)	(0.81644)	(0.81536)	(9.32031)	(0.71665)	**(0.69858)**
		20:70	0.880	0.785	0.936	0.939	1.000	0.961	0.977
			(0.77342)	(0.77331)	(0.77295)	(0.77212)	(9.14646)	(0.68565)	**(0.66794)**
		20:100	0.881	0.771	0.938	0.936	1.000	0.955	0.974
			(0.74751)	(0.75716)	(0.75518)	(0.75486)	(9.01403)	(0.66649)	**(0.64864)**
		50:50	0.954	0.844	0.959	0.957	1.000	0.971	0.972
			(0.61352)	(0.61548)	(0.58437)	(0.58263)	(8.97940)	(0.49362)	**(0.48696)**
		50:70	0.933	0.811	0.944	0.941	1.000	0.963	0.971
			(0.56353)	(0.56351)	(0.54602)	(0.54455)	(8.75846)	(0.51006)	**(0.50256)**
		50:100	0.929	0.831	0.947	0.946	1.000	0.965	0.972
			(0.52100)	(0.51751)	(0.51179)	(0.51040)	(8.80094)	(0.48117)	**(0.47381)**
		70:70	0.945	0.839	0.947	0.949	1.000	0.963	0.970
			(0.50570)	(0.50146)	(0.48671)	(0.48559)	(8.88094)	(0.42697)	**(0.42218)**
		70:100	0.930	0.823	0.935	0.939	1.000	0.954	0.959
			(0.46529)	(0.46119)	(0.45407)	(0.45341)	(8.69342)	(0.43008)	**(0.42495)**
		100:100	0.934	0.822	0.941	0.941	1.000	0.947	0.952
			(0.41834)	(0.42495)	(0.40749)	(0.40682)	(8.68934)	(0.36586)	**(0.36266)**
		200:200	0.949	0.818	0.946	0.949	1.000	0.948	0.947
			(0.29119)	(0.28902)	(0.28719)	(0.28680)	**(8.58164)**	(0.26447)	(0.26291)
	0.1:0.3	20:20	0.909	0.815	0.938	0.935	1.000	0.906	0.924
			(0.72177)	(0.72093)	(0.68863)	(0.68755)	**(5.90382)**	(0.61065)	(0.59462)
		20:50	0.877	0.777	0.930	0.929	1.000	0.924	0.956
			(0.54310)	(0.54690)	(0.55135)	(0.55153)	(5.21853)	(0.51977)	**(0.50599)**
		20:70	0.863	0.763	0.927	0.927	1.000	0.919	0.952
			(0.51732)	(0.51472)	(0.52967)	(0.52954)	(5.22866)	(0.49399)	**(0.48061)**
		20:100	0.846	0.753	0.919	0.925	1.000	0.910	0.949
			(0.50473)	(0.51335)	(0.51565)	(0.51646)	**(5.16800)**	(0.47525)	(0.46231)
		50:50	0.948	0.844	0.946	0.945	1.000	0.920	0.941
			(0.42680)	(0.42396)	(0.41805)	(0.41764)	**(4.96777)**	(0.40396)	(0.39742)
		50:70	0.932	0.808	0.933	0.932	1.000	0.920	0.942
			(0.38657)	(0.38867)	(0.38201)	(0.38182)	**(5.00262)**	(0.37317)	(0.36718)
		50:100	0.930	0.808	0.942	0.941	1.000	0.916	0.942
			(0.35851)	(0.35605)	(0.35723)	(0.35707)	**(4.91902)**	(0.34825)	(0.34254)

TABLE 4-17 (CON)

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		70:70	0.953	0.838	0.950	0.952	1.000	0.933	0.957
			(0.35127)	(0.35200)	(0.34531)	(0.34550)	(4.95755)	(0.33754)	**(0.33309)**
		70:100	0.934	0.833	0.937	0.941	1.000	0.928	0.945
			(0.32264)	(0.32240)	(0.31983)	(0.31985)	**(4.89001)**	(0.31248)	(0.30846)
		100:100	0.943	0.820	0.948	0.948	1.000	0.942	0.955
			(0.29038)	(0.28412)	(0.28710)	(0.28705)	(4.88004)	(0.28182)	**(0.27882)**
		200:200	0.952	0.830	0.955	0.956	1.000	0.940	0.948
			(0.20227)	(0.20068)	(0.20144)	**(0.20125)**	(4.80422)	(0.19817)	(0.19684)
	0.3:0.3	20:20	0.947	0.830	0.950	0.950	0.998	0.980	0.979
			(1.19786)	(1.21006)	(1.11541)	(1.11493)	(4.98623)	(0.77804)	**(0.75843)**
		20:50	0.943	0.840	0.952	0.949	1.000	0.973	0.974
			(0.95651)	(0.96947)	(0.94457)	(0.94682)	(4.45349)	(0.72987)	**(0.71295)**
		20:70	0.929	0.814	0.941	0.942	1.000	0.976	0.964
			(0.88807)	(0.87471)	(0.88458)	(0.88728)	(4.24022)	(0.68168)	(0.66584)
		20:100	0.937	0.837	0.946	0.944	1.000	0.973	0.966
			(0.86243)	(0.86638)	(0.86346)	(0.86501)	(4.30506)	(0.65545)	**(0.64074)**
		50:50	0.943	0.823	0.949	0.950	1.000	0.956	0.957
			(0.68342)	(0.67108)	(0.66409)	(0.66393)	(4.00062)	(0.55238)	**(0.54470)**
		50:70	0.953	0.824	0.952	0.950	1.000	0.967	0.967
			(0.61691)	(0.60869)	(0.60588)	(0.60613)	(3.84093)	(0.54621)	**(0.53861)**
		50:100	0.945	0.808	0.948	0.946	1.000	0.960	0.962
			(0.58038)	(0.57424)	(0.57401)	(0.57446)	(3.77062)	(0.51675)	**(0.50971)**
		70:70	0.954	0.831	0.949	0.945	1.000	0.964	0.964
			(0.56136)	(0.56505)	(0.54989)	(0.54971)	(3.77425)	(0.48170)	**(0.47623)**
		70:100	0.937	0.823	0.940	0.940	1.000	0.944	0.947
			(0.50814)	(0.50373)	(0.50263)	(0.50283)	**(3.71780)**	(0.46603)	(0.46100)
		100:100	0.957	0.861	0.958	0.957	1.000	0.961	0.963
			(0.46012)	(0.46982)	(0.45305)	(0.45283)	(3.67152)	(0.40385)	**(0.40026)**
		200:200	0.957	0.861	0.958	0.957	1.000	0.961	0.963
			(0.46012)	(0.46982)	(0.45305)	(0.45283)	(3.67152)	(0.40385)	**(0.40026)**

TABLE 4-18 Coverage Probabilities and Expected Lengths of 95% Confidence Intervals for ratio of Two CVs under the ZITR Distribution with $(\mu_1, \mu_2) = (1, 0.2)$, $(\lambda_1, \lambda_2) = (1,1)$, and $(\delta_1, \delta_2) = (0.1, 0.1), (0.1, 0.3), (0.3, 0.3)$

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
1:1	0.1:0.1	20:20	0.883	0.819	0.911	0.912	1.000	0.980	0.986
			(1.03421)	(1.04767)	(0.89940)	(0.89171)	(9.93262)	(0.67047)	**(0.65259)**
		20:50	0.875	0.770	0.903	0.906	1.000	0.970	0.981
			(0.82465)	(0.83038)	(0.78263)	(0.78142)	(9.14507)	(0.70865)	**(0.68907)**
		20:70	0.871	0.761	0.900	0.900	1.000	0.963	0.978
			(0.79546)	(0.81177)	(0.76460)	(0.76329)	(9.02480)	(0.69547)	**(0.67624)**
		20:100	0.883	0.780	0.906	0.905	1.000	0.955	0.972
			(0.77947)	(0.78884)	(0.75206)	(0.75197)	(8.93392)	(0.68073)	**(0.66220)**
		50:50	0.938	0.818	0.940	0.939	1.000	0.962	0.965
			(0.60902)	(0.61270)	(0.56906)	(0.56694)	(9.16581)	(0.48613)	**(0.47816)**
		50:70	0.945	0.823	0.956	0.957	1.000	0.975	0.980
			(0.56119)	(0.57088)	(0.53460)	(0.53305)	(9.03713)	(0.50040)	**(0.49175)**
		50:100	0.928	0.822	0.934	0.938	1.000	0.956	0.968
			(0.54212)	(0.55554)	(0.52034)	(0.51936)	(8.89645)	(0.48829)	**(0.47998)**
		70:70	0.949	0.840	0.951	0.949	1.000	0.967	0.968
			(0.50178)	(0.50704)	(0.47859)	(0.47744)	(9.06557)	(0.41799)	**(0.41242)**
		70:100	0.933	0.823	0.935	0.936	1.000	0.963	0.966
			(0.46606)	(0.47032)	(0.44950)	(0.44834)	(8.93967)	(0.42536)	**(0.41954)**

TABLE 4-18 (CON)

λ_1, λ_2	$\delta_1 : \delta_2$	Coverage probability							
		$n_1 : n_2$	PB	BS	GCI	MOVER	AN	MCMC	HPD
		100:100	0.947	0.842	0.949	0.946	1.000	0.960	0.961
			(0.41613)	(0.41318)	(0.40144)	(0.40064)	(8.85206)	(0.36008)	**(0.35635)**
		200:200	0.946	0.846	0.953	0.953	1.000	0.948	0.959
			(0.28883)	(0.28511)	(0.28282)	(0.28269)	(8.77195)	(0.26143)	**(0.25965)**
	0.1:0.3	20:20	0.895	0.802	0.924	0.928	1.000	0.905	0.920
			(0.70250)	(0.70941)	(0.64721)	(0.64551)	**(5.80609)**	(0.59093)	(0.57326)
		20:50	0.865	0.754	0.901	0.905	1.000	0.908	0.943
			(0.55708)	(0.55498)	(0.54201)	(0.54230)	**(5.32051)**	(0.52182)	(0.50671)
		20:70	0.854	0.748	0.900	0.900	1.000	0.930	0.953
			(0.52888)	(0.54221)	(0.51914)	(0.51928)	(5.26468)	(0.49672)	**(0.48263)**
		20:100	0.891	0.767	0.905	0.910	1.000	0.919	0.946
			(0.52816)	(0.52155)	(0.51410)	(0.51498)	**(5.20100)**	(0.48208)	(0.46873)
		50:50	0.932	0.808	0.938	0.942	1.000	0.934	0.954
			(0.41475)	(0.40523)	(0.39830)	(0.39809)	(5.15768)	(0.38608)	**(0.37891)**
		50:70	0.932	0.823	0.937	0.941	1.000	0.928	0.949
			(0.38758)	(0.39037)	(0.37595)	(0.37567)	**(5.17766)**	(0.36700)	(0.36037)
		50:100	0.937	0.804	0.943	0.939	1.000	0.929	0.950
			(0.36646)	(0.35969)	(0.35597)	(0.35577)	(5.17025)	(0.34647)	**(0.34036)**
		70:70	0.951	0.834	0.953	0.953	1.000	0.931	0.949
			(0.34471)	(0.34392)	(0.33566)	**(0.33518)**	(5.13867)	(0.32813)	(0.32326)
		70:100	0.929	0.832	0.943	0.945	1.000	0.938	0.953
			(0.32123)	(0.32185)	(0.31372)	(0.31363)	(5.08335)	(0.30694)	**(0.30254)**
		100:100	0.940	0.829	0.948	0.944	1.000	0.931	0.940
			(0.28384)	(0.28014)	(0.27803)	(0.27787)	**(5.12306)**	(0.27368)	(0.27044)
		200:200	0.938	0.833	0.938	0.940	1.000	0.939	0.944
			(0.19889)	(0.20002)	(0.19655)	(0.19656)	**(5.03722)**	(0.19405)	(0.19267)
	0.3:0.3	20:20	0.943	0.848	0.946	0.944	0.999	0.973	0.972
			(1.16330)	(1.17436)	(1.05306)	(1.05256)	(4.78796)	(0.75739)	**(0.73756)**
		20:50	0.953	0.834	0.953	0.956	1.000	0.975	0.970
			(0.93642)	(0.91201)	(0.89868)	(0.90088)	(4.10481)	(0.71050)	**(0.69392)**
		20:70	0.932	0.841	0.938	0.938	1.000	0.968	0.962
			(0.90192)	(0.89615)	(0.86978)	(0.87144)	(4.09191)	(0.68009)	**(0.66454)**
		20:100	0.949	0.850	0.955	0.950	1.000	0.970	0.971
			(0.87548)	(0.87915)	(0.85095)	(0.85276)	(4.02727)	(0.65778)	**(0.64331)**
		50:50	0.956	0.862	0.957	0.957	1.000	0.966	0.961
			(0.65958)	(0.66833)	(0.63377)	(0.63438)	(4.09313)	(0.52767)	**(0.51999)**
		50:70	0.950	0.851	0.956	0.953	1.000	0.965	0.963
			(0.61006)	(0.61197)	(0.59430)	(0.59397)	(3.87166)	(0.53493)	**(0.52739)**
		50:100	0.953	0.846	0.954	0.954	1.000	0.963	0.958
			(0.57092)	(0.57941)	(0.55986)	(0.55970)	(3.83622)	(0.50468)	**(0.49796)**
		70:70	0.938	0.833	0.935	0.935	1.000	0.944	0.946
			(0.54849)	(0.54044)	(0.53353)	(0.53333)	**(3.96700)**	(0.46416)	(0.45875)
		70:100	0.939	0.825	0.944	0.941	1.000	0.949	0.948
			(0.50512)	(0.50399)	(0.49582)	(0.49572)	**(3.89454)**	(0.45877)	(0.45375)
		100:100	0.944	0.848	0.944	0.944	1.000	0.948	0.949
			(0.45133)	(0.45303)	(0.44147)	(0.44148)	**(3.83052)**	(0.39909)	(0.39546)
		200:200	0.945	0.851	0.950	0.951	1.000	0.953	0.948
			(0.31173)	(0.31389)	(0.30866)	(0.30886)	(3.79956)	**(0.28396)**	(0.28228)

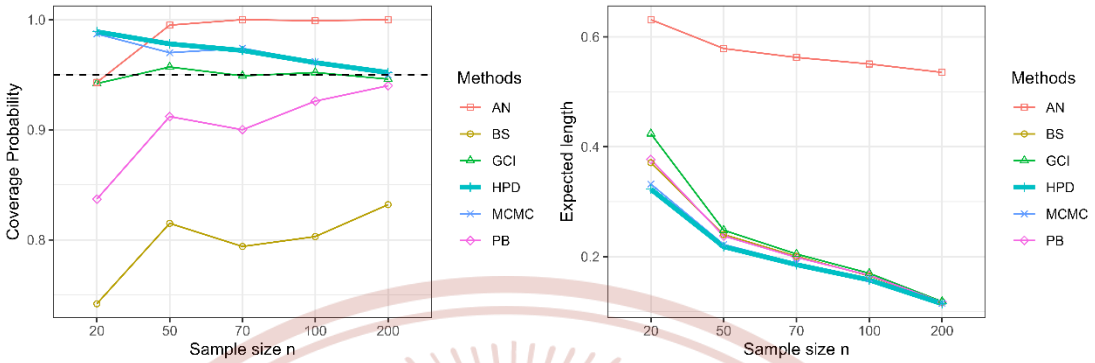


FIGURE 4-1 Graphs comparing the performance of the proposed methods for the single CV parameter in terms of (a) coverage probability and (b) expected length.

The performance of the confidence intervals for the difference of the coefficients of variation was evaluated based on the coverage probability (CP) and expected length (EL), as presented in Tables 4-7 to 4.12 and Figure 4-2. The primary criterion for comparing the methods was whether the coverage probability was sufficiently close to the nominal confidence level of 0.95. Expected length was subsequently used to assess interval precision among methods with acceptable coverage performance.

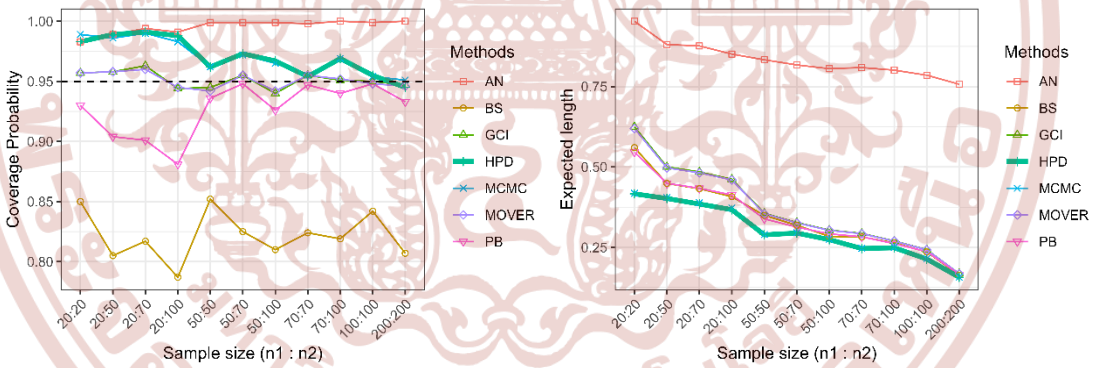


FIGURE 4-2 Graphs comparing the performance of the proposed methods for the difference between CVs parameters in terms of (a) coverage probability and (b) expected length.

In terms of CP, the MCMC, HPD, AN, GCI, and MOVER methods generally produced coverage probabilities close to or exceeding the nominal confidence level of 0.95 across most sample sizes. Meanwhile, the PB method showed improved coverage performance as the sample size increased, resulting in more satisfactory coverage for larger samples.

In terms of interval precision, as measured by EL, the HPD method consistently produced the shortest confidence intervals across most scenarios. However, in some simulation settings, the HPD method yielded coverage probabilities slightly below the nominal level of 0.95. Therefore, its shorter interval lengths should be considered together with its coverage performance rather than evaluated independently. Overall, the HPD method provided a favorable balance between coverage probability and interval length.

Similarly, the confidence intervals for the ratio of the coefficients of variation under the ZITR distribution were evaluated based on CP and EL, as presented in Tables 4-13 to 4-18 and Figure 4-3. The results exhibited patterns similar to those observed for the difference of the coefficients of variation, with the main conclusions remaining consistent across both comparison settings.

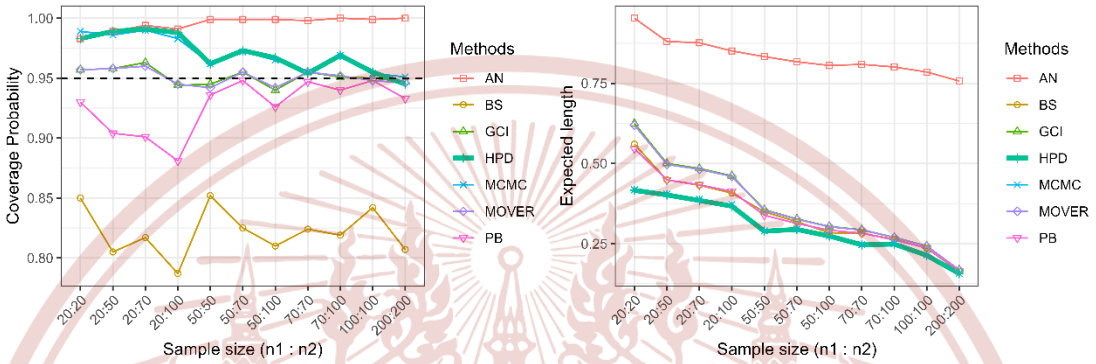


FIGURE 4-3 Graphs comparing the performance of the proposed methods for the ratio CVs parameter in terms of (a) coverage probability and (b) expected length.

In terms of CP, the MCMC, HPD, AN, GCI, and MOVER methods generally achieved coverage probabilities close to or exceeding the nominal level of 0.95 across most simulation scenarios. In addition, all methods tended to produce shorter expected lengths as the sample sizes increased, indicating improved interval precision with increasing sample size.

Overall, the HPD method generally demonstrated the most desirable performance in terms of both coverage probability and expected length under most simulation settings. Although slight undercoverage was observed in some scenarios, the HPD method generally provided a favorable balance between adequate coverage and relatively short confidence intervals compared with the other competing methods.

4.5 An empirical application

This study used road traffic fatality rates per 100,000 population to assess and compare the performance of the proposed methods for estimating the coefficient of variation, the difference between coefficients of variation, and the ratio of coefficients of variation under the ZITR distribution.

4.5.1 Application of the single coefficient of variation for ZITR

This study applies various confidence interval methods for the single coefficient of variation for the ZITR distribution, constructed using MCMC, HPD, AN, GCI, PB, and BS. The proposed methods were applied to road traffic fatality rates per 100,000 population collected at the district level in Phichit, Suphan Buri, and Prachuap Khiri Khan Provinces, Thailand, in January 2026. The road traffic fatality data were obtained from the Thai Road Safety Collaboration (ThaiRSC), while population data

for each district were obtained from the Bureau of Registration Administration, Department of Provincial Administration, Ministry of Interior, Thailand.

Following the definition of the World Health Organization (WHO), the road traffic mortality rate is defined as the number of road traffic deaths per 100,000 population. Accordingly, the mortality rate for each district was calculated as

$$R_i = \frac{D_i}{N_i} \times 100,000,$$

where R_i denotes the road traffic fatality rate for the i th district, D_i is the number of road traffic fatalities in the i th district, and N_i is the population of the i th district.

The statistics include the road traffic fatality rates per 100,000 people for January 2026, categorized by district in the provinces of Phichit, Thailand. The data that were observed are shown in Table 4-19. The positive observations were modeled using six probability distributions commonly used in actuarial science, namely the Rayleigh, Normal, Weibull, Lognormal, Exponential, and Gamma distributions. The Akaike information criterion (AIC) and the Bayesian information criterion (BIC) were employed to evaluate the goodness of fit of the competing models and select the most appropriate distribution for the real-data analysis. The model with the lowest AIC and BIC values was considered to provide the best fit to the data (Akaike, 1974; Schwarz, 1978).

TABLE 4-19 Road Traffic Fatality Rates per 100,000 Population in Districts of Phichit province

Province	Road traffic mortality rates per 100,000 population					
Phichit	5.5602	3.753	0	0	6.9406	6.132
	0	0	3.2625	0	0	3.9941

The AIC and BIC values of the candidate models used to evaluate the suitability of the distributions for the data are presented in Tables 4-20. The findings suggest that the TR distribution offers the most accurate representation of the road traffic death rate data collected in January 2026 from Phichit Province in Thailand. Figure 4-4 displays Q-Q plots and the comparison of the empirical and the fitted CDFs for the three provinces, which lends further support to the suitability of the TR distribution. The observed quantiles are close to the theoretical quantiles and the fitted CDFs are in good agreement with the empirical CDFs, showing that the TR distribution is appropriate for the observed road traffic fatality rate data. Tables 4-21 present the confidence intervals for the CV obtained from the proposed methods under the ZITR model. The results indicate that the shortest interval length is obtained by the HPD method, followed by the MCMC, AN, BS, PB, and GCI methods. Based on the simulation results, considering both the CP and EL, the HPD approach appears to be the most appropriate method for constructing confidence intervals for the coefficient of variation when the underlying population follows the ZITR distribution.

TABLE 4-20 Model Comparison Based on AIC and BIC for Road Traffic Fatality Rates per 100,000 Population across Districts in Phichit Province, January 2026

Criterion	Rayleigh	Normal	Weibull	Lognormal	Exponential	Gamma
AIC	**23.77456**	24.62316	24.52218	24.33657	32.16936	24.36036
BIC	**23.35808**	24.20668	24.1057	23.92009	32.96112	23.94388

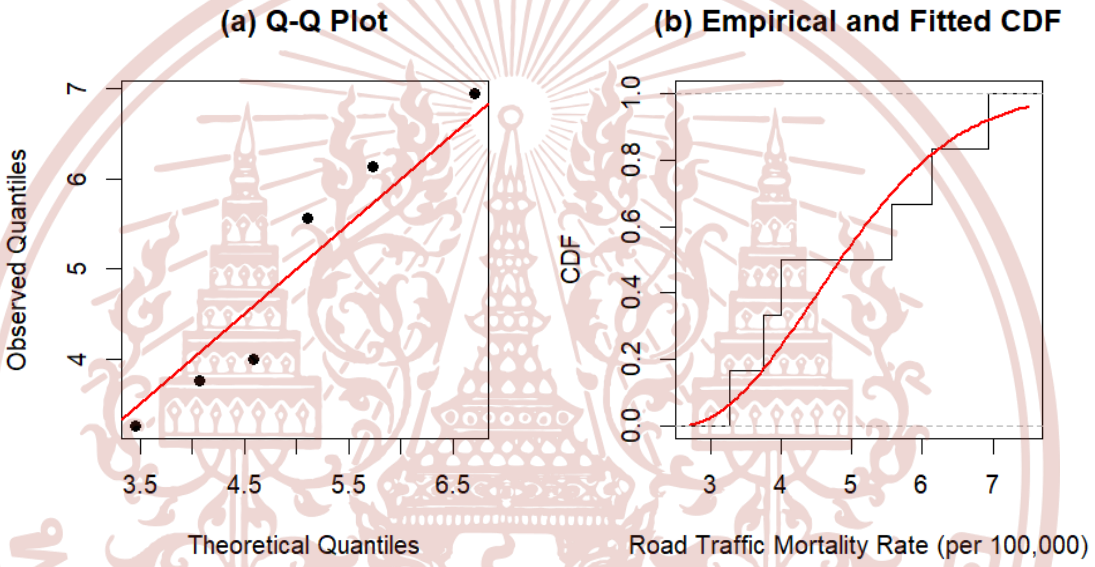


FIGURE 4-4 (a) Q-Q plot of Road Traffic Fatality Data. and (b) CDF of Road Traffic Fatality Rates per 100,000 Population across Districts in Phichit Province, January 2026

TABLE 4-21 Confidence Interval Estimates for the CV under ZITR: January 2026 Road Traffic Fatality Rates in Phichit Province

Method	**Lower**	**Upper**	**Length**
MCMC	0.5611	1.2451	0.6840
HPD	0.5288	1.1989	**0.6701**
AN	0.6355	1.8582	1.2227
PB	0.5457	1.8260	1.2803
BS	0.5463	1.8782	1.3319
GCI	0.6368	2.0521	1.4153

The sample sizes of the provincial datasets are relatively small, which might affect the performance of some interval estimation approaches for the CV. In particular, the AN and GCI approaches might have lower coverage performance under the small sample condition, as indicated in the simulation study. Therefore, the interval estimates

for the CV obtained from these approaches should be interpreted with caution. The performance of the proposed interval estimation approaches was systematically evaluated through the simulation study, while this practical application aims to demonstrate their implementation for the CV under the ZITR distribution using real-world data.

4.5.2 Application of difference and ratio coefficients of variation for ZITR

To demonstrate the practical applicability of the proposed confidence interval methods for the difference and ratio of coefficients of variation under the ZITR distribution, including MCMC, HPD, AN, GCI, MOVER, PB, and BS, road traffic fatality rates per 100,000 population recorded at the district level in Uttaradit and Mae Hong Son Provinces, Thailand, in March 2026 were analyzed. The road traffic fatality data were obtained from the Thai Road Safety Collaboration (ThaiRSC) database and were combined with population data for each district obtained from the Bureau of Registration Administration, Department of Provincial Administration, Ministry of Interior, Thailand. The resulting district-level road traffic fatality rates for Uttaradit and Mae Hong Son Provinces are presented in Table 4-22. Uttaradit and Mae Hong Son provinces were selected because their district-level fatality rates exhibited a substantial proportion of zero observations, making them appropriate for illustrating the proposed methods under a zero-inflated setting. Before constructing the confidence intervals, several candidate distributions were fitted to the data. Based on the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), the zero-inflated two-parameter Rayleigh (ZITR) distribution provided the best fit and was therefore adopted for the subsequent analysis. Then, we constructed confidence intervals for the difference and ratio of coefficients of variation (CVs) based on the proposed methods.

TABLE 4-22 Road Traffic Fatality Rates per 100,000 Population in Districts of Uttaradit and Mae Hong Son province

Province	Road traffic fatality data						
Uttaradit	2.23	0	3.553	0	0	0	4.325
	2.807	0					
Mae Hong Son	3.528	0	8.353	3.774	6.211	0	0

Both datasets have a significant quantity of zero observations, reflecting districts with no recorded incidents throughout the research period. The positive numbers fluctuate across districts, reflecting variations in accident severity within each province.

The study starts by scrutinizing the distribution of the positive observations, considering these properties. The appropriateness of candidate models for the non-zero component is evaluated using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). The findings (refer to Tables 4-23 and 4-24) demonstrate that the two-parameter Rayleigh distribution adequately fits the positive data for both provinces. Furthermore, Figures 4-5 and 4-6 present the Q-Q plots and comparisons

between the empirical and fitted CDFs, providing additional graphical evidence of the adequacy of the two-parameter Rayleigh distribution for the positive observations.

TABLE 4-23 Model Comparison Based on AIC and BIC for Road Traffic Fatality Rates per 100,000 Population across Districts in Uttaradit Province, March 2026

Criterion	Normal	Rayleigh	Weibull	Lognormal	Exponential	Gamma
AIC	13.44253	**12.98544**	13.41914	13.35213	18.37715	13.33844
BIC	12.21512	**11.75803**	12.19172	12.12472	18.76345	12.11103

TABLE 4-24 Model Comparison Based on AIC and BIC for Road Traffic Fatality Rates per 100,000 Population across Districts in Mae Hong Son Province, March 2026

Criterion	Normal	Rayleigh	Weibull	Lognormal	Exponential	Gamma
AIC	20.77278	**19.98839**	20.56379	20.17872	22.58955	20.30145
BIC	19.54537	**18.76097**	19.33638	18.95131	22.97584	19.07404

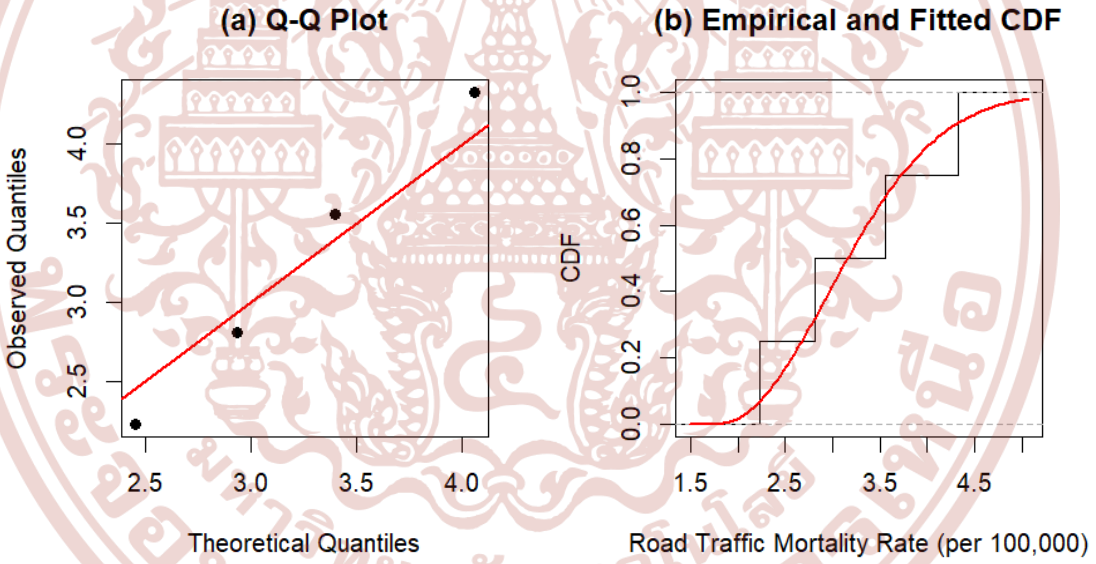


FIGURE 4-5 (a) Q-Q plot of Road Traffic Fatality Data and (b) CDF of Road Traffic Fatality Rates per 100,000 Population in Districts of Uttaradit Province

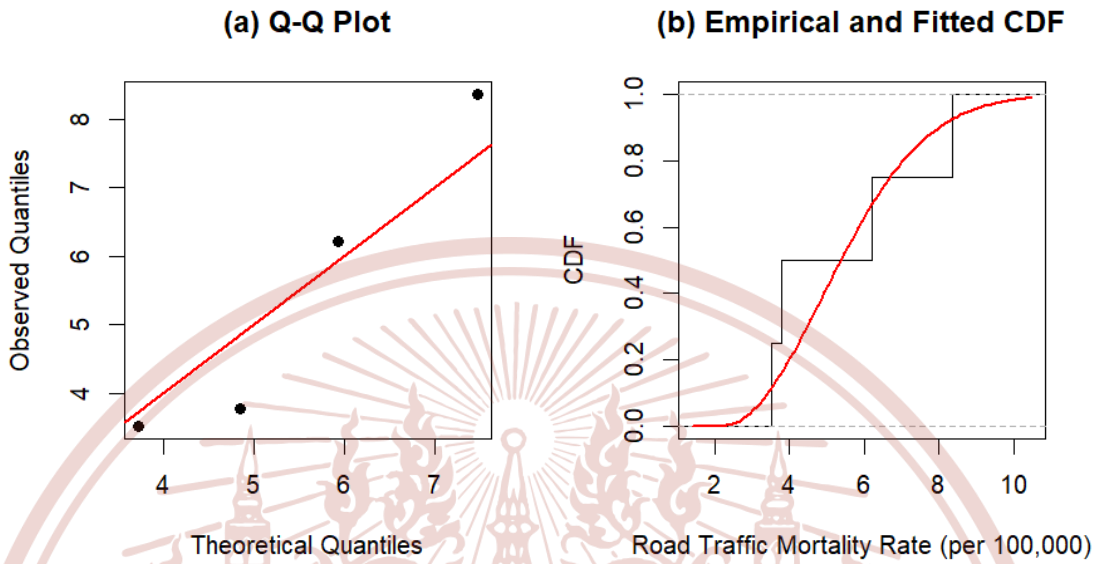


FIGURE 4-6 (a) Q-Q plot of Road Traffic Fatality Data and (b) CDF of Road Traffic Fatality Rates per 100,000 Population in Districts of Mae Hong Son Province

The existence of excess zeros and a positively skewed distribution of the positive fatality rates suggests that the ZITR model is appropriate for these data. The model structure allows the districts to be represented by two distinct components: those with zero recorded road traffic fatality rates and those with positive fatality rates. These structural differences may reflect variations in traffic density, road infrastructure, enforcement measures, healthcare accessibility, or other local characteristics across districts. The above datasets were used to demonstrate the proposed confidence interval procedures for the difference and ratio of the coefficients of variation using observed road traffic fatality rates. The confidence intervals were constructed according to the procedures described in this paper, allowing a direct comparison of the relative variability in road traffic fatality rates between the two provinces under the assumed ZITR model.

The resulting confidence intervals and the associated performance measures are presented in Tables 4-25 and 4-26 for the difference and ratio of the coefficients of variation, respectively. The results illustrate that the proposed methodologies are practical and applicable for analyzing real-world road traffic fatality rate data.

The analysis of district-level road traffic fatality rates (per 100,000 population) from Uttaradit and Mae Hong Son provinces, Thailand, recorded in March 2026, reveals noticeable differences in the performance of the confidence interval methods under the ZITR model for the difference between coefficients of variation. The HPD method produces the shortest confidence interval, which is consistent with the findings from the simulation study. The remaining methods can be ranked in ascending order of interval length as MCMC, AN, BS, PB, GCI, and MOVER. The substantially

wider intervals produced by the latter methods reflect a greater degree of uncertainty in the estimation process. Furthermore, the simulation results demonstrate that the HPD method consistently achieves CP close to the nominal confidence level while maintaining relatively short interval lengths. Therefore, the favorable performance of the HPD method observed in the real-data application is consistent with the simulation results, suggesting that it provides a competitive approach for interval estimation under the ZITR distribution.

TABLE 4-25 Confidence Interval Estimates for the Difference of CVs under ZITR: March 2026 District-Level Road Traffic Fatality Rates in Uttaradit and Mae Hong Son Provinces

Method	Lower	Upper	Length
MCMC	-0.4352	0.6096	1.0448
HPD	-0.4224	0.6202	**1.0426**
AN	-0.1697	1.4464	1.6161
GCI	-1.8364	1.8547	3.6911
MOVER	-1.9269	1.7130	3.6399
PB	-0.6897	1.1797	1.8694
BS	-0.1864	1.9134	2.0998

TABLE 4-26 Confidence Interval Estimates for the Ratio of CVs under ZITR: March 2026 District-Level Road Traffic Fatality Rates in Uttaradit and Mae Hong Son Provinces

Method	Lower	Upper	Length
MCMC	0.5928	2.0387	1.4459
HPD	0.5172	1.8911	**1.3739**
AN	-3.4951	7.8910	11.3861
GCI	0.3391	3.3510	3.0119
MOVER	0.3176	3.1546	2.8370
PB	0.4816	2.8947	2.4131
BS	-0.0823	2.8909	2.9732

A similar pattern is observed for the ratio of coefficients of variation. The HPD method again produces the shortest confidence interval among all competing methods. The remaining methods can be ranked in ascending order of interval length as MCMC, PB, GCI, MOVER, BS, and AN. The wider intervals produced by these methods indicate a greater degree of uncertainty in the estimation process. It is also noteworthy that the AN and BS methods produced negative lower confidence limits for the ratio of the coefficients of variation. Since the ratio of two positive coefficients of variation is itself strictly positive, these intervals extend beyond the admissible parameter space. This limitation arises from the additive construction of these intervals on the original scale and should be taken into account when interpreting the results. In future studies, this issue could be addressed by considering appropriate transformations or alternative interval construction methods that ensure the confidence intervals remain within the admissible parameter space. For example, a log-scale transformation may be considered to avoid

negative lower confidence limits for the ratio of coefficients of variation and to provide more appropriate interval estimates for a strictly positive parameter.

Furthermore, the simulation study demonstrates that the HPD method consistently provides satisfactory coverage probabilities while maintaining relatively short interval lengths. Therefore, the agreement between the simulation results and the real-data analysis suggests that the HPD method is the most suitable approach for constructing confidence intervals for the ratio of coefficients of variation in district-level road traffic fatality rate data exhibiting the characteristics of the ZITR distribution.

4.6 Conclusion

This chapter presents and evaluates several confidence interval estimation methods for the coefficient of variation (CV), the difference between coefficients of variation, and the ratio of coefficients of variation under the Zero-Inflated Two-Parameter Rayleigh (ZITR) distribution. The methods were evaluated through Monte Carlo simulation and real-data applications, with performance assessed based on coverage probability (CP) and expected length (EL).

For the coefficient of variation, six methods were considered: Bayesian MCMC, Bayesian HPD, the approximation normal (AN) method, the generalized confidence interval (GCI), the percentile bootstrap (PB), and the bootstrap with standard error (BS). The simulation results indicate that the Bayesian HPD method generally provides the best overall performance, achieving coverage probabilities close to the nominal level while producing relatively short confidence intervals. The Bayesian MCMC method also performs comparably to HPD under some parameter settings. The real-data applications using road traffic mortality rates per 100,000 population from Phichit, Suphan Buri, and Prachuap Khiri Khan provinces in January 2026 generally support the simulation findings, with HPD often producing shorter interval estimates.

For the difference and ratio of coefficients of variation, the proposed methods, namely MCMC, HPD, and AN, were compared with GCI, MOVER, PB, and BS. The simulation results show that HPD generally provides favorable performance for both parameters, with coverage probabilities close to the nominal level and relatively short expected lengths. For the difference of CVs, the remaining methods can generally be ranked in terms of expected length performance as MCMC, MOVER, GCI, and AN, respectively. For the ratio of CVs, MCMC, GCI, MOVER, and AN show comparable performance after HPD. Although PB and BS may produce shorter intervals in some settings, their coverage probabilities are not consistently close to the nominal level. The real-data application using district-level road traffic fatality rates per 100,000 population from Uttaradit and Mae Hong Son provinces in March 2026 is generally consistent with the simulation findings, with HPD providing relatively short intervals while maintaining satisfactory coverage.

Overall, the findings indicate that the Bayesian HPD method generally provides the most favorable overall performance for confidence interval estimation of the CV, the difference of CVs, and the ratio of CVs under the ZITR distribution. Nevertheless, its performance may be affected by extreme zero-inflation or very small sample sizes, where the limited number of positive observations can increase uncertainty in parameter estimation and interval construction. Further studies could investigate more extreme parameter settings and extend the proposed methods to other zero-inflated lifetime distributions or censored data.

As also observed in the previous chapters, the Bayesian methods require relatively longer computational times than the non-Bayesian methods because MCMC sampling is required for posterior inference and the construction of HPD intervals. Thus, although the Bayesian HPD method generally provides the most favorable statistical performance, its computational cost should also be considered when selecting an interval estimation method. In addition, the performance of the GCI method depends on the construction of an appropriate generalized pivotal quantity (GPQ). Although the GPQ used in this study provided satisfactory results, alternative GPQ constructions may further improve the coverage probabilities and expected lengths. Future research could therefore investigate alternative GPQs for the CV, the difference of CVs, and the ratio of CVs under the ZITR distribution.



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APPENDIX (A)

The R Statistical Program

A1 Functions are used to calculate the coverage probability and the expected length of the confidence interval for the means of the zero-inflated two-parameter Rayleigh distribution

The following R code is used to compute the confidence intervals, the coverage probability, and the expected length of the confidence intervals for the means of zero-inflated two-parameter Rayleigh distributions in Chapter 2.

```
#All code parameter means

# This code was written by Sasipong Kijsason

library(new.dist)
library(VGAM)
library(MASS)
library(R2OpenBUGS)
library(HDInterval)
rZigTRayreigh <- function(x,lambda,mu,prob0){
 x_0 <- rbinom(x,size = 1,prob = 1-prob0)
 x0 <- subset(x_0,x_0 ==0)
 nzero <- length(x0)
 nNonzero <- x-nzero
 x1 <- rtprd(nNonzero,lambda = lambda,mu = mu)
 x <- c(x0,x1)
 return(x)
}
bootsampnew <- function(x,B = 1000){
 keepdata <- vector()#while
 i = 1
 while(i <=B){
  x.B <- sample(x = x,size = length(x),replace = T)
  x.B0test <- subset(x.B,x.B==0)
  x.B1test <- subset(x.B,x.B>0)
  if(length(x.B1test)<2|x.B1test[1]==mean(x.B1test)){
   i <- i
  }else{
   keepdata <- cbind(keepdata,x.B)
   i <- i+1
```

```
    }
  }
  return(keepdata)
}
MLES = function(x){
  n = length(x)
  xb = mean(x); s = sd(x)
  b.hat = sqrt(2/(4-pi))*s  #moment estimate
  fn = function(a){
    ssq = sum((x-a)^2)
    y = 2*n*sum(x-a)/ssq-sum(1/(x-a))
    return(y)
  }
  a0 = min(x)-12*b.hat/sqrt(n); a1 = min(x)
  a.mle = uniroot(fn, interval = c(a0,a1), tol = 10^-5, maxiter = 20)$root
  b.mle = sqrt(.5*sum((x-a.mle)^2)/n)
  return(c(a.mle,b.mle))
}
ComputeGCI <- function(T,n,lamda,mu,delta,x){
  Qmu <- vector()
  Qlambda <- vector()
  Qb <- vector()
  p <- delta
  delta_GPQ <- vector()
  mean_GCI <- vector()
  x1 <- x[x>0]
  n1 <- length(x1)
  deltahat <- 1-(length(x1)/length(x))
  parameters <- MLES(x1)
  #parameters <- fitdist(x,distr = "tprd",lower = c(0.01,0.01),start = list(lambda
=0.01,mu =0.01))
  lambdahat <- parameters[2]
  muhat <- parameters[1]
  #lambdahat <- parameters$estimate[1]
  #muhat <- parameters$estimate[2]
  for(i in 1:T){
    xstar <- rtprd(n1,lambda = 1/2,mu = 0)
    parameterstar <- MLES(xstar)
    #parameterstar <- fitdist(xstar,distr = "tprd",lower = c(0.01,0.001),start =
list(lambda =1,mu =0))
```

```
    lambdastar <- parameterstar[2]
    mustar <- parameterstar[1]
    #lambdastar <- parameterstar$estimate[1]
    #mustar <- parameterstar$estimate[2]
    Qmu[i] <- muhat-(lambdahat*(mustar/lambdastar))
    #Qlambda[i] <- lambdahat/lambdastar
    Qb[i] <- lambdahat/lambdastar
    #Z.VST = 2*(sqrt(n))*(asin(sqrt(deltahat))-asin(sqrt(p)))
    Z.VST <- rnorm(1,0,1)
    delta_GPQ[i] = (sin(asin(sqrt(deltahat))-(Z.VST/(2*sqrt(n)))))^2
    #mean_GCI[i] = momntZITRR(lambda = Qlambda[i],mu = Qmu[i],delta =
delta_GPQ[i] ) # compute GPQ for single mean
  }
  Qlambda <- 1/(2*(Qb^2))
  out <- cbind(mu=Qmu,lambda=Qlambda,delta_GPQ=delta_GPQ)
  #print(out)

}
momntZITRR <- function(lambda,mu,delta){
  over <- function(k,lambda,mu,delta){
   (1-delta)*(1/(lambda^(k/2)))*gamma((k/2)+1)
  }
  Ex <- over(1,lambda,mu,delta)+((1-delta)*mu)
  return(Ex)
}
Binomialmodel1 <- function() {
  for (i in 1:n)
  {
    x[i] ~ dbern(p);
  }
  p ~ dbeta(8,2);
}
Binomialmodel2 <- function() {
  for (i in 1:n)
  {
    x[i] ~ dbern(p);
  }
  p ~ dbeta(8,2);
```

```
}
TRRmodel1 <- function(){
 for(i in 1:n){
  one[i] <- 1
  p[i] <- (2*l*(x[i]-u))*exp((-l)*pow(x[i]-u,2))/10000
  one[i] ~ dbern(p[i])
 }
 u ~ dunif(0,3)
 l ~ dgamma(1,7);
}
TRRmodel2 <- function(){
 for(i in 1:n){
  one[i] <- 1
  p[i] <- (2*l*(x[i]-u))*exp((-l)*pow(x[i]-u,2))/10000
  one[i] ~ dbern(p[i])
 }
 u ~ dunif(0,3)
 l ~ dgamma(1,7);
}
CI.dif.ratio.ZiTRR <- function(M,n,m,lambda1,mu1,p1,lambda2,mu2,p2){
 #M = loop for montecarlo
 #N = number of sample size
 #s = scale parameter of x
 #p = weigh parameter for ZiRR distribution
 meanZITRR1 <- momntZITRR(lambda = lambda1,mu = mu1,delta = p1)
 meanZITRR2 <- momntZITRR(lambda = lambda2,mu = mu2,delta = p2)
 dif.mean <- meanZITRR1-meanZITRR2
 ratio.mean <- meanZITRR1/meanZITRR2
 CP.diff.mean.percentile <- rep(0,M)
 length.diff.mean.percentile <- rep(0,M)
 CP.diff.BS <- rep(0,M)
 length.diff.BS <- rep(0,M)
 CP.diff.mean.GCI <- rep(0,M)
 length.diff.mean.GCI <- rep(0,M)
 CP.dif.GCI.MOVER <- rep(0,M)
 length.dif.GCI.MOVER <- rep(0,M)
 CP.AN.diff.mean <- rep(0,M)
 length.AN.diff.mean <- rep(0,M)
 CP.diff.mean.MOVER <- rep(0,M)
 length.diff.mean.MOVER <- rep(0,M)
```

```
  CP.diff.mean.Baye <- rep(0,M)
  length.diff.mean.Baye <- rep(0,M)
  CP.diff.mean.Baye2 <- rep(0,M)
  length.diff.mean.Baye2 <- rep(0,M)
  ##ratio
  CP.ratio.mean.percentile <- rep(0,M)
  length.ratio.mean.percentile <- rep(0,M)
  CP.ratio.BS <- rep(0,M)
  length.ratio.BS <- rep(0,M)
  CP.ratio.mean.GCI <- rep(0,M)
  length.ratio.mean.GCI <- rep(0,M)
  CP.AN.ratio.mean <- rep(0,M)
  CP.ratio.GCI.MOVER <- rep(0,M)
  length.ratio.GCI.MOVER <- rep(0,M)
  length.AN.ratio.mean <- rep(0,M)
  CP.ratio.mean.MOVER <- rep(0,M)
  length.ratio.mean.MOVER <- rep(0,M)
  CP.ratio.mean.Baye <- rep(0,M)
  length.ratio.mean.Baye <- rep(0,M)
  CP.ratio.mean.Baye2 <- rep(0,M)
  length.ratio.mean.Baye2 <- rep(0,M)
  ##
  CP.standard.CI.mean <- rep(0,M)
  length.standard.CI.mean <- rep(0,M)
  CP.mean.GCI <- rep(0,M)
  length.mean.GCI <- rep(0,M)
  CP.mean.boot <- rep(0,M) # coverage probability of CI based on bootstrap
  length.mean.boot <- rep(0,M)
  #CP.mean.BS <- rep(0,M) # coverage probability of CI based on bootstrap standard
error
  CP.mean.GCI <- rep(0,M)
  CP.AN.CI.mean <- rep(0,M)
  length.AN.CI.mean <- rep(0,M)
  CP.mean.bayesian <- rep(0,M) # coverage probability of CI based on
bayesianMCMC
  CP.BAYE2.mean <- rep(0,M) # coverage probability of CI based on bayesianHPTD
  length.mean.boot <- rep(0,M)
  length.mean.BS <- rep(0,M)
```

```
length.mean.GCI <- rep(0,M)
length.mean.bayesian <- rep(0,M)
length.BAYE2.mean <- rep(0,M)
#expected.ZiRR <- kmomentRR(1,s,p)
B <- 1000 #number of bootstrap sample
z.value <- qnorm(0.975)
scoda <- rep(0,M)
pcoda <- rep(0,M)
baye.mean <- rep(0,M)
mean_GCI <- vector()

###stat iteration
for(i in 1:M){
 deltahat.xn.boot <- rep(0,B)
 lambda.xn.boot <- rep(0,B)
 mu.xn.boot <- rep(0,B)
 deltahat.xm.boot <- rep(0,B)
 lambda.xm.boot <- rep(0,B)
 mu.xm.boot <- rep(0,B)
 xn <- rZigTRayreigh(n,lambda1,mu1,p1)
 xm <- rZigTRayreigh(n,lambda2,mu2,p2)
 xn0 <- subset(xn,xn==0)
 xm0 <- subset(xm,xm==0)
 xn1 <- subset(xn,xn>0)
 xm1 <- subset(xm,xm>0)
 nxn1 <- length(xn1)
 nxm1 <- length(xm1)
 xn01 <- ifelse(xn>0,1,0)
 xm01 <- ifelse(xm>0,1,0)
 #x.B <- bootsamp(x,nsamp = B)
 xn.B <- bootsampnew(xn,1000)
 xm.B <- bootsampnew(xm,1000)
 for(j in 1:B){
  #for n
  xn.B0 <- subset(xn.B[,j],xn.B[,j]==0)
  xn.B1 <- subset(xn.B[,j],xn.B[,j]>0)
  xnB0 <- length(xn.B0)
  xnB1 <- length(xn.B1)
  x.B01 <- ifelse(xn.B[,j]>0,1,0)
  deltahat.xn.boot[j] <- xnB0/n
```

```
    MLExn.B <- MLES(x = xn.B1)
    lambda.xn.boot[j] <- 1/(2*(MLExn.B[2]^2))
    mu.xn.boot[j] <- MLExn.B[1]
    #for m
    xm.B0 <- subset(xm.B[,j],xm.B[,j]==0)
    xm.B1 <- subset(xm.B[,j],xm.B[,j]>0)
    xmB0 <- length(xm.B0)
    xmB1 <- length(xm.B1)
    x.B01 <- ifelse(xm.B[,j]>0,1,0)
    deltahat.xm.boot[j] <- xmB0/m
    MLExm.B <- MLES(x = xm.B1)
    lambda.xm.boot[j] <- 1/(2*(MLExm.B[2]^2))
    mu.xm.boot[j] <- MLExm.B[1]
   }
   xn.B.mean <- momntZITRR(lambda = lambda.xn.boot,mu = mu.xn.boot,delta =
deltahat.xn.boot)
   xm.B.mean <- momntZITRR(lambda = lambda.xm.boot,mu = mu.xm.boot,delta =
deltahat.xm.boot)
   x.B.diff.mean <- xn.B.mean-xm.B.mean
   x.B.ratio.mean <- xn.B.mean/xm.B.mean
   #####Percentile Bootstrap
   L.Boot.mean <- quantile(xn.B.mean,0.025,type = 8)
   U.Boot.mean <- quantile(xn.B.mean,0.975,type = 8)
   CP.mean.boot[i] <-
ifelse(L.Boot.mean<meanZITRR1&meanZITRR1<U.Boot.mean,1,0)
   length.mean.boot[i]          = U.Boot.mean-L.Boot.mean
   L.Boot.diff <- quantile(x.B.diff.mean,0.025,type = 8)
   U.Boot.diff <- quantile(x.B.diff.mean,0.975,type = 8)
   CP.diff.mean.percentile[i] <-
ifelse(L.Boot.diff<dif.mean&dif.mean<U.Boot.diff,1,0)
   length.diff.mean.percentile[i]                = U.Boot.diff-L.Boot.diff
   L.Boot.ratio <- quantile(x.B.ratio.mean,0.025,type = 8)
   U.Boot.ratio <- quantile(x.B.ratio.mean,0.975,type = 8)
   CP.ratio.mean.percentile[i] <-
ifelse(L.Boot.ratio<ratio.mean&ratio.mean<U.Boot.ratio,1,0)
   length.ratio.mean.percentile[i]               = U.Boot.ratio-L.Boot.ratio
   #Bootstrap with SE
   std.B.diff <- qnorm(0.975)*sd(x.B.diff.mean)
```

```
    CI.diff.bs <- x.B.diff.mean+c(-1,1)*std.B.diff
    L.Bs.diff <- CI.diff.bs[1]
    U.Bs.diff <- CI.diff.bs[2]
    CP.diff.BS[i] <- ifelse(L.Bs.diff<dif.mean&dif.mean<U.Bs.diff,1,0)
    length.diff.BS[i] <- U.Bs.diff-L.Bs.diff
    #ratio
    std.B.ratio <- qnorm(0.975)*sd(x.B.ratio.mean)
    CI.ratio.bs <- x.B.ratio.mean+c(-1,1)*std.B.ratio
    L.Bs.ratio <- CI.ratio.bs[1]
    U.Bs.ratio <- CI.ratio.bs[2]
    CP.ratio.BS[i] <- ifelse(L.Bs.ratio<ratio.mean&ratio.mean<U.Bs.ratio,1,0)
    length.ratio.BS[i] <- U.Bs.ratio-L.Bs.ratio
    ## GCI
    es1 <- ComputeGCI(T =2500,n=n,lamda = lambda1,mu = mu1,delta = p1,xn)
    es2 <- ComputeGCI(T =2500,n=m,lamda = lambda2,mu = mu2,delta = p2,xm)
    delta.xn.gpq <- es1[,"delta_GPQ"]
    lambda.xn.gpq <- es1[,"lambda"]
    mu.xn.gpq <- es1[,"mu"]
    delta.xm.gpq <- es2[,"delta_GPQ"]
    lambda.xm.gpq <- es2[,"lambda"]
    mu.xm.gpq <- es2[,"mu"]

    meanxnGPQ <- momntZITRR(lambda = es1[,2],mu = es1[,1],delta = es1[,3])
    meanxmGPQ <- momntZITRR(lambda = es2[,2],mu = es2[,1],delta = es2[,3])
    diffGPQ <- meanxnGPQ-meanxmGPQ
    rationGPQ <- meanxnGPQ/meanxmGPQ
    L.xnGCI <- quantile(meanxnGPQ,0.025,type = 8)
    U.xnGCI <- quantile(meanxnGPQ,0.975,type = 8)
    L.xmGCI <- quantile(meanxmGPQ,0.025,type = 8)
    U.xmGCI <- quantile(meanxmGPQ,0.975,type = 8)
    CP.mean.GCI[i] <- ifelse(L.xnGCI<meanxnGPQ&meanxnGPQ<U.xnGCI,1,0)
    length.mean.GCI[i] <- U.xnGCI-L.xnGCI
    L.FGCI.diff <- quantile(diffGPQ,0.025,type = 8)
    U.FGCI.diff <- quantile(diffGPQ,0.975,type = 8)
    CP.diff.mean.GCI[i] <- ifelse(L.FGCI.diff<dif.mean&dif.mean<U.FGCI.diff,1,0)
    length.diff.mean.GCI[i] <- U.FGCI.diff-L.FGCI.diff
    L.FGCI.ratio <- quantile(rationGPQ,0.025,type = 8)
    U.FGCI.ratio <- quantile(rationGPQ,0.975,type = 8)
    CP.ratio.mean.GCI[i] <-
ifelse(L.FGCI.ratio<ratio.mean&ratio.mean<U.FGCI.ratio,1,0)
```

```
   length.ratio.mean.GCI[i] <- U.FGCI.ratio-L.FGCI.ratio
   #MOVER GCI
   L.MOVER.diff.GCI <- mean(diffGPQ)-sqrt(((mean(meanxnGPQ)-
L.xnGCI)^2)+((U.xmGCI-mean(meanxmGPQ))^2))
   U.MOVER.diff.GCI <- mean(diffGPQ)+sqrt(((U.xnGCI -
mean(meanxnGPQ))^2)+((mean(meanxmGPQ)- L.xmGCI)^2))
   CP.dif.GCI.MOVER[i] <-
ifelse(L.MOVER.diff.GCI<dif.mean&dif.mean<U.MOVER.diff.GCI,1,0)
   length.dif.GCI.MOVER[i] <- U.MOVER.diff.GCI-L.MOVER.diff.GCI
   pd.mean <- mean(meanxnGPQ)*mean(meanxnGPQ)
   Lu <- mean(pd.mean)-sqrt((mean(pd.mean)^2)-
(L.xnGCI*U.xmGCI*((2*mean(meanxnGPQ))-L.xnGCI)*((2*mean(meanxnGPQ))-
U.xmGCI)))
   Ll <- U.xmGCI*((2*mean(meanxnGPQ))-U.xmGCI)
   Uu <- mean(pd.mean)+sqrt((mean(pd.mean)^2)-
(U.xnGCI*L.xmGCI*((2*mean(meanxnGPQ))-U.xnGCI)*((2*mean(meanxnGPQ))-
L.xmGCI)))
   Ul <- L.xmGCI*((2*mean(meanxnGPQ))-L.xmGCI)
   L.CI.mv.ratio              = Lu/Ll
   U.CI.mv.ratio              = Uu/Ul
   CP.ratio.GCI.MOVER[i] <-
ifelse(L.CI.mv.ratio<ratio.mean&ratio.mean<U.CI.mv.ratio,1,0)
   length.ratio.GCI.MOVER[i] <- U.CI.mv.ratio-L.CI.mv.ratio
   #NA
   parax0 <- length(x = xn0)/length(xn)
   paray0 <- length(xm0)/length(xm)
   parax <- MLES(xn1)
   lamdahatx <- 1/(2*(parax[2]^2))
   muhatx <- parax[1]
   paray <- MLES(xm1)
   lamdahaty <- 1/(2*(paray[2]^2))
   muhaty <- paray[1]
   meanxhat <- momntZITRR(lambda = lamdahatx,mu = muhatx,delta = parax0)
   meanyhat <- momntZITRR(lambda = lamdahaty,mu = muhaty,delta = paray0)
    varx <- ((muhatx + gamma(3/2)/sqrt(lamdahatx))^2) *parax0*(1-
parax0)/length(xn) +
    (1-parax0)^2 *1 / ((2*muhatx) +(1/length(xn1))*sum((xn1-muhatx)^(-2))) +(1-
parax0)^2 *gamma(3/2)^2 /(4*lamdahatx*length(xn1))
```

```
   vary <- ((muhaty + gamma(3/2)/sqrt(lamdahaty))^2) *paray0*(1-
paray0)/length(xm)+ (1-paray0)^2 *1 / ((2*muhaty) +(1/length(xm1))*sum((xm1-
muhaty)^(-2))) +(1-paray0)^2 *gamma(3/2)^2 /(4*lamdahaty*length(xm1))
   variance.diff.mean.hat <- varx + vary
   variance.diff.mean.hat <- ((((muhatx+gamma(3/2)/sqrt(lamdahatx))^2)*parax0*(1-
parax0)/length(xn))+((1-parax0)^2)*(1/((2*muhatx)+(1/length(xn1))*sum(xn1-
muhatx)^-2))+((1-parax0)^2)*((-1/2)*lamdahatx^(-
3/2)*gamma(3/2))^2*(lamdahatx^2)/length(xn1))+((((muhaty+gamma(3/2)/sqrt(lamd
ahaty))^2)*paray0*(1-paray0)/length(xm))+((1-
paray0)^2)*(1/((2*muhaty)+(1/length(xm1))*sum(xm1-muhaty)^-2))+((1-
paray0)^2)*((-1/2)*lamdahaty^(-3/2)*gamma(3/2))^2*(lamdahaty^2)/length(xm1))
   variance.ratio.mean.hat1 <-
((((muhatx+gamma(3/2)/sqrt(lamdahatx))/momntZITRR(lambda = lamdahaty,mu =
muhaty,delta = paray0))^2)*parax0*(1-parax0)/length(xn))+((((1-
parax0)/momntZITRR(lambda = lamdahaty,mu = muhaty,delta =
paray0))^2)*(1/((2*muhatx)+(1/length(xn1))*sum(xn1-muhatx)^-2)))+(((((1-
parax0)^2)*((-1/2)*lamdahatx^(-3/2)*gamma(3/2)))/momntZITRR(lambda =
lamdahaty,mu = muhaty,delta = paray0))^2)*(lamdahatx^2)/length(xn1)
   variance.ratio.mean.hat2 <- (((momntZITRR(lambda = lamdahatx,mu =
muhatx,delta =
parax0)*(muhaty+gamma(3/2)/sqrt(lamdahaty))/momntZITRR(lambda =
lamdahaty,mu = muhaty,delta = paray0)^2)^2)*paray0*(1-
paray0)/length(xm))+((momntZITRR(lambda = lamdahatx,mu = muhatx,delta =
parax0)*(1-paray0)/momntZITRR(lambda = lamdahaty,mu = muhaty,delta =
paray0)^2)^2)*(1/((2*muhaty)+(1/length(xm1))*sum(xm1-muhaty)^-
2))+(((momntZITRR(lambda = lamdahatx,mu = muhatx,delta = parax0)*((1-
paray0))*((-1/2)*lamdahaty^(-3/2)*gamma(3/2))/(momntZITRR(lamdahaty,mu =
muhaty,delta = paray0)^2))^2)*(lamdahaty^2)/length(xm1))
   variance.ratio.mean.hat <- variance.ratio.mean.hat1+variance.ratio.mean.hat2
   #varmeanhat <- (((muhat+gamma(3/2)/sqrt(lambdahat))^2)*deltahat*(1-
deltahat)/length(x))+((1-deltahat)^2)*(1/((2*muhat)+(1/length(x1))*sum(x1-muhat)^-
2))+((1-deltahat)^2)*((-1/2)*lambdahat^(-
3/2)*gamma(3/2))^2*(lambdahat^2)/length(x1)
   L.CI.diff.AN <- (meanxhat-meanyhat)-(qnorm(0.975)*sqrt(variance.diff.mean.hat))
   U.CI.diff.AN <- (meanxhat-
meanyhat)+(qnorm(0.975)*sqrt(variance.diff.mean.hat))
   CP.AN.diff.mean[i] <- ifelse(L.CI.diff.AN<dif.mean&dif.mean<U.CI.diff.AN,1,0)
   length.AN.diff.mean[i] <- U.CI.diff.AN-L.CI.diff.AN
```

```
   L.CI.ratio.AN <- (meanxhat/meanyhat)-
(qnorm(0.975)*sqrt(variance.ratio.mean.hat))
   U.CI.ratio.AN <-
(meanxhat/meanyhat)+(qnorm(0.975)*sqrt(variance.ratio.mean.hat))
   CP.AN.ratio.mean[i] <-
ifelse(L.CI.ratio.AN<ratio.mean&ratio.mean<U.CI.ratio.AN,1,0)
   length.AN.ratio.mean[i] <- U.CI.ratio.AN-L.CI.ratio.AN
   #MOVER
   #parax0 <- length(x = xn0)/length(xn)
   #paray0 <- length(xm0)/length(xm)
   #parax <- MLES(xn1)
   #paray <- MLES(xm1)
   #meanxhat <- momntZITRR(lambda = 1/(2*(parax[2]^2)),mu = parax[1],delta =
parax0)
   #meanyhat <- momntZITRR(lambda = 1/(2*(paray[2]^2)),mu = paray[1],delta =
paray0)
   sd.mean.x <- meanxhat/sqrt(2*length(xn))
   sd.mean.y <- meanyhat/sqrt(2*length(xm))
   #STD
   #based on lamba mu
   L.CImux <-  parax[1]-(qnorm(0.975)*sqrt(1/((2*parax[1])+(sum(xn-parax[1])^(-
2))/length(xn1))))
   U.CImux <-  parax[1]+(qnorm(0.975)*sqrt(1/((2*parax[1])+(sum(xn-parax[1])^(-
2))/length(xn1))))
   #L.CImuy <-  paray[1]-(qnorm(0.975)*sqrt(1/((2*parax[1])+(sum(xm-paray[1])^(-
2))/length(xm1))))
   #U.CImuy <-  paray[1]+(qnorm(0.975)*sqrt(1/((2*parax[1])+(sum(xm-paray[1])^(-
2))/length(xm1))))
   L.CIlambdax <- (1/(2*(parax[2]^2)))-
(qnorm(0.975)*(1/(2*(parax[2]^2)))/sqrt(length(xn1)))
   U.CIlambdax <-
(1/(2*(parax[2]^2)))+(qnorm(0.975)*(1/(2*(parax[2]^2)))/sqrt(length(xn1)))
   #L.CIlambday <- (1/(2*(paray[2]^2)))-
(qnorm(0.975)*(1/(2*(paray[2]^2)))/sqrt(length(xm1)))
   #U.CIlambday <-
(1/(2*(paray[2]^2)))+(qnorm(0.975)*(1/(2*(paray[2]^2)))/sqrt(length(xm1)))
   L.CIdeltax <- parax0-qnorm(0.975)*sqrt(parax0*(1-parax0)/length(xn))
   U.CIdeltax <- parax0+qnorm(0.975)*sqrt(parax0*(1-parax0)/length(xn))
```

```
   #L.CIdeltay <- paray0-qnorm(0.975)*sqrt(paray0*(1-paray0)/length(xm))
   #U.CIdeltay <- paray0+qnorm(0.975)*sqrt(paray0*(1-paray0)/length(xm))
   L.CIMLEx <- (1-L.CIdeltax)*((gamma(3/2)/sqrt(L.CIlambdax))+L.CImux)
   U.CIMLEx <- (1-U.CIdeltax)*((gamma(3/2)/sqrt(U.CIlambdax))+U.CImux)
   CP.standard.CI.mean[i] <-
ifelse(L.CIMLEx<meanZITRR1&meanZITRR1<U.CIMLEx,1,0)
   length.standard.CI.mean[i] <- U.CIMLEx-L.CIMLEx
   #L.CIMLEy <- (1-L.CIdeltay)*((gamma(3/2)/sqrt(L.CIlambday))+L.CImuy)
   #U.CIMLEy <- (1-U.CIdeltay)*((gamma(3/2)/sqrt(U.CIlambday))+U.CImuy)
   varmeanhatx  <-(((muhatx + gamma(3/2)/sqrt(lamdahatx))^2) *
    parax0*(1-parax0)/length(xn)) + ((1-parax0)^2) *  (1 / ((2*muhatx)
+(1/length(xn1))*sum((xn1-muhatx)^(-2)))) +((1-parax0)^2) *(gamma(3/2)^2
/(4*lamdahatx*length(xn1)))    L.CI.ANx <-
meanxhat(qnorm(0.975)*sqrt(varmeanxhat))
   U.CI.ANx <- meanxhat+(qnorm(0.975)*sqrt(varmeanxhat))
   CP.AN.CI.mean[i] <-
ifelse(L.CI.ANx<meanZITRR1&meanZITRR1<U.CI.ANx,1,0)
   length.AN.CI.mean[i] <- U.CI.ANx-L.CI.ANx
   #varmeanyhat <- (((muhaty+gamma(3/2)/sqrt(lamdahaty))^2)*paray0*(1-
paray0)/length(xm))+((1-paray0)^2)*(1/((2*muhaty)+(1/length(xm1))*sum(xm1-
muhaty)^-2))+((1-paray0)^2)*((-1/2)*lamdahaty^(-
3/2)*gamma(3/2))^2*(lamdahaty^2)/length(xm1)
   #L.CI.ANy <- meanyhat-(qnorm(0.975)*sqrt(varmeanyhat))
   #U.CI.ANy <- meanyhat+(qnorm(0.975)*sqrt(varmeanyhat))
   #L.CI.mv.diff <- (meanxhat-meanyhat)-sqrt(((meanxhat-
L.CI.ANx)^2)+((U.CI.ANy-meanyhat)^2))
   #U.CI.mv.diff <- (meanxhat-meanyhat)+sqrt(((U.CI.ANx -
meanxhat)^2)+((meanyhat- L.CI.ANy)^2))
   #CP.diff.mean.MOVER[i] <-
ifelse(L.CI.mv.diff<dif.mean&dif.mean<U.CI.mv.diff,1,0)
   #length.diff.mean.MOVER[i] <- U.CI.mv.diff-L.CI.mv.diff
   #Ratio
   #based on lamba mu
   #pd.mean <- meanxhat*meanyhat
   #Lu <- pd.mean-sqrt((pd.mean^2)-(L.CI.ANx*U.CI.ANy*((2*meanxhat)-
L.CI.ANx)*((2*meanyhat)-U.CI.ANy)))
   #Ll <- U.CI.ANy*((2*meanyhat)-U.CI.ANy)
   #Uu <- pd.mean+sqrt((pd.mean^2)-(U.CI.ANx*L.CI.ANy*((2*meanxhat)-
U.CI.ANx)*((2*meanyhat)-L.CI.ANy)))
   #Ul <- L.CI.ANy*((2*meanyhat)-L.CI.ANy)
```

```
   #L.CI.mv.ratio              = Lu/Ll
   #U.CI.mv.ratio              = Uu/Ul
   #CP.ratio.mean.MOVER[i] <-
ifelse(L.CI.mv.ratio<ratio.mean&ratio.mean<U.CI.mv.ratio,1,0)
   #length.ratio.mean.MOVER[i] <- U.CI.mv.ratio-L.CI.mv.ratio
   #Bayesian
   #for p
   # n sample
   zerodatan <- list(x = xn01,n =length(xn01))
   inits1<-function(){ list( p=0) }
   outputpn <- bugs(data = zerodatan, inits = inits1,parameters.to.save =
c("p"),model.file = Binomialmodel1, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
   out.codapn <- read.bugs(outputpn)
   codpn <- as.vector(out.codapn)
   pcodan <- codpn[[1]][,2]
   deltacodan <- 1-pcodan
   # m sample
   zerodatam <- list(x = xm01,n =length(xm01))
   inits2<-function(){ list( p=0) }
   outputpm <- bugs(data = zerodatam, inits = inits2,parameters.to.save =
c("p"),model.file = Binomialmodel2, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
   out.codapm <- read.bugs(outputpm)
   codpm <- as.vector(out.codapm)
   pcodam <- codpm[[1]][,2]
   deltacodam <- 1-pcodam
   #for tprd
   #for n
   TZiRRdatan <- list(x = xn1,n =length(xn1))
   inits3<-function(){ list( u= mu1,l = lambda1) }
   outputsn <- bugs(data = TZiRRdatan, inits = inits3, parameters.to.save =
c("u","l"),model.file = TRRmodel1, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
   out.codasn <- read.bugs(outputsn)
   codsn <- as.vector(out.codasn)
   ucodan <- codsn[[1]][,3]
   lcodan <- codsn[[1]][,2]
```

```
   #for m
   TZiRRdatam <- list(x = xm1,n =length(xm1))
   inits4<-function(){ list( u= mu2,l = lambda2) }
   outputsm <- bugs(data = TZiRRdatam, inits = inits4, parameters.to.save =
c("u","l"),model.file = TRRmodel2, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
   out.codasm <- read.bugs(outputsm)
   codsm <- as.vector(out.codasm)
   ucodam <- codsm[[1]][,3]
   lcodam <- codsm[[1]][,2]
   ###

   baye.mean.xn <- momntZITRR(lambda = lcodan,mu = ucodan,delta = deltacodan)
   baye.mean.xm <- momntZITRR(lambda = lcodam,mu = ucodam,delta =
deltacodam)
   baye.diff.mean <- baye.mean.xn-baye.mean.xm
   baye.ratio.mean <- baye.mean.xn/baye.mean.xm
   L.Baye.mean <- quantile(baye.mean.xn,0.025,type = 8)
   U.Baye.mean <- quantile(baye.mean.xn,0.975,type = 8)
   CP.mean.bayesian[i] <-
ifelse(L.Baye.mean<meanZITRR1&meanZITRR1<U.Baye.mean,1,0)
   length.mean.bayesian[i] <- U.Baye.mean-L.Baye.mean
   L.Baye.diff <- quantile(baye.diff.mean,0.025,type = 8)
   U.Baye.diff <- quantile(baye.diff.mean,0.975,type = 8)
   L.Baye.ratio <- quantile(baye.ratio.mean,0.025,type = 8)
   U.Baye.ratio <- quantile(baye.ratio.mean,0.975,type = 8)
   CP.diff.mean.Baye[i] <- ifelse(L.Baye.diff<dif.mean&dif.mean<U.Baye.diff,1,0)
   length.diff.mean.Baye[i] <- U.Baye.diff-L.Baye.diff
   CP.ratio.mean.Baye[i] <-
ifelse(L.Baye.ratio<ratio.mean&ratio.mean<U.Baye.ratio,1,0)
   length.ratio.mean.Baye[i] <- U.Baye.ratio-L.Baye.ratio
   #HPTD
   #for n
   HPD.beta.uni                              =hdi(baye.mean.xn,0.95)
   L.baye2.mean              =HPD.beta.uni [1]
   U.baye2.mean              =HPD.beta.uni [2]
   CP.BAYE2.mean[i]          =
ifelse(L.baye2.mean<meanZITRR1&meanZITRR1<U.baye2.mean,1,0)    # compute
probability of CI based on BAYE of Baye
   length.BAYE2.mean[i]      = U.baye2.mean- L.baye2.mean
```

```
   #for diff
   HPD.diff.mean                              =hdi(baye.diff.mean,0.95)
   L.baye2.diff=HPD.diff.mean[1]
   U.baye2.diff                =HPD.diff.mean[2]
   CP.diff.mean.Baye2[i]       =
ifelse(L.baye2.diff<dif.mean&dif.mean<U.baye2.diff,1,0)   # compute probability of
CI based on BAYE of Baye
   length.diff.mean.Baye2[i]  = U.baye2.diff- L.baye2.diff  # compute length of CI
based on BAYE of Baye
   #for ratio
   HPD.ratio.mean                             =hdi(baye.ratio.mean,0.95)
   L.baye2.ratio               =HPD.ratio.mean[1]
   U.baye2.ratio               =HPD.ratio.mean[2]
   CP.ratio.mean.Baye2[i]      =
ifelse(L.baye2.ratio<ratio.mean&ratio.mean<U.baye2.ratio,1,0)             # compute
probability of CI based on BAYE of Baye
   length.ratio.mean.Baye2[i] = U.baye2.ratio- L.baye2.ratio              # compute
length of CI based on BAYE of Baye
  }
  #mean
  ACP.mean.Boot <- mean(CP.mean.boot)
  AEP.mean.Boot<- mean(length.mean.boot)
  ACP.mean.GCI <- mean(CP.mean.GCI)
  AEP.mean.GCI <- mean(length.mean.GCI)
  ACP.mean.standard <- mean(CP.standard.CI.mean,na.rm = TRUE)
  AEP.mean.standard <- mean(length.standard.CI.mean,na.rm = TRUE)
  ACP.mean.AN <- mean(CP.AN.CI.mean,na.rm = TRUE)
  AEP.mean.AN <- mean(length.AN.CI.mean,na.rm = TRUE)
  ACP.mean.Baye <- mean(CP.mean.bayesian)
  AEP.mean.Baye <- mean(length.mean.bayesian)
  ACP.mean.Baye2 <- mean(CP.BAYE2.mean)
  AEP.mean.Baye2 <- mean(length.BAYE2.mean)
  cat("CP mean Percentile Boot=",ACP.mean.Boot,"\n")
  cat("Ep mean Percentile Boot=",AEP.mean.Boot,"\n")
  cat("CP mean GCI=",ACP.mean.GCI,"\n")
  cat("EP mean GCI=",AEP.mean.GCI,"\n")
  cat("CP mean std=",ACP.mean.standard,"\n")
  cat("EP mean std=",AEP.mean.standard,"\n")
```

```
cat("CP mean AN=",ACP.mean.AN,"\n")
cat("EP mean AN=",AEP.mean.AN,"\n")
cat("CP mean BayesianMCMC=",ACP.mean.Baye,"\n")
cat("Ep mean BayesianMCMC=",AEP.mean.Baye,"\n")
cat("CP mean BayesianHPTD=",ACP.mean.Baye2,"\n")
cat("Ep mean BayesianHPTD=",AEP.mean.Baye2,"\n")
# diff and ratio
#diff
ACP.diff.Boot <- mean(CP.diff.mean.percentile,na.rm= TRUE)
AEP.diff.Boot<- mean(length.diff.mean.percentile,na.rm= TRUE)
ACP.diff.BS <- mean(CP.diff.BS,na.rm= TRUE)
AEP.diff.BS <- mean(length.diff.BS,na.rm= TRUE)
ACP.diff.NA <- mean(CP.AN.diff.mean,na.rm= TRUE)
AEP.diff.NA <- mean(length.AN.diff.mean,na.rm= TRUE)
ACP.diff.MOVER <- mean(CP.diff.mean.MOVER,na.rm= TRUE)
AEP.diff.MOVER <- mean(length.diff.mean.MOVER,na.rm= TRUE)
ACP.diff.FGCI <- mean(CP.diff.mean.GCI,na.rm= TRUE)
AEP.diff.FGCI <- mean(length.diff.mean.GCI,na.rm= TRUE)
ACP.diff.GCI.MOVER <- mean(CP.dif.GCI.MOVER,na.rm=TRUE)
AEP.diff.GCI.MOVER <- mean(length.dif.GCI.MOVER,na.rm=TRUE)
ACP.diff.Baye <- mean(CP.diff.mean.Baye,na.rm= TRUE)
AEP.diff.Baye <- mean(length.diff.mean.Baye,na.rm= TRUE)
ACP.diff.Baye2 <- mean(CP.diff.mean.Baye2,na.rm= TRUE)
AEP.diff.Baye2 <- mean(length.diff.mean.Baye2,na.rm= TRUE)
cat("CP diff mean Percentile Boot=",ACP.diff.Boot,"\n")
cat("Ep diff mean Percentile Boot=",AEP.diff.Boot,"\n")
cat("CP diff mean Standarderror Boot=",ACP.diff.BS,"\n")
cat("Ep diff mean Standarderror Boot=",AEP.diff.BS,"\n")
cat("CP diff mean GCI=",ACP.diff.FGCI,"\n")
cat("EP diff mean GCI=",AEP.diff.FGCI,"\n")
cat("CP diff mean GCI MOVER=",ACP.diff.GCI.MOVER,"\n")
cat("EP diff mean GCI MOVER=",AEP.diff.GCI.MOVER,"\n")
cat("CP diff mean NA=",ACP.diff.NA,"\n")
cat("EP diff mean NA=",AEP.diff.NA,"\n")
cat("CP diff mean BayesianMCMC=",ACP.diff.Baye,"\n")
cat("Ep diff mean BayesianMCMC=",AEP.diff.Baye,"\n")
cat("CP diff mean BayesianHPTD=",ACP.diff.Baye2,"\n")
cat("Ep diff mean BayesianHPTD=",AEP.diff.Baye2,"\n")
#ratio
ACP.ratio.Boot <- mean(CP.ratio.mean.percentile,na.rm= TRUE)
```

```
 AEP.ratio.Boot<- mean(length.ratio.mean.percentile,na.rm= TRUE)
 ACP.ratio.BS <- mean(CP.ratio.BS,na.rm= TRUE)
 AEP.ratio.BS <- mean(length.ratio.BS,na.rm= TRUE)
 ACP.ratio.FGCI <- mean(CP.ratio.mean.GCI,na.rm= TRUE)
 AEP.ratio.FGCI <- mean(length.ratio.mean.GCI,na.rm= TRUE)
 ACP.ratio.GCI.MOVER <- mean(CP.ratio.GCI.MOVER,na.rm= TRUE)
 AEP.ratio.GCI.MOVER <- mean(length.ratio.GCI.MOVER,na.rm= TRUE)
 ACP.ratio.NA <- mean(CP.AN.ratio.mean,na.rm= TRUE)
 AEP.ratio.NA <- mean(length.AN.ratio.mean,na.rm= TRUE)
 ACP.ratio.MOVER <- mean(CP.ratio.mean.MOVER,na.rm= TRUE)
 AEP.ratio.MOVER <- mean(length.ratio.mean.MOVER,na.rm= TRUE)
 ACP.ratio.Baye <- mean(CP.ratio.mean.Baye,na.rm= TRUE)
 AEP.ratio.Baye <- mean(length.ratio.mean.Baye,na.rm= TRUE)
 ACP.ratio.Baye2 <- mean(CP.ratio.mean.Baye2,na.rm= TRUE)
 AEP.ratio.Baye2 <- mean(length.ratio.mean.Baye2,na.rm= TRUE)
 cat("CP ratio mean Percentile Boot=",ACP.ratio.Boot,"\n")
 cat("Ep ratio mean Percentile Boot=",AEP.ratio.Boot,"\n")
 cat("CP ratio mean Standarderror Boot=",ACP.ratio.BS,"\n")
 cat("Ep ratio mean Standarderror Boot=",AEP.ratio.BS,"\n")
 cat("CP ratio mean GCI=",ACP.ratio.FGCI,"\n")
 cat("EP ratio mean GCI=",AEP.ratio.FGCI,"\n")
 cat("CP ratio GCI MOVER=",ACP.ratio.GCI.MOVER,"\n")
 cat("EP ratio GCI MOVER=",AEP.ratio.GCI.MOVER,"\n")
 cat("CP ratio mean NA=",ACP.ratio.NA,"\n")
 cat("EP ratio mean NA=",AEP.ratio.NA,"\n")
 cat("CP ratio mean BayesianMCMC=",ACP.ratio.Baye,"\n")
 cat("Ep ratio mean BayesianMCMC=",AEP.ratio.Baye,"\n")
 cat("CP ratio mean BayesianHPTD=",ACP.ratio.Baye2,"\n")
 cat("Ep ratio mean BayesianHPTD=",AEP.ratio.Baye2,"\n")
}
```

A2 Functions are used to calculate the coverage probability and the expected length of the confidence interval for the variances of zero-inflated two-parameter Rayleigh distribution

The following R code is used to compute the confidence intervals, the coverage probability, and the expected length of the confidence intervals for the variances of the zero-inflated two-parameter Rayleigh distribution in Chapter 3.

```
##This code was written by Sasipong Kijsason
library(new.dist)
library(VGAM)
library(MASS)
library(R2OpenBUGS)
library(HDInterval)
rZigTRayreigh <- function(x,lambda,mu,prob0){
 x_0 <- rbinom(x,size = 1,prob = 1-prob0)
 x0 <- subset(x_0,x_0 ==0)
 nzero <- length(x0)
 nNonzero <- x-nzero
 x1 <- rtprd(nNonzero,lambda = lambda,mu = mu)
 x <- c(x0,x1)
 return(x)
}
bootsamp <- function(x,B = 1000){
 keepdata <- vector()#while
 i = 1
 while(i <=B){
  x.B <- sample(x = x,size = length(x),replace = T)
  x.B0test <- subset(x.B,x.B==0)
  x.B1test <- subset(x.B,x.B>0)
  if(length(x.B1test)<2|x.B1test[1]==mean(x.B1test)){
   i <- i
  }else{
   keepdata <- cbind(keepdata,x.B)
   i <- i+1
  }
 }
 return(keepdata)
}
MLES = function(x){
```

```
  n = length(x)
  xb = mean(x); s = sd(x)
  b.hat = sqrt(2/(4-pi))*s  #moment estimate
  fn = function(a){
    ssq = sum((x-a)^2)
    y = 2*n*sum(x-a)/ssq-sum(1/(x-a))
    return(y)
  }
  a0 = min(x)-12*b.hat/sqrt(n); a1 = min(x)
  a.mle = uniroot(fn, interval = c(a0,a1), tol = 10^-5, maxiter = 20)$root
  b.mle = sqrt(.5*sum((x-a.mle)^2)/n)
  return(c(a.mle,b.mle))
}
momentZITRl <- function(lambda,mu,delta){
  over <- function(k,lambda,mu,delta){
    (1-delta)*(1/(lambda^(k/2)))*gamma((k/2)+1)
  }
  Ex <- over(1,lambda,mu,delta)+((1-delta)*mu)
  #Ex2 <- over(2,lambda,mu,delta)+(2*mu*(over(1,lambda,mu,delta)+mu))-((1-
delta)*(mu^2))
  Ex2 <- over(2,lambda,mu,delta)+(2*mu*Ex)-((1-delta)*(mu^2))
  Vx <- Ex2-(Ex^2)
  #cat("mean =",Ex,"\n")
  #cat("Variance =",Vx,"\n")
  output <- cbind(Ex,Vx)
}
ComputeGCI <- function(T,n,x){
  Qmu <- vector()
  Qlambda <- vector()
  Qb <- vector()
  delta_GPQ <- vector()
  mean_GCI <- vector()
  x1 <- x[x>0]
  n1 <- length(x1)
  deltahat <- 1-(length(x1)/length(x))
  parameters <- MLES(x1)
  #parameters <- fitdist(x,distr = "tprd",lower = c(0.01,0.01),start = list(lambda
=0.01,mu =0.01))
```

```
  lambdahat <- parameters[2]
  muhat <- parameters[1]
  for(i in 1:T){
    xstar <- rtprd(n1,lambda = 1/2,mu = 0)
    parameterstar <- MLES(xstar)
    #parameterstar <- fitdist(xstar,distr = "tprd",lower = c(0.01,0.001),start =
list(lambda =1,mu =0))
    lambdastar <- parameterstar[2]
    mustar <- parameterstar[1]
    #lambdastar <- parameterstar$estimate[1]
    #mustar <- parameterstar$estimate[2]
    Qmu[i] <- muhat-(lambdahat*(mustar/lambdastar))
    #Qlambda[i] <- lambdahat/lambdastar
    Qb[i] <- lambdahat/lambdastar
    #Z.VST = 2*(sqrt(n))*(asin(sqrt(deltahat))-asin(sqrt(p)))
    Z.VST <- rnorm(1,0,1)
    delta_GPQ[i] = (sin(asin(sqrt(deltahat))-(Z.VST/(2*sqrt(n)))))^2
    #mean_GCI[i] = momntZITRR(lambda = Qlambda[i],mu = Qmu[i],delta =
delta_GPQ[i] ) # compute GPQ for single mean
  }
  Qlambda <- 1/(2*(Qb^2))
  out <- cbind(mu=Qmu,lambda=Qlambda,delta_GPQ=delta_GPQ)
  #print(out)

}
Binomialmodel1 <- function() {
  for (i in 1:n)
  {
    x[i] ~ dbern(p);
  }
  p ~ dbeta(8,2);
}
Binomialmodel2 <- function() {
  for (i in 1:n)
  {
    x[i] ~ dbern(p);
  }
  p ~ dbeta(8,2);
}
TRRmodel1 <- function(){
```

```
  for(i in 1:n){
    one[i] <- 1
    p[i] <- (2*l*(x[i]-u))*exp((-l)*pow(x[i]-u,2))/10000
    one[i] ~ dbern(p[i])
  }
  u ~ dunif(0,1)
  l ~ dgamma(1,2);
}
TRRmodel2 <- function(){
  for(i in 1:n){
    one[i] <- 1
    p[i] <- (2*l*(x[i]-u))*exp((-l)*pow(x[i]-u,2))/10000
    one[i] ~ dbern(p[i])
  }
  u ~ dunif(0,1)
  l ~ dgamma(1,2);
}
CI.Variance.ZiTR <- function(M,n1,lambda1,mu1,p1,n2,lambda2,mu2,p2){
  #M = loop for montecarlo
  #N = number of sample size
  #s = scale parameter of x
  #p = weigh parameter for ZiRR distribution
  momentZITR1 <- momentZITRl(lambda1,mu1,delta = p1)
  momentZITR2 <- momentZITRl(lambda2,mu2,delta = p2)
  meanZITR1 <- momentZITR1[,1]
  varianceZITR1 <- momentZITR1[,2]
  meanZITR2 <- momentZITR2[,1]
  varianceZITR2 <- momentZITR2[,2]
  diff.variance <- varianceZITR1-varianceZITR2
  ratio.variance <- varianceZITR1/varianceZITR2
  CP.single.variance.boot <- rep(0,M)
  length.variance.boot          <-  rep(0,M)
  CP.diff.variance.boot <- rep(0,M)
  length.diff.variance.boot     <-  rep(0,M)
  CP.ratio.variance.boot <- rep(0,M)
  length.ratio.variance.boot <-  rep(0,M)
  CP.variance.BS <- rep(0,M)
  length.variance.BS <- rep(0,M)
```

```
CP.diff.BS <- rep(0,M)
length.diff.BS <- rep(0,M)
CP.ratio.BS <- rep(0,M)
length.ratio.BS <- rep(0,M)
CP.variance.GCI <- rep(0,M)
length.variance.GCI <- rep(0,M)
CP.diff.variance.GCI <- rep(0,M)
length.diff.variance.GCI <- rep(0,M)
CP.ratio.variance.GCI <- rep(0,M)
length.ratio.variance.GCI <- rep(0,M)
CP.dif.GCI.MOVER <- rep(0,M)
length.dif.GCI.MOVER <- rep(0,M)
CP.ratio.GCI.MOVER <- rep(0,M)
length.ratio.GCI.MOVER <- rep(0,M)
CP.AN.CI.variance <- rep(0,M)
length.AN.CI.variance <- rep(0,M)
CP.AN.diff.var <- rep(0,M)
length.AN.diff.var <- rep(0,M)
CP.AN.ratio.var <- rep(0,M)
length.AN.ratio.var <- rep(0,M)
CP.variance.Baye <- rep(0,M)
length.variance.Baye <- rep(0,M)
CP.diff.variance.Baye <- rep(0,M)
length.diff.variance.Baye <- rep(0,M)
CP.ratio.variance.Baye <- rep(0,M)
length.ratio.variance.Baye <- rep(0,M)
CP.BAYE2.variance <-  rep(0,M)
length.BAYE2.variance <-  rep(0,M)
CP.diff.variance.Baye2 <- rep(0,M)
length.diff.variance.Baye2 <- rep(0,M)
CP.ratio.variance.Baye2 <- rep(0,M)
length.ratio.variance.Baye2 <- rep(0,M)
#expected.ZiRR <- kmomentRR(1,s,p)
B <- 1000 #number of bootstrap sample
z.value <- qnorm(0.975)
scoda <- rep(0,M)
pcoda <- rep(0,M)
baye.variance <- rep(0,M)
mean_GCI <- rep(0,M)
CP.standard.CI.mu <- rep(0,M)
```

```
length.standard.CI.mu <- rep(0,M)
CP.standard.CI.lambda <- rep(0,M)
length.standard.CI.lambda <- rep(0,M)
CP.standard.CI.delta <- rep(0,M)
length.standard.CI.delta <- rep(0,M)
CP.standard.CI.variance <- rep(0,M)
length.standard.CI.variance <- rep(0,M)
CP.AN.CI.variance <- rep(0,M)
length.AN.CI.variance <- rep(0,M)

###stat iteration
for(i in 1:M){
  deltahat1.boot <- rep(0,B)
  lambda1.boot <- rep(0,B)
  mu1.boot <- rep(0,B)
  deltahat2.boot <- rep(0,B)
  lambda2.boot <- rep(0,B)
  mu2.boot <- rep(0,B)
  x1 <- rZigTRayreigh(n1,lambda = lambda1,mu = mu1,prob0 = p1)
  x10 <- subset(x1,x1==0)
  x11 <- subset(x1,x1>0)
  n11 <- length(x11)
  x001 <- ifelse(x1>0,1,0)
  x.B1 <- bootsamp(x1,B)
  x2 <- rZigTRayreigh(n2,lambda = lambda2,mu = mu2,prob0 = p2)
  x20 <- subset(x2,x2==0)
  x21 <- subset(x2,x2>0)
  n21 <- length(x2)
  x002 <- ifelse(x2>0,1,0)
  x.B2 <- bootsamp(x2,B)
  for(j in 1:B){
    x.B10 <- subset(x.B1[,j],x.B1[,j]==0)
    x.B11 <- subset(x.B1[,j],x.B1[,j]>0)
    n10 <- length(x.B10)
    n11 <- length(x.B11)
    x.B001 <- ifelse(x.B1[,j]>0,1,0)
    deltahat1.boot[j] <- n10/n1
    MLEx.B1 <- MLES(x = x.B11)
```

```
    lambda1.boot[j] <- 1/(2*(MLEx.B1[2]^2))
    mu1.boot[j] <- MLEx.B1[1]
    x.B20 <- subset(x.B2[,j],x.B2[,j]==0)
    x.B21 <- subset(x.B2[,j],x.B2[,j]>0)
    n20 <- length(x.B20)
    n21 <- length(x.B21)
    x.B002 <- ifelse(x.B2[,j]>0,1,0)
    deltahat2.boot[j] <- n20/n2
    MLEx.B2 <- MLES(x = x.B21)
    lambda2.boot[j] <- 1/(2*(MLEx.B2[2]^2))
    mu2.boot[j] <- MLEx.B2[1]
  }
  x.B.moment1 <- momentZITRl(lambda = lambda1.boot,mu = mu1.boot,delta =
deltahat1.boot)
  x.B.variance1 <- x.B.moment1[,2]
  x.B.moment2 <- momentZITRl(lambda = lambda2.boot,mu = mu2.boot,delta =
deltahat2.boot)
  x.B.variance2 <- x.B.moment2[,2]
  x.B.diff.variance <- x.B.variance1-x.B.variance2
  x.B.ratio.variance <- x.B.variance1/x.B.variance2
  #####Percentile Bootstrap
  L.PB.variance1 <- quantile(x.B.variance1,0.025,type = 8)
  U.PB.variance1 <- quantile(x.B.variance1,0.975,type = 8)
  L.PB.variance2 <- quantile(x.B.variance2,0.025,type = 8)
  U.PB.variance2 <- quantile(x.B.variance2,0.975,type = 8)
  L.PB.diff <- quantile(x.B.diff.variance,0.025,type = 8)
  U.PB.diff <- quantile(x.B.diff.variance,0.975,type = 8)
  L.PB.ratio <- quantile(x.B.ratio.variance,0.025,type = 8)
  U.PB.ratio <- quantile(x.B.ratio.variance,0.975,type = 8)
  CP.single.variance.boot[i] <-
ifelse(L.PB.variance1<varianceZITR1&varianceZITR1<U.PB.variance1,1,0)
  length.variance.boot[i]       = U.PB.variance1-L.PB.variance1
  CP.diff.variance.boot[i] <-
ifelse(L.PB.diff<diff.variance&diff.variance<U.PB.diff,1,0)
  length.diff.variance.boot[i]                  = U.PB.diff-L.PB.diff
  CP.ratio.variance.boot[i] <-
ifelse(L.PB.ratio<ratio.variance&ratio.variance<U.PB.ratio,1,0)
  length.ratio.variance.boot[i]                 = U.PB.ratio-L.PB.ratio
  #Bootstrap with SE
  std.B1 <- qnorm(0.975)*sd(x.B.variance1)
```

```
    std.B2 <- qnorm(0.975)*sd(x.B.variance2)
    CI.variance1.bs <- x.B.variance1+c(-1,1)*std.B1
    CI.variance2.bs <- x.B.variance2+c(-1,1)*std.B2
    L.Bs.variance <- CI.variance1.bs[1]
    U.Bs.variance <- CI.variance1.bs[2]
    CP.variance.BS[i] <-
ifelse(L.Bs.variance<varianceZITR1&varianceZITR1<U.Bs.variance,1,0)
    length.variance.BS[i] <- U.Bs.variance-L.Bs.variance
    std.B.diff <- qnorm(0.975)*sd(x.B.diff.variance)
    CI.diff.variance.bs <- x.B.diff.variance+c(-1,1)*std.B.diff
    L.Bs.diff <- CI.diff.variance.bs[1]
    U.Bs.diff <- CI.diff.variance.bs[2]
    CP.diff.BS[i] <- ifelse(L.Bs.diff<diff.variance&diff.variance<U.Bs.diff,1,0)
    length.diff.BS[i] <- U.Bs.diff-L.Bs.diff
    std.B.ratio <- qnorm(0.975)*sd(x.B.ratio.variance)
    CI.ratio.variance.bs <- x.B.ratio.variance+c(-1,1)*std.B.ratio
    L.Bs.ratio <- CI.ratio.variance.bs[1]
    U.Bs.ratio <- CI.ratio.variance.bs[2]
    CP.ratio.BS[i] <- ifelse(L.Bs.ratio<ratio.variance&ratio.variance<U.Bs.ratio,1,0)
    length.ratio.BS[i] <- U.Bs.ratio-L.Bs.ratio

    parameters1 <- MLES(x11)#compute MLE
    lambdahat1 <- 1/(2*(parameters1[2]^2))
    muhat1 <- parameters1[1]
    parameters2 <- MLES(x21)#compute MLE
    lambdahat2 <- 1/(2*(parameters2[2]^2))
    muhat2 <- parameters2[1]
    #deltaprimehat <- optim(0.01,fn = ll.binom,x = x01,method = "BFGS") #compute
MLE delta
    deltahat1 <- 1-(n11/length(x1))
    deltahat2 <-1-(n21/length(x2))
    ## GCI
    es1 <- ComputeGCI(T =2500,n=n1,x1)
    es2 <- ComputeGCI(T=2500,n=n2,x2)
    delta.gpq1 <- es1[,"delta_GPQ"]
    lambda.gpq1 <- es1[,"lambda"]
    mu.gpq1 <- es1[,"mu"]
    delta.gpq2 <- es2[,"delta_GPQ"]
```

```
    lambda.gpq2 <- es2[,"lambda"]
    mu.gpq2 <- es2[,"mu"]
    momentGPQ1 <- momentZITRl(lambda = es1[,2],mu = es1[,1],delta = es1[,3])
    varianceGPQ1 <- momentGPQ1[,2]
    momentGPQ2 <- momentZITRl(lambda = es2[,2],mu = es2[,1],delta = es2[,3])
    varianceGPQ2 <- momentGPQ2[,2]
    diffGPQ <- varianceGPQ1-varianceGPQ2
    rationGPQ <- varianceGPQ1/varianceGPQ2
    L.GCI.variance1 <- quantile(varianceGPQ1,0.025,type = 8)
    U.GCI.variance1 <- quantile(varianceGPQ1,0.975,type = 8)
    L.GCI.variance2 <- quantile(varianceGPQ2,0.025,type = 8)
    U.GCI.variance2 <- quantile(varianceGPQ2,0.975,type = 8)
    CP.variance.GCI[i] <-
ifelse(L.GCI.variance1<varianceZITR1&varianceZITR1<U.GCI.variance1,1,0)
    length.variance.GCI[i] <- U.GCI.variance1-L.GCI.variance1
    L.GCI.diff <- quantile(diffGPQ,0.025,type = 8)
    U.GCI.diff <- quantile(diffGPQ,0.975,type = 8)
    CP.diff.variance.GCI[i] <-
ifelse(L.GCI.diff<diff.variance&diff.variance<U.GCI.diff,1,0)
    length.diff.variance.GCI[i] <- U.GCI.diff-L.GCI.diff
    L.GCI.ratio <- quantile(rationGPQ,0.025,type = 8)
    U.GCI.ratio <- quantile(rationGPQ,0.975,type = 8)
    CP.ratio.variance.GCI[i] <-
ifelse(L.GCI.ratio<ratio.variance&ratio.variance<U.GCI.ratio,1,0)
    length.ratio.variance.GCI[i] <- U.GCI.ratio-L.GCI.ratio
    #MOVER
    L.MOVER.diff.GCI <- mean(diffGPQ)-sqrt(((mean(varianceGPQ1)-
L.GCI.variance1)^2)+((U.GCI.variance2-mean(varianceGPQ2))^2))
    U.MOVER.diff.GCI <- mean(diffGPQ)+sqrt(((U.GCI.variance1 -
mean(varianceGPQ1))^2)+((mean(varianceGPQ2)- L.GCI.variance2)^2))
    CP.dif.GCI.MOVER[i] <-
ifelse(L.MOVER.diff.GCI<diff.variance&diff.variance<U.MOVER.diff.GCI,1,0)
    length.dif.GCI.MOVER[i] <- U.MOVER.diff.GCI-L.MOVER.diff.GCI
    pd.mean <- mean(varianceGPQ1)*mean(varianceGPQ2)
    Lu <- mean(pd.mean)-sqrt((mean(pd.mean)^2)-
(L.GCI.variance1*U.GCI.variance2*((2*mean(varianceGPQ1))-
L.GCI.variance1)*((2*mean(varianceGPQ2))-U.GCI.variance2)))
    Ll <- U.GCI.variance2*((2*mean(varianceGPQ2))-U.GCI.variance2)
```

```
   Uu <- mean(pd.mean)+sqrt((mean(pd.mean)^2)-
(U.GCI.variance1*L.GCI.variance2*((2*mean(varianceGPQ1))-
U.GCI.variance1)*((2*mean(varianceGPQ2))-L.GCI.variance2)))
   Ul <- L.GCI.variance2*((2*mean(varianceGPQ2))-L.GCI.variance2)
   L.CI.mv.ratio                = Lu/Ll
   U.CI.mv.ratio                = Uu/Ul
   CP.ratio.GCI.MOVER[i] <-
ifelse(L.CI.mv.ratio<ratio.variance&ratio.variance<U.CI.mv.ratio,1,0)
   length.ratio.GCI.MOVER[i] <- U.CI.mv.ratio-L.CI.mv.ratio

   #delta method, Approximation normal
   MLEhat1 <- momentZITRl(lambda = lambdahat1,mu = muhat1,delta = deltahat1)
   MLEhat2 <- momentZITRl(lambda = lambdahat2,mu = muhat2,delta = deltahat2)
   varhat1 <- MLEhat1[,2]
   varhat2 <- MLEhat2[,2]
   varmeanhat1 <- (((((2*pi*(1-deltahat1)-
4)/(4*lambdahat1)+((muhat1*sqrt(pi/lambdahat1)*(1-(2*deltahat1)))+(muhat1^2)*(1-
(2*deltahat1))))^2)*(deltahat1*(1-deltahat1))/length(x1))
            +((((sqrt(pi/lambdahat1)*deltahat1*(1-
deltahat1))+2*muhat1*deltahat1*(1-deltahat1))^2)*(1/(2*muhat1+(sum((x11-
muhat1)^(-2)))/length(x11))))
            +((((((1-deltahat1)^2)*pi)-4*(1-
deltahat1))/(4*(lambdahat1^2)))+(0.5*(muhat1/(lambdahat1^1.5))*sqrt(pi)*deltahat1
*(1-deltahat1)))^2*((lambdahat1^2)/length(x11)))
   varmeanhat2 <- (((((2*pi*(1-deltahat2)-
4)/(4*lambdahat2)+((muhat2*sqrt(pi/lambdahat2)*(1-(2*deltahat2)))+(muhat2^2)*(1-
(2*deltahat2))))^2)*(deltahat2*(1-deltahat2))/length(x2))
            +((((sqrt(pi/lambdahat2)*deltahat2*(1-
deltahat2))+2*muhat2*deltahat2*(1-deltahat2))^2)*(1/(2*muhat2+(sum((x21-
muhat2)^(-2)))/length(x21))))
            +((((((1-deltahat2)^2)*pi)-4*(1-
deltahat2))/(4*(lambdahat2^2)))+(0.5*(muhat2/(lambdahat2^1.5))*sqrt(pi)*deltahat2
*(1-deltahat2)))^2*((lambdahat2^2)/length(x21)))
   L.CI.AN <- varhat1-(qnorm(0.975)*sqrt(varmeanhat1))
   U.CI.AN <- varhat1+(qnorm(0.975)*sqrt(varmeanhat1))
   CP.AN.CI.variance[i] <-
ifelse(L.CI.AN<varianceZITR1&varianceZITR1<U.CI.AN,1,0)
   length.AN.CI.variance[i] <- U.CI.AN-L.CI.AN
```

```
   variance.diff.var.hat <- varmeanhat1+varmeanhat2
   variancedelta1 <- (deltahat1*(1-deltahat1))/length(x1)
   variancedelta2 <- (deltahat2*(1-deltahat2))/length(x2)
   variancemu1 <- (1/((2*muhat1)+(1/length(x11))*sum(x11-muhat1)^(-2)))
   variancemu2 <- (1/((2*muhat2)+(1/length(x21))*sum(x21-muhat2)^(-2)))
   variancelambda1 <- (lambdahat1^2)/length(x11)
   variancelambda2 <- (lambdahat2^2)/length(x21)
   variance.ratio.var.hat1.delta <- ((((2*pi*(1-deltahat1))-
4+(4*sqrt(lambdahat1*pi)*muhat1*(1-
(2*deltahat1)))+(4*lambdahat1*(muhat1^2)*(1-
2*deltahat1)))/(4*lambdahat1*varhat2))^2)*variancedelta1
   variance.ratio.var.hat1.mu <- ((((4*sqrt(lambdahat1*pi)*deltahat1*(1-
deltahat1))+(8*muhat1*deltahat1*lambdahat1*(1-
deltahat1)))/(4*lambdahat1*varhat2))^2)*variancemu1
   #variance.ratio.var.hat1.lambda <-
((((4*lambdahat1*varmeanhat2*(((2*muhat1*sqrt(pi)*deltahat1*(1-
deltahat1))/sqrt(lambdahat1))+4*deltahat1*(1-deltahat1)*(muhat1^2)))-
(4*varmeanhat2*(4*(1-deltahat1)-((1-
deltahat1)^2)*pi+4*sqrt(lambdahat1*pi)*muhat1*deltahat1*(1-
deltahat1)+(4*lambdahat1*(muhat1^2)*deltahat1*(1-
deltahat1)))))/((4*lambdahat1*varmeanhat2)^2))^2)*variancelambda1
   variance.ratio.var.hat1.lambda <- (((1-deltahat1)*(4-((1-
deltahat1)*pi)+(2*muhat1*deltahat1*sqrt(pi*lambdahat2))))/(4*(lambdahat1^2)*varh
at2))*variancelambda1
   variance.ratio.var.hat2.delta <- (((-1)*varhat1*((2*pi*(1-deltahat2)-
4+4*sqrt(lambdahat2*pi)*muhat2*(1-
(2*deltahat2)))+(4*lambdahat2*(muhat2)^2)*(1-
2*deltahat2))/(4*lambdahat2*(varhat2^2)))^2)*variancedelta2
   variance.ratio.var.hat2.mu <- ((varhat1*((4*sqrt(lambdahat2*pi)*deltahat2*(1-
deltahat2))+(8*muhat2*deltahat2*lambdahat2*(1-
deltahat2)))/(4*lambdahat2*(varhat2^2)))^2)*variancemu2
   #variance.ratio.var.hat2.lambda <-
(((((4*varhat1)*((((4*muhat2*sqrt(pi)*deltahat2*(1-
deltahat2)))/sqrt(lambdahat2))+4*(muhat2^2)*deltahat2*(1-
deltahat2)))/(lambdahat2*(varhat2^2)))+((4*varhat1)/(lambdahat2*varhat2)))^2)*vari
ancelambda2
   variance.ratio.var.hat2.lambda <-((varhat1/(lambdahat2*varhat2) -
(varhat1*((2*muhat2*sqrt(pi)*deltahat2*(1-deltahat2))/sqrt(lambdahat2) +
4*(muhat2^2)*deltahat2*(1-
deltahat2)))/(4*lambdahat2*(varhat2^2)))^2)*variancelambda2
```

```
   variance.ratio.var.hat <-
variance.ratio.var.hat1.delta+variance.ratio.var.hat1.mu+variance.ratio.var.hat1.lambd
a+variance.ratio.var.hat2.delta+variance.ratio.var.hat2.mu+variance.ratio.var.hat2.lam
bda
   L.CI.diff.AN <- (varhat1-varhat2)-(qnorm(0.975)*sqrt(variance.diff.var.hat))
   U.CI.diff.AN <- (varhat1-varhat2)+(qnorm(0.975)*sqrt(variance.diff.var.hat))
   CP.AN.diff.var[i] <-
ifelse(L.CI.diff.AN<diff.variance&diff.variance<U.CI.diff.AN,1,0)
   length.AN.diff.var[i] <- U.CI.diff.AN-L.CI.diff.AN
   L.CI.ratio.AN <- (varhat1/varhat2)-(qnorm(0.975)*sqrt(variance.ratio.var.hat))
   U.CI.ratio.AN <- (varhat1/varhat2)+(qnorm(0.975)*sqrt(variance.ratio.var.hat))
   CP.AN.ratio.var[i] <-
ifelse(L.CI.ratio.AN<ratio.variance&ratio.variance<U.CI.ratio.AN,1,0)
   length.AN.ratio.var[i] <- U.CI.ratio.AN-L.CI.ratio.AN
   ##############
   #Bayesian
   # n sample
   zerodatan <- list(x = x001,n =length(x001))
   inits1<-function(){ list( p=0) }
   outputpn <- bugs(data = zerodatan, inits = inits1,parameters.to.save =
c("p"),model.file = Binomialmodel1, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
   out.codapn <- read.bugs(outputpn)
   codpn <- as.vector(out.codapn)
   pcodan <- codpn[[1]][,2]
   deltacodan <- 1-pcodan
   # m sample
   zerodatam <- list(x = x002,n =length(x002))
   inits2<-function(){ list( p=0) }
   outputpm <- bugs(data = zerodatam, inits = inits2,parameters.to.save =
c("p"),model.file = Binomialmodel2, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
   out.codapm <- read.bugs(outputpm)
   codpm <- as.vector(out.codapm)
   pcodam <- codpm[[1]][,2]
   deltacodam <- 1-pcodam
   #for tprd
   #for n
```

```
   TZiRRdatan <- list(x = x11,n =length(x11))
   inits3<-function(){ list( u= mu1,l = lambda1) }
   outputsn <- bugs(data = TZiRRdatan, inits = inits3, parameters.to.save =
c("u","l"),model.file = TRRmodel1, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
   out.codasn <- read.bugs(outputsn)
   codsn <- as.vector(out.codasn)
   ucodan <- codsn[[1]][,3]
   lcodan <- codsn[[1]][,2]
   #for m
   TZiRRdatam <- list(x = x21,n =length(x21))
   inits4<-function(){ list( u= mu2,l = lambda2) }
   outputsm <- bugs(data = TZiRRdatam, inits = inits4, parameters.to.save =
c("u","l"),model.file = TRRmodel2, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
   out.codasm <- read.bugs(outputsm)
   codsm <- as.vector(out.codasm)
   ucodam <- codsm[[1]][,3]
   lcodam <- codsm[[1]][,2]
   ###

   baye.var.x1 <- momentZITRl(lambda = lcodan,mu = ucodan,delta = deltacodan)
   baye.var.x2 <- momentZITRl(lambda = lcodam,mu = ucodam,delta = deltacodam)
   baye.diff.var <- baye.var.x1[,2]-baye.var.x2[,2]
   baye.ratio.var <- baye.var.x1[,2]/baye.var.x2[,2]
   L.Baye.var <- quantile(baye.var.x1[,2],0.025,type = 8)
   U.Baye.var <- quantile(baye.var.x1[,2],0.975,type = 8)
   L.Baye.diff <- quantile(baye.diff.var,0.025,type = 8)
   U.Baye.diff <- quantile(baye.diff.var,0.975,type = 8)
   L.Baye.ratio <- quantile(baye.ratio.var,0.025,type = 8)
   U.Baye.ratio <- quantile(baye.ratio.var,0.975,type = 8)
   CP.variance.Baye[i] <-
ifelse(L.Baye.var<varianceZITR1&varianceZITR1<U.Baye.var,1,0)
   length.variance.Baye[i] <- U.Baye.var-L.Baye.var
   CP.diff.variance.Baye[i] <-
ifelse(L.Baye.diff<diff.variance&diff.variance<U.Baye.diff,1,0)
   length.diff.variance.Baye[i] <- U.Baye.diff-L.Baye.diff
   CP.ratio.variance.Baye[i] <-
ifelse(L.Baye.ratio<ratio.variance&ratio.variance<U.Baye.ratio,1,0)
   length.ratio.variance.Baye[i] <- U.Baye.ratio-L.Baye.ratio
```

```
  #HPTD
  HPD.beta.variance                    =hdi(baye.var.x1[,2],0.95)
  L.baye2.variance          =HPD.beta.variance [1]
  U.baye2.variance          =HPD.beta.variance [2]
  CP.BAYE2.variance[i]      =
ifelse(L.baye2.variance<varianceZITR1&varianceZITR1<U.baye2.variance,1,0)
                # compute probability of CI based on BAYE of Baye
  length.BAYE2.variance[i]  = U.baye2.variance- L.baye2.variance        # compute
length of CI based on BAYE of Baye
  #for diff
  HPD.diff.variance                    =hdi(baye.diff.var,0.95)
  L.baye2.diff=HPD.diff.variance[1]
  U.baye2.diff              =HPD.diff.variance[2]
  CP.diff.variance.Baye2[i]  =
ifelse(L.baye2.diff<diff.variance&diff.variance<U.baye2.diff,1,0)          # compute
probability of CI based on BAYE of Baye
  length.diff.variance.Baye2[i]        = U.baye2.diff- L.baye2.diff  # compute
length of CI based on BAYE of Baye
  #for ratio
  HPD.ratio.variance                   =hdi(baye.ratio.var,0.95)
  L.baye2.ratio             =HPD.ratio.variance[1]
  U.baye2.ratio             =HPD.ratio.variance[2]
  CP.ratio.variance.Baye2[i] =
ifelse(L.baye2.ratio<ratio.variance&ratio.variance<U.baye2.ratio,1,0)      # compute
probability of CI based on BAYE of Baye
  length.ratio.variance.Baye2[i]       = U.baye2.ratio- L.baye2.ratio
 }
 ACP.variance.PB <- mean(CP.single.variance.boot)
 AEP.variance.PB<- mean(length.variance.boot)
 ACP.diff.PB <- mean(CP.diff.variance.boot)
 AEP.diff.PB<- mean(length.diff.variance.boot)
 ACP.ratio.PB <- mean(CP.ratio.variance.boot)
 AEP.ratio.PB<- mean(length.ratio.variance.boot)
 ACP.variance.BS <- mean(CP.variance.BS)
 AEP.variance.BS <- mean(length.variance.BS)
 ACP.diff.BS <- mean(CP.diff.BS)
 AEP.diff.BS <- mean(length.diff.BS)
 ACP.ratio.BS <- mean(CP.ratio.BS)
```

```
AEP.ratio.BS <- mean(length.ratio.BS)
ACP.variance.GCI <- mean(CP.variance.GCI)
AEP.variance.GCI <- mean(length.variance.GCI)
ACP.diff.GCI <- mean(CP.diff.variance.GCI)
AEP.diff.GCI <- mean(length.diff.variance.GCI)
ACP.ratio.GCI <- mean(CP.ratio.variance.GCI)
AEP.ratio.GCI <- mean(length.ratio.variance.GCI)
ACP.diff.MOVER <- mean(CP.dif.GCI.MOVER)
AEP.diff.MOVER <- mean(length.dif.GCI.MOVER)
ACP.ratio.MOVER <- mean(CP.ratio.GCI.MOVER,na.rm = TRUE)
AEP.ratio.MOVER <- mean(length.ratio.GCI.MOVER,na.rm = TRUE)

#
ACP.variance.AN <- mean(CP.AN.CI.variance)
AEP.variance.AN <- mean(length.AN.CI.variance)
ACP.diff.AN <- mean(CP.AN.diff.var)
AEP.diff.AN <- mean(length.AN.diff.var)
ACP.ratio.AN <- mean(CP.AN.ratio.var,na.rm = TRUE)
AEP.ratio.AN <- mean(length.AN.ratio.var,na.rm = TRUE)
#
ACP.variance.Baye <- mean(CP.variance.Baye)
AEP.variance.Baye <- mean(length.variance.Baye)
ACP.diff.Baye <- mean(CP.diff.variance.Baye)
AEP.diff.Baye <- mean(length.diff.variance.Baye)
ACP.ratio.Baye <- mean(CP.ratio.variance.Baye)
AEP.ratio.Baye <- mean(length.ratio.variance.Baye)
ACP.variance.Baye2 <- mean(CP.BAYE2.variance)
AEP.variance.Baye2 <- mean(length.BAYE2.variance)
ACP.diff.Baye2 <- mean(CP.diff.variance.Baye2)
AEP.diff.Baye2 <- mean(length.diff.variance.Baye2)
ACP.ratio.Baye2 <- mean(CP.ratio.variance.Baye2)
AEP.ratio.Baye2 <- mean(length.ratio.variance.Baye2)
cat("CP variance Percentile Boot=",ACP.variance.PB,"\n")
cat("Ep variance Percentile Boot=",AEP.variance.PB,"\n")
cat("CP diff Percentile Boot=",ACP.diff.PB,"\n")
cat("Ep diff Percentile Boot=",AEP.diff.PB,"\n")
cat("CP ratio Percentile Boot=",ACP.ratio.PB,"\n")
cat("Ep ratio Percentile Boot=",AEP.ratio.PB,"\n")

cat("CP variance Standarderror Boot=",ACP.variance.BS,"\n")
```

```
 cat("Ep variance Standarderror Boot=",AEP.variance.BS,"\n")
 cat("CP diff Standarderror Boot=",ACP.diff.BS,"\n")
 cat("Ep diff Standarderror Boot=",AEP.diff.BS,"\n")
 cat("CP ratio Standarderror Boot=",ACP.ratio.BS,"\n")
 cat("Ep ratio Standarderror Boot=",AEP.ratio.BS,"\n")
 cat("CP variance FGCI=",ACP.variance.GCI,"\n")
 cat("EP variance FGCI=",AEP.variance.GCI,"\n")
 cat("CP diff GCI=",ACP.diff.GCI,"\n")
 cat("EP diff GCI=",AEP.diff.GCI,"\n")
 cat("CP ratio GCI=",ACP.ratio.GCI,"\n")
 cat("EP ratio GCI=",AEP.ratio.GCI,"\n")
 cat("CP diff MOVER=",ACP.diff.MOVER,"\n")
 cat("EP diff MOVER=",AEP.diff.MOVER,"\n")
 cat("CP ratio MOVER=",ACP.ratio.MOVER,"\n")
 cat("EP ratio MOVER=",AEP.ratio.MOVER,"\n")
 #
 cat("CP variance AN=",ACP.variance.AN,"\n")
 cat("EP variance AN=",AEP.variance.AN,"\n")
 cat("CP diff AN=",ACP.diff.AN,"\n")
 cat("EP diff AN=",AEP.diff.AN,"\n")
 cat("CP ratio AN=",ACP.ratio.AN,"\n")
 cat("EP ratio AN=",AEP.ratio.AN,"\n")
 #
 cat("CP variance BayesianMCMC=",ACP.variance.Baye,"\n")
 cat("Ep variance BayesianMCMC=",AEP.variance.Baye,"\n")
 cat("CP diff BayesianMCMC=",ACP.diff.Baye,"\n")
 cat("Ep diff BayesianMCMC=",AEP.diff.Baye,"\n")
 cat("CP ratio BayesianMCMC=",ACP.ratio.Baye,"\n")
 cat("Ep ratio BayesianMCMC=",AEP.ratio.Baye,"\n")
 cat("CP variance BayesianHPTD=",ACP.variance.Baye2,"\n")
 cat("Ep variance BayesianHPTD=",AEP.variance.Baye2,"\n")
 cat("CP diff BayesianHPTD=",ACP.diff.Baye2,"\n")
 cat("Ep diff BayesianHPTD=",AEP.diff.Baye2,"\n")
 cat("CP ratio BayesianHPTD=",ACP.ratio.Baye2,"\n")
 cat("Ep ratio BayesianHPTD=",AEP.ratio.Baye2,"\n")
 CP.varianceall <-
c(ACP.variance.PB,ACP.variance.BS,ACP.variance.GCI,ACP.variance.AN,ACP.vari
ance.Baye,ACP.variance.Baye2)
```

```
  EL.varianceall <-
c(AEP.variance.PB,AEP.variance.BS,AEP.variance.GCI,AEP.variance.AN,AEP.vari
ance.Baye,AEP.variance.Baye2)
  CP.diffall <-
c(ACP.diff.PB,ACP.diff.BS,ACP.diff.GCI,ACP.diff.MOVER,ACP.diff.AN,ACP.diff.
Baye,ACP.diff.Baye2)
  EL.diffall <-
c(AEP.diff.PB,AEP.diff.BS,AEP.diff.GCI,AEP.diff.MOVER,AEP.diff.AN,AEP.diff.
Baye,AEP.diff.Baye2)
  CP.ratioall <-
c(ACP.ratio.PB,ACP.ratio.BS,ACP.ratio.GCI,ACP.ratio.MOVER,ACP.ratio.AN,ACP
.ratio.Baye,ACP.ratio.Baye2)
  EL.ratioall <-
c(AEP.ratio.PB,AEP.ratio.BS,AEP.ratio.GCI,AEP.ratio.MOVER,AEP.ratio.AN,AEP.
ratio.Baye,AEP.ratio.Baye2)
  print(CP.varianceall)
  print(EL.varianceall)
  print(CP.diffall)
  print(EL.diffall)
  print(CP.ratioall)
  print(EL.ratioall)
}
```

A3 Functions are used to calculate the coverage probability and the expected length of the confidence interval for the coefficients of variation for the zero-inflated two-parameter Rayleigh distribution

The following R code is used to compute the confidence intervals, the coverage probability, and the expected length of the confidence intervals for the coefficient of variation for the zero-inflated two-parameter Rayleigh distribution in Chapter 4.

```
##This code was written by Sasipong Kijsason
library(new.dist)
library(VGAM)
library(MASS)
library(R2OpenBUGS)
library(HDInterval)
rZigTRayreigh <- function(x,lambda,mu,prob0){
 x_0 <- rbinom(x,size = 1,prob = 1-prob0)
 x0 <- subset(x_0,x_0 ==0)
 nzero <- length(x0)
 nNonzero <- x-nzero
 x1 <- rtprd(nNonzero,lambda = lambda,mu = mu)
 x <- c(x0,x1)
 return(x)
}
bootsamp <- function(x,B = 1000){
 keepdata <- vector()#while
 i = 1
 while(i <=B){
  x.B <- sample(x = x,size = length(x),replace = T)
  x.B0test <- subset(x.B,x.B==0)
  x.B1test <- subset(x.B,x.B>0)
  if(length(x.B1test)<2|x.B1test[1]==mean(x.B1test)){
   i <- i
  }else{
   keepdata <- cbind(keepdata,x.B)
   i <- i+1
  }
 }
 return(keepdata)
}
```

```
MLES = function(x){
  n = length(x)
  xb = mean(x); s = sd(x)
  b.hat = sqrt(2/(4-pi))*s  #moment estimate
  fn = function(a){
    ssq = sum((x-a)^2)
    y = 2*n*sum(x-a)/ssq-sum(1/(x-a))
    return(y)
  }
  a0 = min(x)-12*b.hat/sqrt(n); a1 = min(x)
  a.mle = uniroot(fn, interval = c(a0,a1), tol = 10^-5, maxiter = 20)$root
  b.mle = sqrt(.5*sum((x-a.mle)^2)/n)
  return(c(a.mle,b.mle))
}
momentZITRl <- function(lambda,mu,delta){
  over <- function(k,lambda,mu,delta){
    (1-delta)*(1/(lambda^(k/2)))*gamma((k/2)+1)
  }
  Ex <- over(1,lambda,mu,delta)+((1-delta)*mu)
  #Ex2 <- over(2,lambda,mu,delta)+(2*mu*(over(1,lambda,mu,delta)+mu))-((1-
delta)*(mu^2))
  Ex2 <- over(2,lambda,mu,delta)+(2*mu*Ex)-((1-delta)*(mu^2))
  Vx <- Ex2-(Ex^2)
  #cat("mean =",Ex,"\n")
  #cat("Variance =",Vx,"\n")
  output <- cbind(Ex,Vx)
}
EX <- momentZITRl(1/(2*(1.7854820^2)),mu = 0.2384616,delta = 0.24)
ComputeGCI <- function(T,n,x){
  Qmu <- vector()
  Qlambda <- vector()
  Qb <- vector()
  delta_GPQ <- vector()
  mean_GCI <- vector()
  x1 <- x[x>0]
  n1 <- length(x1)
  deltahat <- 1-(length(x1)/length(x))
  parameters <- MLES(x1)
  #parameters <- fitdist(x,distr = "tprd",lower = c(0.01,0.01),start = list(lambda
=0.01,mu =0.01))
```

```
  lambdahat <- parameters[2]
  muhat <- parameters[1]
  #lambdahat <- parameters$estimate[1]
  #muhat <- parameters$estimate[2]
  for(i in 1:T){
    xstar <- rtprd(n1,lambda = 1/2,mu = 0)
    parameterstar <- MLES(xstar)
    #parameterstar <- fitdist(xstar,distr = "tprd",lower = c(0.01,0.001),start =
list(lambda =1,mu =0))
    lambdastar <- parameterstar[2]
    mustar <- parameterstar[1]
    #lambdastar <- parameterstar$estimate[1]
    #mustar <- parameterstar$estimate[2]
    Qmu[i] <- muhat-(lambdahat*(mustar/lambdastar))
    #Qlambda[i] <- lambdahat/lambdastar
    Qb[i] <- lambdahat/lambdastar
    #Z.VST = 2*(sqrt(n))*(asin(sqrt(deltahat))-asin(sqrt(p)))
    Z.VST <- rnorm(1,0,1)
    delta_GPQ[i] = (sin(asin(sqrt(deltahat))-(Z.VST/(2*sqrt(n)))))^2
    #mean_GCI[i] = momntZITRR(lambda = Qlambda[i],mu = Qmu[i],delta =
delta_GPQ[i] ) # compute GPQ for single mean
  }
  Qlambda <- 1/(2*(Qb^2))
  out <- cbind(mu=Qmu,lambda=Qlambda,delta_GPQ=delta_GPQ)
  #print(out)

}
Binomialmodel1 <- function() {
  for (i in 1:n)
  {
    x[i] ~ dbern(p);
  }
  p ~ dbeta(8,2);
}
Binomialmodel2 <- function() {
  for (i in 1:n)
  {
    x[i] ~ dbern(p);
```

```
  }
  p ~ dbeta(8,2);
}
TRRmodel1 <- function(){
  for(i in 1:n){
    one[i] <- 1
    p[i] <- (2*l*(x[i]-u))*exp((-l)*pow(x[i]-u,2))/10000
    one[i] ~ dbern(p[i])
  }
  u ~ dunif(0,1)
  l ~ dgamma(1,2);
}
TRRmodel2 <- function(){
  for(i in 1:n){
    one[i] <- 1
    p[i] <- (2*l*(x[i]-u))*exp((-l)*pow(x[i]-u,2))/10000
    one[i] ~ dbern(p[i])
  }
  u ~ dunif(0,1)
  l ~ dgamma(1,2);
}
CI.CVs.ZiTR <- function(M,n1,lambda1,mu1,p1,n2,lambda2,mu2,p2){
  #M = loop for montecarlo
  #N = number of sample size
  #s = scale parameter of x
  #p = weigh parameter for ZiRR distribution
  momentZITR1 <- momentZITRl(lambda1,mu1,delta = p1)
  momentZITR2 <- momentZITRl(lambda2,mu2,delta = p2)
  meanZITR1 <- momentZITR1[,1]
  varianceZITR1 <- momentZITR1[,2]
  meanZITR2 <- momentZITR2[,1]
  varianceZITR2 <- momentZITR2[,2]
  CVZITR1 <- sqrt(varianceZITR1)/meanZITR1
  CVZITR2 <- sqrt(varianceZITR2)/meanZITR2
  diff.CV <- CVZITR1-CVZITR2
  ratio.CV <- CVZITR1/CVZITR2
  CP.single.CV.boot <- rep(0,M)
  length.CV.boot          <-  rep(0,M)
  CP.diff.CV.boot <- rep(0,M)
  length.diff.CV.boot     <-  rep(0,M)
```

```
CP.ratio.CV.boot <- rep(0,M)
length.ratio.CV.boot <-  rep(0,M)
CP.CV.BS <- rep(0,M)
length.CV.BS <- rep(0,M)
CP.diff.BS <- rep(0,M)
length.diff.BS <- rep(0,M)
CP.ratio.BS <- rep(0,M)
length.ratio.BS <- rep(0,M)
CP.CV.GCI <- rep(0,M)
length.CV.GCI <- rep(0,M)
CP.diff.CV.GCI <- rep(0,M)
length.diff.CV.GCI <- rep(0,M)
CP.ratio.CV.GCI <- rep(0,M)
length.ratio.CV.GCI <- rep(0,M)
CP.dif.GCI.MOVER <- rep(0,M)
length.dif.GCI.MOVER <- rep(0,M)
CP.ratio.GCI.MOVER <- rep(0,M)
length.ratio.GCI.MOVER <- rep(0,M)
CP.AN.CI.CV <- rep(0,M)
length.AN.CI.CV <- rep(0,M)
CP.AN.diff.CV <- rep(0,M)
length.AN.diff.CV <- rep(0,M)
CP.AN.ratio.CV <- rep(0,M)
length.AN.ratio.CV <- rep(0,M)
CP.CV.Baye <- rep(0,M)
length.CV.Baye <- rep(0,M)
CP.diff.CV.Baye <- rep(0,M)
length.diff.CV.Baye <- rep(0,M)
CP.ratio.CV.Baye <- rep(0,M)
length.ratio.CV.Baye <- rep(0,M)
CP.BAYE2.CV <-  rep(0,M)
length.BAYE2.CV <-  rep(0,M)
CP.diff.CV.Baye2 <- rep(0,M)
length.diff.CV.Baye2 <- rep(0,M)
CP.ratio.CV.Baye2 <- rep(0,M)
length.ratio.CV.Baye2 <- rep(0,M)
#expected.ZiRR <- kmomentRR(1,s,p)
B <- 1000 #number of bootstrap sample
```

```
z.value <- qnorm(0.975)
scoda <- rep(0,M)
pcoda <- rep(0,M)
baye.variance <- rep(0,M)
mean_GCI <- rep(0,M)
CP.standard.CI.CV <- rep(0,M)
length.standard.CI.CV <- rep(0,M)
CP.AN.CI.CV <- rep(0,M)
length.AN.CI.CV <- rep(0,M)

###stat iteration
for(i in 1:M){
  deltahat1.boot <- rep(0,B)
  lambda1.boot <- rep(0,B)
  mu1.boot <- rep(0,B)
  deltahat2.boot <- rep(0,B)
  lambda2.boot <- rep(0,B)
  mu2.boot <- rep(0,B)
  x1 <- rZigTRayreigh(n1,lambda = lambda1,mu = mu1,prob0 = p1)
  x10 <- subset(x1,x1==0)
  x11 <- subset(x1,x1>0)
  n11 <- length(x11)
  x001 <- ifelse(x1>0,1,0)
  x.B1 <- bootsamp(x1,B)
  x2 <- rZigTRayreigh(n2,lambda = lambda2,mu = mu2,prob0 = p2)
  x20 <- subset(x2,x2==0)
  x21 <- subset(x2,x2>0)
  n21 <- length(x2)
  x002 <- ifelse(x2>0,1,0)
  x.B2 <- bootsamp(x2,B)
  for(j in 1:B){
    x.B10 <- subset(x.B1[,j],x.B1[,j]==0)
    x.B11 <- subset(x.B1[,j],x.B1[,j]>0)
    n10 <- length(x.B10)
    n11 <- length(x.B11)
    x.B001 <- ifelse(x.B1[,j]>0,1,0)
    deltahat1.boot[j] <- n10/n1
    MLEx.B1 <- MLES(x = x.B11)
    lambda1.boot[j] <- 1/(2*(MLEx.B1[2]^2))
    mu1.boot[j] <- MLEx.B1[1]
```

```
    x.B20 <- subset(x.B2[,j],x.B2[,j]==0)
    x.B21 <- subset(x.B2[,j],x.B2[,j]>0)
    n20 <- length(x.B20)
    n21 <- length(x.B21)
    x.B002 <- ifelse(x.B2[,j]>0,1,0)
    deltahat2.boot[j] <- n20/n2
    MLEx.B2 <- MLES(x = x.B21)
    lambda2.boot[j] <- 1/(2*(MLEx.B2[2]^2))
    mu2.boot[j] <- MLEx.B2[1]
  }
  x.B.moment1 <- momentZITRl(lambda = lambda1.boot,mu = mu1.boot,delta =
deltahat1.boot)
  x.B.mean1 <- x.B.moment1[,1]
  x.B.variance1 <- x.B.moment1[,2]
  x.B.CV1 <- sqrt(x.B.variance1)/x.B.mean1
  x.B.moment2 <- momentZITRl(lambda = lambda2.boot,mu = mu2.boot,delta =
deltahat2.boot)
  x.B.mean2 <- x.B.moment2[,1]
  x.B.variance2 <- x.B.moment2[,2]
  x.B.CV2 <- sqrt(x.B.variance2)/x.B.mean2
  x.B.diff.CV <- x.B.CV1-x.B.CV2
  x.B.ratio.CV <- x.B.CV1/x.B.CV2
  #####Percentile Bootstrap
  L.PB.CV1 <- quantile(x.B.CV1,0.025,type = 8)
  U.PB.CV1 <- quantile(x.B.CV1,0.975,type = 8)
  L.PB.CV2 <- quantile(x.B.CV2,0.025,type = 8)
  U.PB.CV2 <- quantile(x.B.CV2,0.975,type = 8)
  L.PB.diff <- quantile(x.B.diff.CV,0.025,type = 8)
  U.PB.diff <- quantile(x.B.diff.CV,0.975,type = 8)
  L.PB.ratio <- quantile(x.B.ratio.CV,0.025,type = 8)
  U.PB.ratio <- quantile(x.B.ratio.CV,0.975,type = 8)
  CP.single.CV.boot[i] <- ifelse(L.PB.CV1<CVZITR1&CVZITR1<U.PB.CV1,1,0)
  length.CV.boot[i]          = U.PB.CV1-L.PB.CV1
  CP.diff.CV.boot[i] <- ifelse(L.PB.diff<diff.CV&diff.CV<U.PB.diff,1,0)
  length.diff.CV.boot[i]     = U.PB.diff-L.PB.diff
  CP.ratio.CV.boot[i] <- ifelse(L.PB.ratio<ratio.CV&ratio.CV<U.PB.ratio,1,0)
  length.ratio.CV.boot[i]    = U.PB.ratio-L.PB.ratio
  #Bootstrap with SE
```

```
  std.B1 <- qnorm(0.975)*sd(x.B.CV1)
  std.B2 <- qnorm(0.975)*sd(x.B.CV2)
  CI.CV1.bs <- x.B.CV1+c(-1,1)*std.B1
  CI.CV2.bs <- x.B.CV2+c(-1,1)*std.B2
  L.Bs.CV <- CI.CV1.bs[1]
  U.Bs.CV <- CI.CV1.bs[2]
  CP.CV.BS[i] <- ifelse(L.Bs.CV<CVZITR1&CVZITR1<U.Bs.CV,1,0)
  length.CV.BS[i] <- U.Bs.CV-L.Bs.CV
  std.B.diff <- qnorm(0.975)*sd(x.B.diff.CV)
  CI.diff.CV.bs <- x.B.diff.CV+c(-1,1)*std.B.diff
  L.Bs.diff <- CI.diff.CV.bs[1]
  U.Bs.diff <- CI.diff.CV.bs[2]
  CP.diff.BS[i] <- ifelse(L.Bs.diff<diff.CV&diff.CV<U.Bs.diff,1,0)
  length.diff.BS[i] <- U.Bs.diff-L.Bs.diff
  std.B.ratio <- qnorm(0.975)*sd(x.B.ratio.CV)
  CI.ratio.CV.bs <- x.B.ratio.CV+c(-1,1)*std.B.ratio
  L.Bs.ratio <- CI.ratio.CV.bs[1]
  U.Bs.ratio <- CI.ratio.CV.bs[2]
  CP.ratio.BS[i] <- ifelse(L.Bs.ratio<ratio.CV&ratio.CV<U.Bs.ratio,1,0)
  length.ratio.BS[i] <- U.Bs.ratio-L.Bs.ratio

  parameters1 <- MLES(x11)#compute MLE
  lambdahat1 <- 1/(2*(parameters1[2]^2))
  muhat1 <- parameters1[1]
  parameters2 <- MLES(x21)#compute MLE
  lambdahat2 <- 1/(2*(parameters2[2]^2))
  muhat2 <- parameters2[1]
  #deltaprimehat <- optim(0.01,fn = ll.binom,x = x01,method = "BFGS") #compute
MLE delta
  deltahat1 <- 1-(n11/length(x1))
  deltahat2 <-1-(n21/length(x2))
  ## GCI
  es1 <- ComputeGCI(T =2500,n=n1,x1)
  es2 <- ComputeGCI(T=2500,n=n2,x2)
  delta.gpq1 <- es1[,"delta_GPQ"]
  lambda.gpq1 <- es1[,"lambda"]
  mu.gpq1 <- es1[,"mu"]
  delta.gpq2 <- es2[,"delta_GPQ"]
  lambda.gpq2 <- es2[,"lambda"]
  mu.gpq2 <- es2[,"mu"]
```

```
    #L.FGCI.delta <- quantile(delta.gpq,0.025,type = 8)
    #U.FGCI.delta <- quantile(delta.gpq,0.975,type = 8)
    #L.FGCI.lambda <- quantile(lambda.gpq,0.025,type = 8)
    #U.FGCI.lambda <- quantile(lambda.gpq,0.975,type = 8)
    #L.FGCI.mu <- quantile(mu.gpq,0.025,type = 8)
    #U.FGCI.mu <- quantile(mu.gpq,0.975,type = 8)
    #CP.delta.FGCI[i] <- ifelse(L.FGCI.delta<p&p<U.FGCI.delta,1,0)
    #length.delta.FGCI[i] <-  U.FGCI.delta-L.FGCI.delta
    #CP.lambda.FGCI[i] <-
ifelse(L.FGCI.lambda<lambda&lambda<U.FGCI.lambda,1,0)
    #length.lambda.FGCI[i] <- U.FGCI.lambda-L.FGCI.lambda
    #CP.mu.FGCI[i] <- ifelse(L.FGCI.mu<mu&mu<U.FGCI.mu,1,0)
    #length.mu.FGCI[i] <- U.FGCI.mu-L.FGCI.mu
    momentGPQ1 <- momentZITRl(lambda = es1[,2],mu = es1[,1],delta = es1[,3])
    varianceGPQ1 <- momentGPQ1[,2]
    meanGPQ1 <- momentGPQ1[,1]
    CVGPQ1 <- sqrt(varianceGPQ1)/meanGPQ1
    momentGPQ2 <- momentZITRl(lambda = es2[,2],mu = es2[,1],delta = es2[,3])
    varianceGPQ2 <- momentGPQ2[,2]
    meanGPQ2 <- momentGPQ2[,1]
    CVGPQ2 <- sqrt(varianceGPQ2)/meanGPQ2
    diffGPQ <- CVGPQ1-CVGPQ2
    rationGPQ <- CVGPQ1/CVGPQ2
    L.GCI.CV1 <- quantile(CVGPQ1,0.025,type = 8)
    U.GCI.CV1 <- quantile(CVGPQ1,0.975,type = 8)
    L.GCI.CV2 <- quantile(CVGPQ2,0.025,type = 8)
    U.GCI.CV2 <- quantile(CVGPQ2,0.975,type = 8)
    CP.CV.GCI[i] <- ifelse(L.GCI.CV1<CVZITR1&CVZITR1<U.GCI.CV1,1,0)
    length.CV.GCI[i] <- U.GCI.CV1-L.GCI.CV1
    L.GCI.diff <- quantile(diffGPQ,0.025,type = 8)
    U.GCI.diff <- quantile(diffGPQ,0.975,type = 8)
    CP.diff.CV.GCI[i] <- ifelse(L.GCI.diff<diff.CV&diff.CV<U.GCI.diff,1,0)
    length.diff.CV.GCI[i] <- U.GCI.diff-L.GCI.diff
    L.GCI.ratio <- quantile(rationGPQ,0.025,type = 8)
    U.GCI.ratio <- quantile(rationGPQ,0.975,type = 8)
    CP.ratio.CV.GCI[i] <- ifelse(L.GCI.ratio<ratio.CV&ratio.CV<U.GCI.ratio,1,0)
    length.ratio.CV.GCI[i] <- U.GCI.ratio-L.GCI.ratio
    #MOVER
```

```
   L.MOVER.diff.GCI <- mean(diffGPQ)-sqrt(((mean(CVGPQ1)-
L.GCI.CV1)^2)+((U.GCI.CV2-mean(CVGPQ2))^2))
   U.MOVER.diff.GCI <- mean(diffGPQ)+sqrt(((U.GCI.CV1 -
mean(CVGPQ1))^2)+((mean(CVGPQ2)- L.GCI.CV2)^2))
   CP.dif.GCI.MOVER[i] <-
ifelse(L.MOVER.diff.GCI<diff.CV&diff.CV<U.MOVER.diff.GCI,1,0)
   length.dif.GCI.MOVER[i] <- U.MOVER.diff.GCI-L.MOVER.diff.GCI
   pd.mean <- mean(CVGPQ1)*mean(CVGPQ2)
   Lu <- mean(pd.mean)-sqrt((mean(pd.mean)^2)-
(L.GCI.CV1*U.GCI.CV2*((2*mean(CVGPQ1))-L.GCI.CV1)*((2*mean(CVGPQ2))-
U.GCI.CV2)))
   Ll <- U.GCI.CV2*((2*mean(CVGPQ2))-U.GCI.CV2)
   Uu <- mean(pd.mean)+sqrt((mean(pd.mean)^2)-
(U.GCI.CV1*L.GCI.CV2*((2*mean(CVGPQ1))-
U.GCI.CV1)*((2*mean(CVGPQ2))-L.GCI.CV2)))
   Ul <- L.GCI.CV2*((2*mean(CVGPQ2))-L.GCI.CV2)
   L.CI.mv.ratio              = Lu/Ll
   U.CI.mv.ratio              = Uu/Ul
   CP.ratio.GCI.MOVER[i] <-
ifelse(L.CI.mv.ratio<ratio.CV&ratio.CV<U.CI.mv.ratio,1,0)
   length.ratio.GCI.MOVER[i] <- U.CI.mv.ratio-L.CI.mv.ratio

   #delta method, Approximation normal
   MLEhat1 <- momentZITRl(lambda = lambdahat1,mu = muhat1,delta = deltahat1)
   MLEhat2 <- momentZITRl(lambda = lambdahat2,mu = muhat2,delta = deltahat2)
   varhat1 <- MLEhat1[,2]
   varhat2 <- MLEhat2[,2]
   meanhat1 <- MLEhat1[,1]
   meanhat2 <- MLEhat2[,1]
   CVhat1 <- sqrt(varhat1)/meanhat1
   CVhat2 <- sqrt(varhat2)/meanhat2
   varmeanhat1 <- ((muhat1+gamma(3/2)/sqrt(lambdahat1))^2*deltahat1*(1 -
deltahat1)/length(x1))+
    ((1-deltahat1)^2)*(1/((2*muhat1) + (1/length(x11))*sum((x11- muhat1)^(-2))))+
    ((1-deltahat1)^2)*((-1/2*lambdahat1^(-
3/2)*gamma(3/2))^2)*(lambdahat1^2)/length(x11)
   varmeanhat2 <- ((muhat2+gamma(3/2)/sqrt(lambdahat2))^2*deltahat2*(1 -
deltahat2)/length(x1))+
    ((1-deltahat2)^2)*(1/((2*muhat2) + (1/length(x11))*sum((x11- muhat2)^(-2))))+
```

```
    ((1-deltahat2)^2)*((-1/2*lambdahat2^(-
3/2)*gamma(3/2))^2)*(lambdahat2^2)/length(x11)
  varvariancehat1 <- (((((2*pi*(1-deltahat1)-
4)/(4*lambdahat1)+((muhat1*sqrt(pi/lambdahat1)*(1-(2*deltahat1)))+(muhat1^2)*(1-
(2*deltahat1))))^2)*(deltahat1*(1-deltahat1))/length(x1))
            +((((sqrt(pi/lambdahat1)*deltahat1*(1-
deltahat1))+2*muhat1*deltahat1*(1-deltahat1))^2)*(1/(2*muhat1+(sum((x11-
muhat1)^(-2)))/length(x11))))
            +((((((1-deltahat1)^2)*pi)-4*(1-
deltahat1))/(4*(lambdahat1^2)))+(0.5*(muhat1/(lambdahat1^1.5))*sqrt(pi)*deltahat1
*(1-deltahat1)))^2*((lambdahat1^2)/length(x11)))
  varvariancehat2 <- (((((2*pi*(1-deltahat2)-
4)/(4*lambdahat2)+((muhat2*sqrt(pi/lambdahat2)*(1-(2*deltahat2)))+(muhat2^2)*(1-
(2*deltahat2))))^2)*(deltahat2*(1-deltahat2))/length(x2))
            +((((sqrt(pi/lambdahat2)*deltahat2*(1-
deltahat2))+2*muhat2*deltahat2*(1-deltahat2))^2)*(1/(2*muhat2+(sum((x21-
muhat2)^(-2)))/length(x21))))
            +((((((1-deltahat2)^2)*pi)-4*(1-
deltahat2))/(4*(lambdahat2^2)))+(0.5*(muhat2/(lambdahat2^1.5))*sqrt(pi)*deltahat2
*(1-deltahat2)))^2*((lambdahat2^2)/length(x21)))
  divCVdelta1 <-  (((meanhat1/(2*sqrt(varhat1)))*(2*pi*(1-deltahat1)-
4)/(4*lambdahat1)+((muhat1*sqrt(pi/lambdahat1)*(1-(2*deltahat1)))+(muhat1^2)*(1-
(2*deltahat1))))-(sqrt(varhat1))*(-
1)*(muhat1+(gamma(3/2)/sqrt(lambdahat1))))/(meanhat1^2)
  divCVmu1 <- (((meanhat1/(2*sqrt(varhat1)))*(sqrt(pi/lambdahat1)*deltahat1*(1-
deltahat1))+2*muhat1*deltahat1*(1-deltahat1))-(sqrt(varhat1)*(1-
deltahat1)))/(meanhat1^2)
  divCVlambda1 <- (((meanhat1/(2*sqrt(varhat1)))*((((((1-deltahat1)^2)*pi)-4*(1-
deltahat1))/(4*(lambdahat1^2)))+(0.5*(muhat1/(lambdahat1^1.5))*sqrt(pi)*deltahat1
*(1-deltahat1))))-(sqrt(varhat1)*(1-deltahat1)*((-1/2*lambdahat1^(-
3/2)*gamma(3/2)))))/(meanhat1^2)
  varianceCVhat1 <- (divCVdelta1^2)*((deltahat1*(1-
deltahat1))/length(x1))+((divCVmu1^2)*(1/((2*muhat1) + (1/length(x11))*sum((x11-
muhat1)^(-2)))))+((divCVlambda1^2)*(lambdahat1^2)/length(x11))
  divCVdelta2 <-  (((meanhat2/(2*sqrt(varhat2)))*(2*pi*(1-deltahat2)-
4)/(4*lambdahat2)+((muhat2*sqrt(pi/lambdahat2)*(1-(2*deltahat2)))+(muhat2^2)*(1-
(2*deltahat2))))-(sqrt(varhat2))*(-
1)*(muhat2+(gamma(3/2)/sqrt(lambdahat2))))/(meanhat2^2)
```

```
   divCVmu2 <- (((meanhat2/(2*sqrt(varhat2)))*(sqrt(pi/lambdahat2)*deltahat2*(1-
deltahat2))+2*muhat2*deltahat2*(1-deltahat2))-(sqrt(varhat2)*(1-
deltahat2)))/(meanhat2^2)
   divCVlambda2 <- (((meanhat2/(2*sqrt(varhat2)))*((((((1-deltahat2)^2)*pi)-4*(1-
deltahat2))/(4*(lambdahat2^2)))+(0.5*(muhat2/(lambdahat2^1.5))*sqrt(pi)*deltahat2
*(1-deltahat2))))-(sqrt(varhat1)*(1-deltahat2)*((-1/2*lambdahat2^(-
3/2)*gamma(3/2)))))/(meanhat2^2)
   varianceCVhat2 <- (divCVdelta2^2)*((deltahat2*(1-
deltahat2))/length(x2))+((divCVmu2^2)*(1/((2*muhat2) + (1/length(x21))*sum((x21-
muhat2)^(-2)))))+((divCVlambda2^2)*(lambdahat2^2)/length(x21))
   L.CI.AN <- CVhat1-(qnorm(0.975)*sqrt(varianceCVhat1))
   U.CI.AN <- CVhat1+(qnorm(0.975)*sqrt(varianceCVhat1))
   CP.AN.CI.CV[i] <- ifelse(L.CI.AN<CVZITR1&CVZITR1<U.CI.AN,1,0)
   length.AN.CI.CV[i] <- U.CI.AN-L.CI.AN
   variance.diff.CV.hat <- varianceCVhat1+varianceCVhat2
   variancedelta1 <- (deltahat1*(1-deltahat1))/length(x1)
   variancedelta2 <- (deltahat2*(1-deltahat2))/length(x2)
   variancemu1 <- (1/((2*muhat1)+(1/length(x11))*sum(x11-muhat1)^(-2)))
   variancemu2 <- (1/((2*muhat2)+(1/length(x21))*sum(x21-muhat2)^(-2)))
   variancelambda1 <- (lambdahat1^2)/length(x11)
   variancelambda2 <- (lambdahat2^2)/length(x21)
   variance.ratio.CV.hat1.delta1 <-
(((((CVhat2*meanhat1*(1/(2*sqrt(varhat1))))*(((2*pi*(1-deltahat1)-
4)/(4*lambdahat1)+((muhat1*sqrt(pi/lambdahat1)*(1-(2*deltahat1)))+(muhat1^2)*(1-
(2*deltahat1))))^2))-(varhat1*(-
1)*(muhat1+(gamma(3/2)/sqrt(lambdahat1)))))/(meanhat1^2))/(CVhat2^2))
   variance.ratio.CV.hat1.mu1 <-
(((CVhat2*meanhat1*(1/(2*sqrt(varhat1))))*(((sqrt(pi)*deltahat1*(1-
deltahat1))/sqrt(lambdahat1))+2*muhat1*deltahat1*(1-deltahat1))-(sqrt(varhat1)*(1-
deltahat1)))/(meanhat1^2))/(CVhat2^2)
   variance.ratio.CV.hat1.lambda1 <-
(((CVhat2*(((meanhat1/(2*sqrt(varhat1)))*((((((1-deltahat1)^2)*pi)-4*(1-
deltahat1))/(4*(lambdahat1^2)))+(0.5*(muhat1/(lambdahat1^1.5))*sqrt(pi)*deltahat1
*(1-deltahat1))))-(sqrt(varhat1)*(1-deltahat1)*((-1/2*lambdahat1^(-
3/2)*gamma(3/2)))))/(meanhat1^2)))/(CVhat2^2))
   variance.ratio.CV.hat2.delta2 <-
(((CVhat1*(meanhat2/(2*sqrt(varhat2)))*(2*pi*(1-deltahat2)-
4)/(4*lambdahat2)+((muhat2*sqrt(pi/lambdahat2)*(1-(2*deltahat2)))+(muhat2^2)*(1-
(2*deltahat2))))-(sqrt(varhat2))*(-
1)*(muhat2+(gamma(3/2)/sqrt(lambdahat2))))/(meanhat2^2))/(CVhat2^2)
```

```
   variance.ratio.CV.hat2.mu2 <-
(((CVhat1*meanhat2*(1/(2*sqrt(varhat2))))*(((sqrt(pi)*deltahat2*(1-
deltahat2))/sqrt(lambdahat2))+2*muhat2*deltahat2*(1-deltahat2))-(sqrt(varhat2)*(1-
deltahat2)))/(meanhat2^2))/(CVhat2^2)
   variance.ratio.CV.hat2.lambda2 <-
(((CVhat1*(((meanhat2/(2*sqrt(varhat2)))*((((((1-deltahat2)^2)*pi)-4*(1-
deltahat2))/(4*(lambdahat2^2)))+(0.5*(muhat2/(lambdahat2^1.5))*sqrt(pi)*deltahat2
*(1-deltahat2))))-(sqrt(varhat2)*(1-deltahat2)*((-1/2*lambdahat2^(-
3/2)*gamma(3/2))))))/(meanhat2^2)))/(CVhat2^2))
   variance.ratio.CV.hat <-
((variance.ratio.CV.hat1.delta1^2)*variancedelta1)+((variance.ratio.CV.hat1.mu1^2)*
variancemu1)+((variance.ratio.CV.hat1.lambda1^2)*variancelambda1)+((variance.rat
io.CV.hat2.delta2^2)*variancedelta2)+((variance.ratio.CV.hat2.mu2^2)*variancemu2
)+((variance.ratio.CV.hat2.lambda2^2)*variancelambda2)
   #variance.ratio.var.hat1.delta <- ((((2*pi*(1-deltahat1))-
4+(4*sqrt(lambdahat1*pi)*muhat1*(1-
(2*deltahat1)))+(4*lambdahat1*(muhat1^2)*(1-
2*deltahat1)))/(4*lambdahat1*varhat2))^2)*variancedelta1
   #variance.ratio.var.hat1.mu <- ((((4*sqrt(lambdahat1*pi)*deltahat1*(1-
deltahat1))+(8*muhat1*deltahat1*lambdahat1*(1-
deltahat1)))/(4*lambdahat1*varhat2))^2)*variancemu1
   ##variance.ratio.var.hat1.lambda <-
((((4*lambdahat1*varmeanhat2*(((2*muhat1*sqrt(pi)*deltahat1*(1-
deltahat1))/sqrt(lambdahat1))+4*deltahat1*(1-deltahat1)*(muhat1^2)))-
(4*varmeanhat2*(4*(1-deltahat1)-((1-
deltahat1)^2)*pi+4*sqrt(lambdahat1*pi)*muhat1*deltahat1*(1-
deltahat1)+(4*lambdahat1*(muhat1^2)*deltahat1*(1-
deltahat1)))))/((4*lambdahat1*varmeanhat2)^2))^2)*variancelambda1
   #variance.ratio.var.hat1.lambda <- (((1-deltahat1)*(4-((1-
deltahat1)*pi)+(2*muhat1*deltahat1*sqrt(pi*lambdahat2))))/(4*(lambdahat1^2)*varh
at2))*variancelambda1
   #variance.ratio.var.hat2.delta <- (((-1)*varhat1*((2*pi*(1-deltahat2)-
4+4*sqrt(lambdahat2*pi)*muhat2*(1-
(2*deltahat2)))+(4*lambdahat2*(muhat2)^2)*(1-
2*deltahat2))/(4*lambdahat2*(varhat2^2)))^2)*variancedelta2
   #variance.ratio.var.hat2.mu <- ((varhat1*((4*sqrt(lambdahat2*pi)*deltahat2*(1-
deltahat2))+(8*muhat2*deltahat2*lambdahat2*(1-
deltahat2)))/(4*lambdahat2*(varhat2^2)))^2)*variancemu2
```

```
   ##variance.ratio.var.hat2.lambda <-
(((((4*varhat1)*((((4*muhat2*sqrt(pi)*deltahat2*(1-
deltahat2)))/sqrt(lambdahat2))+4*(muhat2^2)*deltahat2*(1-
deltahat2)))/(lambdahat2*(varhat2^2)))+((4*varhat1)/(lambdahat2*varhat2)))^2)*vari
ancelambda2
   #variance.ratio.var.hat2.lambda <-((varhat1/(lambdahat2*varhat2) -
(varhat1*((2*muhat2*sqrt(pi)*deltahat2*(1-deltahat2))/sqrt(lambdahat2) +
4*(muhat2^2)*deltahat2*(1-
deltahat2)))/(4*lambdahat2*(varhat2^2)))^2)*variancelambda2
   #variance.ratio.var.hat <-
variance.ratio.var.hat1.delta+variance.ratio.var.hat1.mu+variance.ratio.var.hat1.lambd
a+variance.ratio.var.hat2.delta+variance.ratio.var.hat2.mu+variance.ratio.var.hat2.lam
bda
   L.CI.diff.AN <- (CVhat1-CVhat2)-(qnorm(0.975)*sqrt(variance.diff.CV.hat))
   U.CI.diff.AN <- (CVhat1-CVhat2)+(qnorm(0.975)*sqrt(variance.diff.CV.hat))
   CP.AN.diff.CV[i] <- ifelse(L.CI.diff.AN<diff.CV&diff.CV<U.CI.diff.AN,1,0)
   length.AN.diff.CV[i] <- U.CI.diff.AN-L.CI.diff.AN
   L.CI.ratio.AN <- (CVhat1/CVhat2)-(qnorm(0.975)*sqrt(variance.ratio.CV.hat))
   U.CI.ratio.AN <- (CVhat1/CVhat2)+(qnorm(0.975)*sqrt(variance.ratio.CV.hat))
   CP.AN.ratio.CV[i] <- ifelse(L.CI.ratio.AN<ratio.CV&ratio.CV<U.CI.ratio.AN,1,0)
   length.AN.ratio.CV[i] <- U.CI.ratio.AN-L.CI.ratio.AN
   ##############
   #Bayesian
   # n sample
   zerodatan <- list(x = x001,n =length(x001))
   inits1<-function(){ list( p=0) }
   outputpn <- bugs(data = zerodatan, inits = inits1,parameters.to.save =
c("p"),model.file = Binomialmodel1, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
   out.codapn <- read.bugs(outputpn)
   codpn <- as.vector(out.codapn)
   pcodan <- codpn[[1]][,2]
   deltacodan <- 1-pcodan
   # m sample
   zerodatam <- list(x = x002,n =length(x002))
   inits2<-function(){ list( p=0) }
   outputpm <- bugs(data = zerodatam, inits = inits2,parameters.to.save =
c("p"),model.file = Binomialmodel2, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
   out.codapm <- read.bugs(outputpm)
```

```
    codpm <- as.vector(out.codapm)
    pcodam <- codpm[[1]][,2]
    deltacodam <- 1-pcodam
    #for tprd
    #for n
    TZiRRdatan <- list(x = x11,n =length(x11))
    inits3<-function(){ list( u= mu1,l = lambda1) }
    outputsn <- bugs(data = TZiRRdatan, inits = inits3, parameters.to.save =
c("u","l"),model.file = TRRmodel1, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
    out.codasn <- read.bugs(outputsn)
    codsn <- as.vector(out.codasn)
    ucodan <- codsn[[1]][,3]
    lcodan <- codsn[[1]][,2]
    #for m
    TZiRRdatam <- list(x = x21,n =length(x21))
    inits4<-function(){ list( u= mu2,l = lambda2) }
    outputsm <- bugs(data = TZiRRdatam, inits = inits4, parameters.to.save =
c("u","l"),model.file = TRRmodel2, n.chains = 1,n.iter = 20000,n.burnin =
5000,debug=FALSE,codaPkg=TRUE)
    out.codasm <- read.bugs(outputsm)
    codsm <- as.vector(out.codasm)
    ucodam <- codsm[[1]][,3]
    lcodam <- codsm[[1]][,2]
    ###

    baye.x1 <- momentZITRl(lambda = lcodan,mu = ucodan,delta = deltacodan)
    baye.x2 <- momentZITRl(lambda = lcodam,mu = ucodam,delta = deltacodam)
    baye.CV.x1 <- sqrt(baye.x1[,2])/baye.x1[,1]
    baye.CV.x2 <- sqrt(baye.x2[,2])/baye.x2[,1]
    baye.diff.CV <- baye.CV.x1-baye.CV.x2
    baye.ratio.CV <- baye.CV.x1/baye.CV.x2
    L.Baye.CV <- quantile(baye.CV.x1,0.025,type = 8)
    U.Baye.CV <- quantile(baye.CV.x1,0.975,type = 8)
    L.Baye.diff <- quantile(baye.diff.CV,0.025,type = 8)
    U.Baye.diff <- quantile(baye.diff.CV,0.975,type = 8)
    L.Baye.ratio <- quantile(baye.ratio.CV,0.025,type = 8)
    U.Baye.ratio <- quantile(baye.ratio.CV,0.975,type = 8)
```

```
   CP.CV.Baye[i] <- ifelse(L.Baye.CV<CVZITR1&CVZITR1<U.Baye.CV,1,0)
   length.CV.Baye[i] <- U.Baye.CV-L.Baye.CV
   CP.diff.CV.Baye[i] <- ifelse(L.Baye.diff<diff.CV&diff.CV<U.Baye.diff,1,0)
   length.diff.CV.Baye[i] <- U.Baye.diff-L.Baye.diff
   CP.ratio.CV.Baye[i] <- ifelse(L.Baye.ratio<ratio.CV&ratio.CV<U.Baye.ratio,1,0)
   length.ratio.CV.Baye[i] <- U.Baye.ratio-L.Baye.ratio
   #HPTD
   HPD.beta.CV                    =hdi(baye.CV.x1,0.95)
   L.baye2.CV=HPD.beta.CV [1]
   U.baye2.CV=HPD.beta.CV [2]
   CP.BAYE2.CV[i]          =
ifelse(L.baye2.CV<CVZITR1&CVZITR1<U.baye2.CV,1,0)          # compute
probability of CI based on BAYE of Baye
   length.BAYE2.CV[i]      = U.baye2.CV- L.baye2.CV   # compute length of CI
based on BAYE of Baye
   #for diff
   HPD.diff.CV                    =hdi(baye.diff.CV,0.95)
   L.baye2.diff=HPD.diff.CV[1]
   U.baye2.diff            =HPD.diff.CV[2]
   CP.diff.CV.Baye2[i]     =
ifelse(L.baye2.diff<diff.CV&diff.CV<U.baye2.diff,1,0)      # compute probability of
CI based on BAYE of Baye
   length.diff.CV.Baye2[i]    = U.baye2.diff- L.baye2.diff  # compute length of CI
based on BAYE of Baye
   #for ratio
   HPD.ratio.CV                   =hdi(baye.ratio.CV,0.95)
   L.baye2.ratio           =HPD.ratio.CV[1]
   U.baye2.ratio           =HPD.ratio.CV[2]
   CP.ratio.CV.Baye2[i]    =
ifelse(L.baye2.ratio<ratio.CV&ratio.CV<U.baye2.ratio,1,0)          # compute
probability of CI based on BAYE of Baye
   length.ratio.CV.Baye2[i]   = U.baye2.ratio- L.baye2.ratio
  }
  ACP.CV.PB <- mean(CP.single.CV.boot)
  AEP.CV.PB<- mean(length.CV.boot)
  ACP.diff.PB <- mean(CP.diff.CV.boot)
  AEP.diff.PB<- mean(length.diff.CV.boot)
  ACP.ratio.PB <- mean(CP.ratio.CV.boot)
  AEP.ratio.PB<- mean(length.ratio.CV.boot)
  ACP.CV.BS <- mean(CP.CV.BS)
```

```
AEP.CV.BS <- mean(length.CV.BS)
ACP.diff.BS <- mean(CP.diff.BS)
AEP.diff.BS <- mean(length.diff.BS)
ACP.ratio.BS <- mean(CP.ratio.BS)
AEP.ratio.BS <- mean(length.ratio.BS)
ACP.CV.GCI <- mean(CP.CV.GCI)
AEP.CV.GCI <- mean(length.CV.GCI)
ACP.diff.GCI <- mean(CP.diff.CV.GCI)
AEP.diff.GCI <- mean(length.diff.CV.GCI)
ACP.ratio.GCI <- mean(CP.ratio.CV.GCI)
AEP.ratio.GCI <- mean(length.ratio.CV.GCI)
ACP.diff.MOVER <- mean(CP.dif.GCI.MOVER)
AEP.diff.MOVER <- mean(length.dif.GCI.MOVER)
ACP.ratio.MOVER <- mean(CP.ratio.GCI.MOVER,na.rm = TRUE)
AEP.ratio.MOVER <- mean(length.ratio.GCI.MOVER,na.rm = TRUE)

#
ACP.CV.AN <- mean(CP.AN.CI.CV,na.rm = TRUE)
AEP.CV.AN <- mean(length.AN.CI.CV,na.rm = TRUE)
ACP.diff.AN <- mean(CP.AN.diff.CV,na.rm = TRUE)
AEP.diff.AN <- mean(length.AN.diff.CV,na.rm = TRUE)
ACP.ratio.AN <- mean(CP.AN.ratio.CV,na.rm = TRUE)
AEP.ratio.AN <- mean(length.AN.ratio.CV,na.rm = TRUE)
#
ACP.CV.Baye <- mean(CP.CV.Baye)
AEP.CV.Baye <- mean(length.CV.Baye)
ACP.diff.Baye <- mean(CP.diff.CV.Baye)
AEP.diff.Baye <- mean(length.diff.CV.Baye)
ACP.ratio.Baye <- mean(CP.ratio.CV.Baye)
AEP.ratio.Baye <- mean(length.ratio.CV.Baye)
ACP.CV.Baye2 <- mean(CP.BAYE2.CV)
AEP.CV.Baye2 <- mean(length.BAYE2.CV)
ACP.diff.Baye2 <- mean(CP.diff.CV.Baye2)
AEP.diff.Baye2 <- mean(length.diff.CV.Baye2)
ACP.ratio.Baye2 <- mean(CP.ratio.CV.Baye2)
AEP.ratio.Baye2 <- mean(length.ratio.CV.Baye2)
cat("CP CV Percentile Boot=",ACP.CV.PB,"\n")
cat("Ep CV Percentile Boot=",AEP.CV.PB,"\n")
```

```
cat("CP diff Percentile Boot=",ACP.diff.PB,"\n")
cat("Ep diff Percentile Boot=",AEP.diff.PB,"\n")
cat("CP ratio Percentile Boot=",ACP.ratio.PB,"\n")
cat("Ep ratio Percentile Boot=",AEP.ratio.PB,"\n")

cat("CP CV Standarderror Boot=",ACP.CV.BS,"\n")
cat("Ep CV Standarderror Boot=",AEP.CV.BS,"\n")
cat("CP diff Standarderror Boot=",ACP.diff.BS,"\n")
cat("Ep diff Standarderror Boot=",AEP.diff.BS,"\n")
cat("CP ratio Standarderror Boot=",ACP.ratio.BS,"\n")
cat("Ep ratio Standarderror Boot=",AEP.ratio.BS,"\n")
cat("CP CV GCI=",ACP.CV.GCI,"\n")
cat("EP CV GCI=",AEP.CV.GCI,"\n")
cat("CP diff GCI=",ACP.diff.GCI,"\n")
cat("EP diff GCI=",AEP.diff.GCI,"\n")
cat("CP ratio GCI=",ACP.ratio.GCI,"\n")
cat("EP ratio GCI=",AEP.ratio.GCI,"\n")
cat("CP diff MOVER=",ACP.diff.MOVER,"\n")
cat("EP diff MOVER=",AEP.diff.MOVER,"\n")
cat("CP ratio MOVER=",ACP.ratio.MOVER,"\n")
cat("EP ratio MOVER=",AEP.ratio.MOVER,"\n")
#
cat("CP CV AN=",ACP.CV.AN,"\n")
cat("EP CV AN=",AEP.CV.AN,"\n")
cat("CP diff AN=",ACP.diff.AN,"\n")
cat("EP diff AN=",AEP.diff.AN,"\n")
cat("CP ratio AN=",ACP.ratio.AN,"\n")
cat("EP ratio AN=",AEP.ratio.AN,"\n")
#
cat("CP CV BayesianMCMC=",ACP.CV.Baye,"\n")
cat("Ep CV BayesianMCMC=",AEP.CV.Baye,"\n")
cat("CP diff BayesianMCMC=",ACP.diff.Baye,"\n")
cat("Ep diff BayesianMCMC=",AEP.diff.Baye,"\n")
cat("CP ratio BayesianMCMC=",ACP.ratio.Baye,"\n")
cat("Ep ratio BayesianMCMC=",AEP.ratio.Baye,"\n")
cat("CP CV BayesianHPTD=",ACP.CV.Baye2,"\n")
cat("Ep CV BayesianHPTD=",AEP.CV.Baye2,"\n")
cat("CP diff BayesianHPTD=",ACP.diff.Baye2,"\n")
cat("Ep diff BayesianHPTD=",AEP.diff.Baye2,"\n")
cat("CP ratio BayesianHPTD=",ACP.ratio.Baye2,"\n")
```

```
  cat("Ep ratio BayesianHPTD=",AEP.ratio.Baye2,"\n")
  CP.CVall <-
c(ACP.CV.PB,ACP.CV.BS,ACP.CV.GCI,ACP.CV.AN,ACP.CV.Baye,ACP.CV.Bay
e2)
  EL.CVall <-
c(AEP.CV.PB,AEP.CV.BS,AEP.CV.GCI,AEP.CV.AN,AEP.CV.Baye,AEP.CV.Baye
2)
  CP.diffall <-
c(ACP.diff.PB,ACP.diff.BS,ACP.diff.GCI,ACP.diff.MOVER,ACP.diff.AN,ACP.diff.
Baye,ACP.diff.Baye2)
  EL.diffall <-
c(AEP.diff.PB,AEP.diff.BS,AEP.diff.GCI,AEP.diff.MOVER,AEP.diff.AN,AEP.diff.
Baye,AEP.diff.Baye2)
  CP.ratioall <-
c(ACP.ratio.PB,ACP.ratio.BS,ACP.ratio.GCI,ACP.ratio.MOVER,ACP.ratio.AN,ACP
.ratio.Baye,ACP.ratio.Baye2)
  EL.ratioall <-
c(AEP.ratio.PB,AEP.ratio.BS,AEP.ratio.GCI,AEP.ratio.MOVER,AEP.ratio.AN,AEP.
ratio.Baye,AEP.ratio.Baye2)
  print(CP.CVall)
  print(EL.CVall)
  print(CP.diffall)
  print(EL.diffall)
  print(CP.ratioall)
  print(EL.ratioall)}
CI.CVs.ZiTR(M = 10,n1 = 100,lambda1 = 0.2,mu1 = 0.2,p1 = 0.1,n2 =50,lambda2 =
0.2,mu2 = 0.2,p2 = 0.1)
```

APPENDIX (B)

Publications

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Article

Confidence Intervals for the Parameter Mean of Zero-Inflated Two-Parameter Rayleigh Distribution

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Abstract

The Rayleigh distribution is a continuous probability distribution that is inherently asymmetric and commonly used to model right-skewed data. It holds significant importance across a wide range of scientific and engineering disciplines and exhibits structural relationships with several other asymmetric probability distributions, for example, Weibull and exponential distribution. This research proposes techniques for establishing credible intervals and confidence intervals for the single mean of the zero-inflated two-parameter Rayleigh distribution. The study introduces methods such as the percentile bootstrap, generalized confidence interval, standard confidence interval, approximate normal using the delta method, Bayesian credible interval, and Bayesian highest posterior density. The effectiveness of the proposed methods is assessed by evaluating coverage probability and expected length through Monte Carlo simulations. The results indicate that the Bayesian highest posterior density method outperforms the other approaches. Finally, the study applies the proposed methods to construct confidence intervals for the single mean using real-world data on COVID-19 total deaths in Singapore during October 2022.

Keywords: Bayesian Markov Chain Monte Carlo; Bayesian highest posterior density; the generalized confidence interval; bootstrap method; delta method

check for updates

Academic Editors: Pedro José Fernández de Córdoba Castellá, Juan Carlos Castro-Palacio, Miguel Enrique Iglesias Martínez and Shufei Wu

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1. Introduction

Characterized by its asymmetry, the Rayleigh distribution is a continuous probability model particularly effective for describing data with a pronounced right skew. In various fields, it is common to analyze data comprising values greater than zero, such as mortality rates, the number of deaths caused by diseases, or fatalities resulting from accidents. Such data are characteristically asymmetric due to the presence of a long right tail, indicating that although extreme values may be observed, they occur infrequently. The Rayleigh distribution was first introduced by Rayleigh [1]. It is extensively applied in various fields, including medical research, quality control, and reliability engineering. For example, Arikan et al. [2] explored the power of wind energy, optimal turbine selection, and associated costs in Turkey using the Rayleigh distribution. Similarly, Almongy et al. [3] analyzed real mortality rate data from COVID-19 in Italy, Mexico, and the Netherlands across different time periods. Kilai et al. [4] used the Rayleigh distribution to study mortality rates from COVID-19 data in Italy and Canada. Kilany et al. [5] applied the power Rayleigh distribution to describe hydrologic data and the total COVID-19 deaths in Egypt. Finally, Seal et al. [6] analyzed the strength of carbon fibers, measured in Giga-Pascals, and applied the findings to predict the lifetime of T8 fluorescent lamps.

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Article

Confidence Intervals for the Difference and Ratio Means of Zero-Inflated Two-Parameter Rayleigh Distribution

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Abstract

The analysis of road traffic accidents often reveals asymmetric patterns, providing insights that support the development of preventive measures, reduce fatalities, and improve road safety interventions. The Rayleigh distribution, a continuous distribution with inherent asymmetry, is well suited for modeling right-skewed data and is widely used in scientific and engineering fields. It also shares structural characteristics with other skewed distributions, such as the Weibull and exponential distributions, and is particularly effective for analyzing right-skewed accident data. This study considers several approaches for constructing confidence intervals, including the percentile bootstrap, bootstrap with standard error, generalized confidence interval, method of variance estimates recovery, normal approximation, Bayesian Markov Chain Monte Carlo, and Bayesian highest posterior density methods. Their performance was evaluated through Monte Carlo simulation based on coverage probabilities and expected lengths. The results show that the HPD method achieved coverage probabilities at or above the nominal confidence level while providing the shortest expected lengths. Finally, all proposed confidence intervals were applied to fatalities recorded during the seven hazardous days of Thailand's Songkran festival in 2024 and 2025.

Keywords: bootstrap; generalized confidence interval; method of variance estimates recovery; delta method; Bayesian credible interval; zero-inflated two-parameter Rayleigh distribution

Check for updates

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1. Introduction

In practical studies involving mortality rates and road traffic accidents, the observed datasets typically consist of non-negative values and often contain a substantial proportion of zeros. Such data characteristics commonly lead to right-skewed and asymmetric distributions, driven by an excess of zero counts and the low probability of severe events. This pattern is consistent with empirical evidence reported in accident data analyses, where a high frequency of zero observations and occasional extreme values are frequently observed due to the rarity of severe crashes [1]. For instance, road-traffic fatality records may include numerous zero observations, representing days or locations with no recorded deaths, alongside occasional extreme values arising from severe accidents. This pronounced asymmetry presents challenges for statistical inference, as zero-inflated and highly skewed data structures are frequently encountered in real-world applications and can substantially influence analytical results and policy interpretations. These challenges have motivated the development and use of specialized statistical models, such as zero-inflated distributions,

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Confidence Intervals for Parameter Variance of Zero-Inflated Two-Parameter Rayleigh Distribution

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Abstract

This study develops confidence and credible intervals for the variance of the zero-inflated two-parameter Rayleigh distribution, a flexible model for non-negative data with excess zeros. Seven approaches are proposed: Bayesian Markov chain Monte Carlo (MCMC), Bayesian highest posterior density (HPD), the standard confidence interval, the approximation normal, the percentile bootstrap, the bootstrap method with standard error, and the generalized confidence interval (GCI). Their performance is assessed through Monte Carlo simulation using coverage probability (CP) and expected length (EL). The results show that the Bayesian HPD interval performs best overall, attaining coverage close to the nominal level while yielding shorter intervals than the alternatives, especially for small samples. The methods are illustrated with road traffic fatality data from Chiang Mai Province, Thailand, recorded in March 2024. These findings support the practical usefulness of the HPD approach for variance interval estimation in zero-inflated continuous models.

Keywords: Bayesian; highest posterior density; resampling; delta method; pivotal quantities

Check for updates

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1. Introduction

Road traffic fatalities constitute a major public health concern worldwide, particularly in regions with high rates of traffic accidents. Reports from international health organizations indicate that road accidents remain one of the leading causes of death, imposing substantial social and economic burdens [1]. Analyses of traffic fatality data should not be limited to the average number of deaths; variability must also be considered to better understand changes in mortality patterns across time and location. The variance of accident-related fatalities provides important insight into the dispersion and instability of death counts, which cannot be fully captured by the mean alone [2]. Therefore, statistical methods that account for both variability and the underlying distributional characteristics of the data are essential.

Variance is not merely a descriptive measure but a key quantity for statistical inference and practical interpretation in studies of road traffic fatalities. In the presence of excess zeros, overall variability arises from two distinct sources: structural zeros and the distribution of positive outcomes [3,4]. As a result, the variance reflects the combined effects of the mixing proportion and the underlying continuous distribution, giving it a more complex structure than the mean. In many mixture and zero-inflated models, this structure depends nonlinearly on the model parameters, which complicates its sampling behavior and reduces the reliability of normal approximation methods, particularly for small sample sizes [5]. From a transportation safety perspective, large variation in fatality counts may

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Interval Estimation for the Difference and Ratio of Variances Under the Zero-Inflated Two-Parameter Rayleigh Distribution

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Abstract

The zero-inflated two-parameter Rayleigh (ZITR) distribution provides a flexible framework for modeling data with excess zeros and positive observations following a two-parameter Rayleigh distribution. It is particularly suitable for right-skewed data and has applications in areas such as road traffic mortality and survival analysis. This study develops and compares several methods for constructing confidence intervals for the difference and ratio of variances from two independent ZITR populations. The considered methods include Bayesian approaches based on Markov Chain Monte Carlo (MCMC) and highest posterior density (HPD) intervals, as well as the generalized confidence interval (GCI), method of variance estimates recovery (MOVER), approximate normal (AN), percentile bootstrap (PB), and bootstrap with standard error (BS). The performance of these methods is evaluated via Monte Carlo simulations under various parameter settings and sample sizes, using coverage probability and expected interval length as performance criteria. The results indicate that the Bayesian HPD method generally performs well across a wide range of scenarios. A real-data application using road traffic mortality data from January 2025 in Chanthaburi and Narathiwat provinces is also presented, demonstrating the practical usefulness of the proposed approaches for comparing the variance structure between the two regions.

Keywords: Bayesian credible interval; Gibbs sampling; pivotal quantities; delta method; resampling

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1. Introduction

Probability distributions form a fundamental basis of statistical analysis, as they describe the behavior of data or random variables and summarize key characteristics such as dispersion, skewness, and kurtosis through a set of unknown parameters [1]. The choice of an appropriate distribution depends largely on the nature of the data under study. For continuous right-skewed data, commonly used distributions include the Rayleigh, Weibull, exponential, Gamma, and lognormal distributions [2,3].

The coexistence of excess zeros and occasional extreme values poses challenges for statistical analysis, particularly when high estimation accuracy is required. Applying standard continuous distributions without accounting for the large proportion of zeros may lead to biased inference and inflated variability. A fundamental framework for handling such

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Bayesian Estimation of the Difference and Ratio of the Coefficients of Variation Under the Zero-Inflated Two-Parameter Rayleigh Distribution

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Abstract

The coefficient of variation (CV) is a widely used unit-free measure for comparing variability across two or more populations, even when the data are measured on different scales. In this study, we investigate confidence and credible interval estimation for the difference and ratio of CVs from the zero-inflated two-parameter Rayleigh (ZITR) distribution, which accommodates excess zeros and positively skewed observations through a two-component mixture structure. Several interval estimation methods are proposed, including the Bayesian Credible Interval (BCI), Bayesian Highest Posterior Density (HPD), and Approximate Normal (AN) approaches. Their performances are compared with established methods, namely the Generalized Confidence Interval (GCI), Method of Variance Estimates Recovery (MOVER), Percentile Bootstrap (PB), and Bootstrap with Standard Error (BS). A Monte Carlo simulation study is conducted to evaluate the methods in terms of coverage probability (CP), Monte Carlo standard error (MCSE) of the estimated coverage probability, and expected length (EL). The simulation results indicate that the HPD method provides the best overall performance, achieving coverage probabilities close to the nominal level with small MCSE values, indicating stable simulation estimates, while maintaining relatively short interval lengths. The proposed confidence interval methods were further applied to district-level road traffic fatality rates (per 100,000 population) in Uttaradit and Mae Hong Son provinces, Thailand. The road traffic fatality data included fatalities occurring in March 2026. The fatality rates were calculated based on the district-level population size to enable comparisons across districts with different population scales.

Keywords: Bayesian method; gibbs sampling; highest posterior density; prior distribution; delta method; pivotal quantities; resampling method

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1. Introduction

Parameter estimation is a fundamental task in statistical inference as it allows researchers to make inferences about the characteristics of a population from random samples, when the underlying parameters are not known [1]. Estimation methods are used to summarize and infer important characteristics of a distribution, such as measures of central tendency and variability, from observed data. Therefore, selecting an appropriate probability distribution and determining suitable estimation methods are essential tasks, especially in fields where high accuracy is required, such as insurance, medicine, engineering, and risk analysis.

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Article

Bayesian Estimation for the Single Coefficient of Variation of Zero-Inflated Two-Parameter Rayleigh Distribution

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Abstract

Real data such as road traffic mortality rates and lifetime observations are often zero-inflated and right-skewed. The zero-inflated two-parameter Rayleigh (ZITR) distribution is employed to model such data in this study. The coefficient of variation (CV) is a statistical measure that is used to quantify the relative dispersion of a population, by comparing the standard deviation with the mean. It is widely used to evaluate variability and facilitate comparisons among datasets with different scales or measurement units. This study develops and evaluates seven methods for constructing confidence intervals for the single CV of the ZITR distribution. Three proposed approaches, including Bayesian Markov chain Monte Carlo (MCMC), Bayesian highest posterior density (HPD), and approximate normal (AN) methods, are compared with three existing approaches: generalized confidence interval (GCI), percentile bootstrap (PB), and bootstrap with standard error (BS). Monte Carlo simulations are employed to assess the efficacy of these methods in terms of expected length (EL) and coverage probability (CP). The simulation results show that the HPD method gives acceptable CP with shorter interval lengths than other methods. Moreover, the proposed methods are illustrated with road traffic mortality rates per 100,000 population collected in January 2026 from the Phichit, Suphan Buri, and Prachuap Khiri Khan provinces in Thailand. The results indicate the applicability of the proposed methods for analyzing zero-inflated and right-skewed data in this real-data example.

Keywords: resampling; pivotal quantities; generalized confidence interval; prior distribution; highest posterior density; delta method; bayesian credible interval; gibbs sampling

MSC: 62F25

Check for updates

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1. Introduction

Estimation of population parameters is an important part of statistical inference. A point estimate provides a single summary result. However, it does not account for uncertainty due to the sampling procedure. Interval estimation solves this problem by giving a confidence interval (CI) that includes a range of possible values for the parameter of interest. These intervals explicitly account for variability arising from the sampling process, thereby offering a more comprehensive framework for statistical inference. Therefore, it is important to construct suitable confidence intervals to make reliable statistical inferences [1].

In many practical situations, a large number of zero observations may cause serious problems in statistical analysis, especially when accurate estimation is required and zero outcomes are a natural consequence of the data-generating process. Continuous

Statistical Computing for Variance Zero-inflated Two-parameter Rayleigh Distribution

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Abstract. Parameter variance is used to measure the dispersion of data or the deviation of individual data points from the mean. A high variance indicates that the data are widely spread around the mean, reflecting greater variability within the dataset. This study introduces four novel methods for constructing confidence intervals for the variance of the Zero-Inflated Two-Parameter Rayleigh distribution. These include the percentile bootstrap, the bootstrap method with standard error, the standard method based on a large sample, and generalized confidence interval approaches. A simulation-based comparison was conducted using coverage probability and expected length as performance criteria. The findings indicate that the generalized confidence interval and standard method achieved coverage probabilities closest to the nominal confidence level. Among these methods, the generalized confidence interval demonstrated the highest efficiency. Additionally, the proposed methods were applied to real-world data on COVID-19 mortality rates in Malaysia during September 2021.

Keywords: Pivotal quantities, Resampling, Rayleigh distribution, Variance

1 Introducetion

COVID-19 is a highly infectious and severe disease that was first reported in December 2019, originating in China. The pandemic has had a profound impact on both the global economy and public health systems. Due to its high transmission rate, data on COVID-19 infections and deaths have been collected from various countries. The World Health Organization (WHO) has compiled data on infection rates and mortality; however, these statistics may not fully reflect the true severity of the disease. This is because many infected individuals may be asymptomatic and, therefore, go undetected, leading to a potential underestimation of COVID-19-related deaths. Nevertheless, the WHO [1] has proposed using mortality rates as a means of assessing the severity of the disease. Analyzing mortality rates based on infection and death statistics from different countries during the pandemic is essential for understanding the overall impact of the disease. In this context, the variance in mortality rates can be used to

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Confidence interval for the difference between variances of the zero-inflated two-parameter Rayleigh distribution

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Abstract. The zero-inflated two-parameter Rayleigh (ZITR) distribution is a continuous, right-skewed distribution with a probability mass at zero, making it suitable for modeling data that include both zero and positive continuous values. It is applied in various fields, such as reliability engineering, quality control, COVID-19 mortality, and traffic accident fatalities. Constructing confidence intervals for the difference between variances of ZITR distributions is important for comparing two distinct populations and supporting resource allocation and traffic safety planning. This study develops confidence intervals for variance differences using four methods: Generalized Confidence Interval (GCI), the method of variance estimates recovery (MOVER), Percentile bootstrap (PB), and Bootstrap with standard error (BS). Monte Carlo simulations were conducted to evaluate the coverage probability and expected length of the intervals. Additionally, real data on traffic accident fatalities in January 2024 from Chachoengsao and Nakhon Si Thammarat provinces were analyzed to illustrate the practical application of these methods. The results provide guidance for selecting appropriate interval estimation techniques for variance differences in zero-inflated, skewed data.

1. Introduction

Zero-inflated models were introduced by Lambert [1] to handle data with excess zeros. Building on this approach, Fuxiang et al. [2] introduced a zero-inflated Rayleigh model to analyze right-skewed data characterized by a large proportion of zeros. The Rayleigh distribution [3,4] is widely used to model non-negative variables and can be extended with a location parameter. Its statistical properties and inferential behavior have been examined in detail [5], and recent studies have focused on parameter estimation under various methodological frameworks [6]. Applications of the Rayleigh distribution span several fields, including COVID-19 mortality, engineering, and energy research [7–11].

Variance reflects the dispersion of data and is fundamental to comparing variability across populations [12]. Differences in variance reveal relative heterogeneity between groups beyond mean-based comparisons. Existing studies have developed confidence interval approaches for variance differences in both general two-sample settings [13] and specific distributional models [14].

This study estimates confidence intervals for the difference between variances under the zero-inflated two-parameter Rayleigh distribution using the generalized confidence interval, method of variance estimates recovery (MOVER), percentile bootstrap, and bootstrap standard error approaches. The methods are applied to 2024 accident-related mortality data from Chachoengsao and Nakhon Si Thammarat provinces, selected for their distinct demographic and transportation profiles and the high prevalence of zero observations, which makes the data suitable for zero-inflated modelling.

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